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From Aspecto-Temporal Analysis of a Natural Language to Logical Modeling Using Combinatory Logic

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Abstract

Thecombinatory logic of Haskell Curry is a relatively adequate formalism for linguistics and more generally, for behavioral science, cognitive science, computer science and philosophy. To machine-readable treatment of a natural language, this paper presents the logical formalism of operators using the combinatory logic and the applications of their representations. The combinatory logic — which, along with Alonzo Church's lambda calculus and functional programming languages is a part of the family of applicative languages — is provided with a basic applicative operation, where the first argument is an operator, the second an operand, and the result is an applicative expression that can function as an operator or operand. This study assumes that the basic units of natural language are operators and operands, respectively. To the treatment of tenses and aspects in the linguistic domain, especially in the aspecto-temporal theory and a topological approach, the application of an operator to an operand is necessary. This article treats the logical formalization of elementary aspecto-temporal operators, which is based on the applicative operation. Additionally, it demonstrates in using natural deduction how an utterance can be formalized logically as an applicative expression. Using these methodologies (combinatory logic and natural deduction), this study aims to show how natural language calculates and computerizes expressions involving semantic domains including tense and aspect beyond the morpho-syntactic side.

Keywords—Aspecto-temporal theory, Curry combinatory logic, GRammar of ApplicativeCognitive and Enunciative Operations, Natural deduction

1. Introduction

If the human being has the capacity to perform intuitionally aspecto-temporal analyses of natural languages, machines must be guided by explicit rules to obtain similar results. All natural languages express tenses and aspects (grammatical aspect in linguistics), however linguistically they are often analyzed separately. In practice, the separative analysis of tense and aspect is very different from the use of language by an actual enunciator[13].As can be seen in English, French and Korean where expressions such as ‘past(tense) perfect (aspect)’ and ‘present (tense) progressive (aspect)’ forms are used(in[13], [14]), it is a fact that the enunciator expresses the tense and the grammatical aspect at the same time when enunciating. Therefore, tense and aspect are not dealt with separately, and the perspective that analyzes both linguistically at the same time is called the aspecto-temporal theory. This study was chosen to deal with the tense and aspect of linguistics at the same time, and to convert it into a formula that can be accurately understood by machines. This paper proposes a formal aspecto-temporal analysis based on theoretical computational model ‘Applicative, Cognitive and

EnunciativeGRammar (ACEGR, in French GRACE, in [9], [11], [12])' to perform combinatorial calculations from an applicative expression. As the main method of analysis, combinatory logic of Curry (in [3], [4]), an applicative system, was selected. It is based on the combination of operators and operands, and can be applied to linguistics (in [17], [18]) by considering language units as operators and operands, respectively. In addition, the combinatory logic is similar to a programming language, given that a computer program is a complex operator composed of multiple elementary programs.Indeed, the concept of compilers developed by John Backus starts from the applicative system; therefore, functional programming languages are based on combinatory logic. This study focuses on the application of combinatory logic to a natural language for, ultimately,establish a generalized theoretical method on the application of (elementary and derivatives) combinators to aspecto-temporal linguistic units. To make a program in an applicative language, operators are always combined with operands to produce a result, and the result becomes an operator again and requires a composition that is combined with other operands.This work, with the result of formal application using combinatory logic and natural deduction, shows that natural languages can be also transformed into a machine-understandable and expression.

2. Linguistic and Logical Methodology

2.1 Linguistic Computational Model

How ensures that computers process natural languages? Existing literature shows that automatic language processing uses two general methods (in[7], [9]).The first qualifies as a traditional method and entails in providing the machine with linguistic analysis of a domain (grammar, logical description of a phenomenon, and so on) with a set of rules for analysis and enunciative productions. The second, which has predominated in recent years, uses a large body of data (documents annotated manually by linguists based on a morphological, syntactic, grammatical, semantic analyses) as inputs, from which the machine extracts the processing rules using statistical models of a machine learning paradigm. The symbolic and statistical approaches of the two respective methods have divided the natural language processing community into two camps.

The theoretical framework of this study is based on the linguistic-computational model, which is the'applicative, cognitive and enunciative grammar' (in French, 'GRammaire Applicative, Cognitive et Enunciative-GRACE'), particularly on the aspecto-temporal theory level (in [19]). The central concern of this model is the applicative operation, that is, when an operator applies to its operand. In particular, it takes charge of the semantic and detailed analysis of certain grammatical categories as well as the taking into account of the meaning attached to lexical units of natural languages.In this model, the passage from one level to the other is affected by an operational process, analogous to a computer process of "compilation" between programming languages, by means of "intermediate representations".In this compilation process, a natural language is then assimilated to "a high-level language", the latter then being represented in a "formal language" whose operations have become fully explicit instructions and executable by a computer machine.The linguistic-computational architecture of the model GRACE (Fig. 1) is accordingly based on linguistic analysis and aims at a computer realization. Seven levels of representations which are explicitly articulated by processes of change of representation.

Level 1) "Superficial morpho-syntactic configurations (syntagmatic structure)level" of diversity of languages, where the particular characteristics of a natural language are described (for example, word order, morphological cases, and so on).

Level 2) "Logico-grammatical representations level" expressed by typed applicative expressions in the formalism of Categorical Grammars (by a syntactic analysis of sentences). At this level, an application representation is built where operators apply to operands.

Level 3)"Level of analysis of grammatical operators" to treat voices and thematizations, using combinators of Combinatory Logic.

Level 4) "Analysis and representations of enunciative conditions level" to describe grammatical categories of tense, aspect, and modalities.

Level 5) "Level of formal representations of the meanings of lexical predicates" by Semantico-Cognitive Schemes.

Level 6) "Integration of enunciative conditions level" (integration of level 4 and 5) with Semantico-Cognitive Schemes.

Level 7) "Level of cognitive representations" which establish relationships between, on the one hand, cognitive language activity and on the other hand, intentional cognitive activities of perception and action.

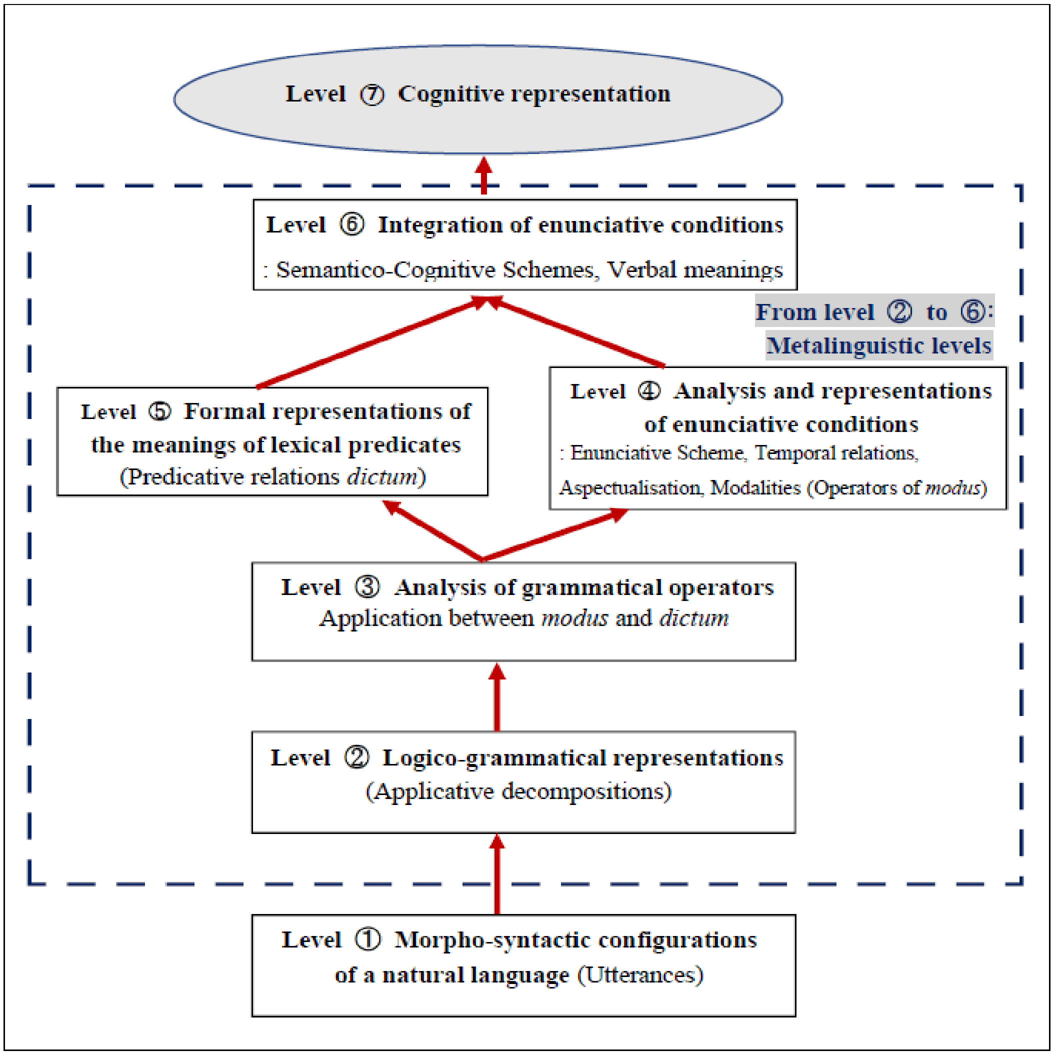

Fig. 1General architecture of GRACE

In the cognitive and computational architecture of GRACE, the morpho-syntactic configurations (level 1) concatenate, which means that the linguistic expressions (sentences, phrases, lexes, words, morphemes) are constructed by an operation of simple syntagmatic juxtaposition of smaller linguistic units.The formal representations of the other levels of representations (from level 2 to 6) are all applicative expressions, that is, formal expressions formed by applying operators, of different types, to operands.These representations use the applicative formalism of λ-calculus and Combinatory Logic without bound variables of Haskell Curry [3] with the functional types of Alonzo Church [2]. The level 7 introduces another mode of representations constructed, among other things, by the activity of perception and action. At this last level, although conditioned by a natural language, its representations leave the symbolic universe to ensure correspondences with iconic figures and images.

Each level of GRACE is articulated for transform the representations. An utterance is the input of a categorial grammar, which results in an applicative expression. This applicative expression is decomposed into two parts: *modus* and *dictum*. The *dictum* means a predicative relation in which a verbal predicate is applied to different terms. The *modus* is a complex operator, such as linguistic grammatical aspectual operator, modality operator, or temporal relations, which is composed using combinators with enunciative operators. The applicative representations of *dictum* and *modus* are merged with those of lexical operators into a complex representation of the semantico-cognitive level (in [9: p269]). This model is analogous to program compilation and, therefore, can be implemented on machines using functional programming languages such as Haskell, F#, Caml, and Lisp. It uses aspecto-temporal operators to analyze aspecto-temporal and enunciative phenomena that contains a propositional predicative relation with aspectual values (state, event, process), and relations on topological intervals (open, closed, semi-open forms) (in [8]).

2.2 Combinatory Logic of Cyrry (CL)

The GRACE model uses the combinatory logic of Curry (in [3], [4], [12]), which is a unique formalism that represents each utterance in an applicative expression where various types of operators apply to operands.The combinatory logic was used by Sebastian Shaumyan (in [17], [18]) to describe and analyze linguistic phenomena, and this approach has spread to the cognitive semantic domain of GRACE as well. The applicative is a fundamental characteristic of the combinatory logic: an operator applies to an operand to obtain a result. There are combinators which are abstract operators that compose or transform the operand it applies to. Combinatorial expressions are a part of applicative expressions. Elementary combinators come together to build more complex combinators. The meaning of each combiner is defined by its action in the form of a β–reduction (by introduction and elimination rules). The β–reduction verifies the confluence property, called the Church-Rosser theorem (in [2]), which states that reduced form, if it exists, is unique. Consider an operator "X" which applies to an operand "Y" to result in "Z". The application program from "X" to "Y" is denoted by a simple (non-associative) juxtaposition which prefixes the operator to its operand: "Z = X Y"[2]. Some elementary combinators with their elimination rules (β–reduction) and introduction rules (β–expansion) are presented in [Table I].

Table I. Elimination And Introduction rulesOf Elementary Combinators

Elementary combinators	Elimination rules(β–reduction)	Introduction rules(β–expansion)
I	$\mathbf{I}\ X \rightarrow_\beta X$	$X \leftarrow_\beta \mathbf{I}\ X$
B	$\mathbf{B}\ X\ Y\ Z \rightarrow_\beta X\ (Y\ Z)$	$X\ (Y\ Z) \leftarrow_\beta \mathbf{B}\ X\ Y\ Z$
B²	$\mathbf{B^2}\ X\ Y\ Z\ U \rightarrow_\beta X\ (Y\ Z\ U)$	$X\ (Y\ Z\ U) \leftarrow_\beta \mathbf{B^2}\ X\ Y\ Z\ U$
C	$\mathbf{C}\ X\ Y\ Z \rightarrow_\beta X\ Z\ Y$	$X\ Z\ Y \leftarrow_\beta \mathbf{C}\ X\ Y\ Z$
C*	$\mathbf{C^*}\ X\ Y \rightarrow_\beta Y\ X$	$Y\ X \leftarrow_\beta \mathbf{C^*}\ X\ Y$

[2] To simplify the writings and to avoid superfluous parentheses, the definitional expression "X Y Z $=_{def}$ (X Y) Z" is proposed. The two expressions "X Y Z" and "X (Y Z)" are not equivalent and do not mean the same application program.

K	**K** X Y $\rightarrow_\beta$ X	X $\leftarrow_\beta$**K** X Y
S	**S** X Y Z $\rightarrow_\beta$ X Z (Y Z)	X Z (Y Z) $\leftarrow_\beta$**S** X Y Z
Φ	**Φ** X Y Z U $\rightarrow_\beta$ X (Y U) (Z U)	X (Y U) (Z U)$\leftarrow_\beta$**Φ** X Y Z U
Ψ	**Ψ** X Y Z U $\rightarrow_\beta$ X (Y Z) (Y U)	X (Y Z) (Y U) $\leftarrow_\beta$**Ψ** X Y Z U

In [Table I], **I**, **B**, **B²**, **C**, and so on are metaoperators called combinators, and X, Y, Z, U are their arguments which can be any linguistic units. Combinator **I** signifies identity. Combinators **B** and its extension **B²** are functional operators of the composition. Combinator **C** represents a conversion of the two arguments of a binary operator, and **C*** changes the role between an operator and an operand. Combinator **K** is an operator for elimination or introduction, and **S** represents both the functional composition parameterized with the duplication of an argument. Combinator **Φ** represents the entanglement of two operators applied to the same operand, that is, the parallel composition of operators acting on a common operand. Combinator **Ψ** represents composition of arguments by distribution.

Combinatory logic takes advantage of the formal analysis of natural languages (in [11]). First, as in natural languages, there are no linked variables in the combinatory logic. Therefore, there is an ease of management of operations, unlike lambda-calculus, which requires the management of linked variables to avoid telescoping, especially during substitutions. Second, the combinatory logic allows the construction of complex operators from elementary operators and predicates, creating a possibility of showing how a lexical predicate or a grammatical operator can be the result of an integration process and the formal potentials of relatively complex operators. Third, the combinatory logic has the flexibility that other formalisms, including lambda-calculus, do not have, such as addressing quantification, determination, or typicality issues. Finally, the combinatory logic appears as an *Ur logik* (primal logic) at the confluence of several formalisms and at the articulation of several disciplinary fields, such as linguistics, philosophy, and computer science.

3. Formal Processing of an Aspecto-Temporal Analysis

The aspecto-temporal theory of the GRACE model explicitly analyzes and represents the enunciative operations of an abstract enunciator (in [6]). This theory proposes a formal and operational representation, which makes it implementable and simulated on the computer level. This section presents general descriptions of the aspecto-temporal category, based on the linguistic works.

Tense and aspect are closely related to a grammatical category. The aspect refers to an aim of the predicative relation represented by the enunciator and tense refers to relations which situate either the process in the referential of the enunciator, or in another referential system. In many languages, particularly in Indo-European languages, considering grammatical marks from a purely temporal or aspectual angle would make it almost impossible to process grammatical markers (in [15]).For the grammatical category of tense and aspect, topological interval diagrams which represent areas of validation of a predicative relation is necessary. The main semantic values of aspecto-temporal markers in languages can therefore be visualized according to topological intervals (open, closed, or semi-closed). A predicative relation is the result of the operations of prediction, of determination, of thematization, and it can manifest itself in the form of three main aspectual values: state, event, and process.

- Linguistic aspectual value “State” represents a stable situation without change, without movement and there is neither a first moment which would express the initial start of the state, nor a last moment which would express the end of the state.
- Aspectual value “Event”represents a transition between a situation in the previous state and a later state, necessarily involving the consideration of an initial change.
- Aspectual value “Process” represents an evolving situation with necessarily an initial change, that is the beginning of the process.

Based on this aspecto-temporal theory, the analysis of an utterance begins with an abstract enunciative schema:

The abstract enunciator “I” utters something (“what is said”) = “I-SAY”

This act of enunciation takes a linguistic aspectual value“process” (using the metalinguistic operator “PROC”) which takes place over a left-closed and right-opened interval (using the symbol “J0”), having been unaccomplished:PROC$_{J0}$ (I-SAY (...)). A metalinguistic operator of this schema “I-SAY” is the result of conversion of the two operators “I” and “SAID”, using permutation combinator **C**of two arguments: [I-SAY = **C** SAID I]. The operator “SAID” expresses the metalinguistic predicate of utterance, and the operator “I” represents the abstract enunciator that can substitute any occurrence of a concrete act of enunciation. The meaning of the enunciative schema is “An abstract enunciator takes charge (I-SAY) 'what is said'” with a certain aspectual aim “ASP$_I$”. This metalinguistic operator “ASP$_I$” signifies an aspectualized predicative relation “ASP” realized in the temporal interval, symbolized “I.” This act of enunciation is an unaccomplished process that takes place over an interval “J0” (in [5]).

Enunciative Schema

PROC$_{J0}$ (I-SAY (& (ASP$_I$ (predicative elation)) ([I **relation** J^0])))

Now, consider the following utterance.

The postman traverses his route.

The formal metalinguistic representation of utterance (a) is constructed as the following stages:

- Its applicative expression of the predicative relation (a) is <*traverse*$_2$*his_routeThe_postman*>.
- The linguistic aspectualization of this predicative relation is in the form of an aspectual operator of process “PROC$_{J1}$” which is one of the values of the abstract aspectual operator “ASP$_I$”, with the realization during the processual interval “J1”. All intervals of process use the symbol “J” and are numbered according to the number of appearances. The right boundary line of this interval “(J1)” is always unaccomplished since it is in process.This aspectual operator constructs an unaccomplished

process from a predicative relation that is not aspectualized.

- The interlocking of the unaccomplished process relating to the predicative relation in the enunciative process, expressed by “$PROC_{J0}$(...)”, is carried out in the interval “J1.”
- A temporal identification between enunciative process and predicative process is established by a concomitance of right boundaries lines of each interval “J1” and “J0”. In the temporal relational formula, the symbol “δ” represents the right boundary line of an interval, and “=” symbolizes the concomitant relation: [δ(J1) = δ(J0)].

The following applicative expression expresses the result of the enunciative and aspecto-temporal analysis of (a). To simplify the writings, the symbol “P2” designates the binary predicate associated with the transitive verb, called*traverse₂*, and the two nominal actants (or the two terms) “A1” and “A2” associated with two respective nominal phrases *The_postman*and *his_route*:

$PROC_{J0}$ (I-SAY (& ($PROC_{J1}$ (P_2 A^2 A^1)) ([$\delta(J^1) = \delta(J^0)$])))

This expression indicates that the enunciative process “$PROC_{J0}$ (I-SAY (...))” is carried out on the interval “J0,” including two metalinguistic clauses: (i) $PROC_{J1}$ (P_2 A^2 A^1) and (ii) [$\delta(J^1) = \delta(J^0)$].

Component (i) represents the aspectualized predicative relation in the form of a predicative process which is carried out over a topological interval “J1.” Component (ii) expresses a temporal condition that relates to the intervals of realization of two processes (enunciative and predicative process), whose two open right boundaries “δ(J1)” and “δ(J0)” coincide. This condition is represented as an infix notation, denoted by two separator symbols “[” and “]”. This temporal condition means that two processes (the predicative and the enunciative) are not accomplished at the same right boundary; rather they are concomitant at this right boundary[3]. The following calculus presents the integrative process in the form of a “natural deduction (in the Gentzen’s style, [10])” using combinators, leading to a definition expressed by an equivalence between operators, that is, by a “compositional reunification” which defines a unique operator as the result of a functional composition of elementary operators (at step **3** in the following natural deduction).

Step **1.** $PROC_{J0}$ (SAID (& ($PROC_{J1}$ (P_2 A^2 A^1)) ([δ(J1) = δ(J0)])) I) - hypothesis

Step **2.** $PROC_{J0}$ (**C** SAID I (& ($PROC_{J1}$ (P_2 A^2 A^1)) ([δ(J1) = δ(J0)]))) - introduction. **C**

Step **3.** [I-SAY $=_{def}$ **C** SAID I] - definition. I-SAY

Step **4.** $PROC_{J0}$ (I-SAY (& ($PROC_{J1}$ (P_2 A^2 A^1)) ([$\delta(J^1)=\delta(J^0)$]))) - replacement 2, 3

Step **5.** **B** $PROC_{J0}$ I-SAY (& ($PROC_{J1}$ (P_2 A^2 A^1)) ([$\delta(J^1)=\delta(J^0)$])) - introduction. **B**

Step **6.** [$ENONC_{J0}$ $=_{def}$ **B** $PROC_{J0}$ I-SAY] - definition. $ENONC_{J0}$

[3] Note that nothing is said about the relationship between the two left start boundaries “γ(J1)” and “γ(J0),” as the aspecto-temporal grammaticalization operated by languages essentially relates to the concomitance between two unaccomplished right boundaries.

Step **7.** $ENONC_{J0}$ (& ($PROC_{J1}$ (P_2 A^2 A^1)) ([$\delta(J^1)=\delta(J^0)$])) -replacement 5, 6 (normal form)

In step **1**, the expression of the conjunction between the predicative process and the relative temporal condition is included in the enunciative process. The predicative process takes place in the interval, J1 and the enunciative process in another interval, J0. Step **2** introduces an elementary combinator **C** which allows to permute two arguments of metalinguistic enunciative operator “I” and the conjunction between predicative process and the temporal condition, in order to group the operators “SAID” and “I” to define a new metalinguistic operator, called “taking charge by the enunciator,” symbolized as “I-SAY” (in step **3**). After replacing the definition of the “taking charge by the enunciator” operator, the expression of step **4** is considered as the instantiation by the predicative relation (P_2 A^2 A^1), with its temporal coordinate of the general enunciative schema “$PROC_{J0}$ (I-SAY (...)).” Step **5** introduces the intrinsic compositional combinator **B** which makes it possible to compose the aspectual operator “$PROC_{J0}$” with the operator “I-SAY”, hence the definition, given in step **6**, of the enunciative operator “$ENONC_{J0}$.” After substitution, the resulting expression of step **7** can be considered as an expansion (more precisely, a β-expansion in the context of the combinatory logic) in step **1**. In the aspectual perspective, the expressions in steps **1** and **7** are equivalent, that is as abstract metalinguistic paraphrases in the metalanguage of logico-grammatical representations. From this deduction, which is taken a starting point, we introduce some more integrated aspectual operators.

4. Metalinguistic Operators

This section develops grammaticalized aspectual operators more complex than the aspectual operators (“$PROC_{J0}$,” “$PROC_{J1}$,” et cetera), which is denoted by “UNAC-PRST” (unaccomplished present). It develops also an aspectual pre-morphological “prest–process” operator to which it can directly associate a morphological trace at the level of the morpho-syntactic configurations of the GRACE model. This shows reveal the differences between the grammaticalized aspectual operator and the morphologized operator with deduction calculations. For the analogue analyses of specific examples in French, [9] and [16: Chapter III].

4.1 Grammaticalized and Morphologized Aspectual Operators

In this analysis, the aspect “unaccomplished present,” symbolized as a metalinguistic operator “UNAC-PRST”, is not associated with the lexical predicate, it relates to the entire predicative relation instead. Its linguistic trace can therefore be expressed at the level of morpho-syntactic configurations, in the form of specific verbal morphemes (preverbs, suffixes, affixes, and the like), adverbs, or even specific aspectual particles. The lexical predicate is categorized as a verb at the morphosyntactic configuration level. The specific morphemes, which are constituents of the conjugated verb (unlike Asian languages such as Korean and Chinese [16], often in Indo-European languages[20]) are combined and integrated into a morpheme that specifies grammatical tense for the singular or pluralized person. The grammaticalized aspectual operator “UNAC-PRST” which integrates the temporal conditions is in fact an aspecto-temporal operator, with certain temporal constraints being closely linked to the choice of the more elementary aspectual operator ($PROC_{J1}$). Meanwhile, the second metalinguistic operator “prest–process,” which has the only lexical predicate as its operand, remains more specific to languages that express the verbal aspect attached to the verb. The following natural deduction calculus is built on the previous deduction (it starts with the step**7**, in succession):

Step **7.** $ENONC_{J0}$ (& ($PROC_{J1}$ (P_2 A^2 A^1)) ([$\delta(J^1)=\delta(J^0)$])) - hypothesis

Step **8.** $\mathbf{B^2}$ $ENONC_{J0}$ & ($PROC_{J1}$ (P_2 A^2 A^1)) **(**[$\delta(J^1)=\delta(J^0)$]**)** - introduction. $\mathbf{B^2}$

Step **9.** **B** ($\mathbf{B^2}$ $ENONC_{J0}$ &) $PROC_{J1}$ (P_2 A^2 A^1) **(**[$\delta(J^1)=\delta(J^0)$]**)** - introduction. **B**

Step **10**. $\mathbf{C_2}$ **B** ($\mathbf{B^2}$ $ENONC_{J0}$ &) $PROC_{J1}$ **(**[$\delta(J^1)=\delta(J^0)$]**)** (P_2 A^2 A^1) - introduction. $\mathbf{C_2}$

Step **11.** $\mathbf{B^2}$ ($\mathbf{C_2}$ **B**) $\mathbf{B^2}$ $ENONC_{J0}$ & $PROC_{J1}$ **(**[$\delta(J^1)=\delta(J^0)$]**)** (P_2 A^2 A^1) - introduction. $\mathbf{B^2}$

Step **12**. [$UNAC\text{-}PRST_{J1\,J0}$=$_{def}$ $\mathbf{B^2}$ ($\mathbf{C_2}$ **B**) $\mathbf{B^2}$ $ENONC_{J0}$ & $PROC_{J1}$ **(**[$\delta(J^1)=\delta(J^0)$]**)**]-definition.

Step **13**. [UNAC-PRST=$_{def}$ $\exists J^0$ J^1{$\mathbf{B^2}$($\mathbf{C_2B}$) $\mathbf{B^2}$ $ENONC_{J0}$ & $PROC_{J1}$ **(**[$\delta(J^1)=\delta(J^0)$]**)**}]

- introduction. ∃

Step **14**. UNAC-PRST(P_2 A^2 A^1) - replacement 13, 12 and 12, 11.

Step **15.** $\mathbf{B^2}$ UNAC-PRSTP_2 A^2 A^1 - introduction. $\mathbf{B^2}$

Step **16**. [$prest_{-process}$ =$_{def}$ $\mathbf{B^2}$ UNAC-PRST] - definition.$prest_{-process}$

Step **17**. ($prest_{-process}$ (P_2)) A^2 A^1 - replacement 16, 15.

Step **18**. [$prest_{-process}$ = $_{def}$ **C*** *suff-prest*] - definition.$prest_{-process}$

Step **19.** **C*** *suff-prest* (P_2) A^2 A^1 - replacement 18, 17.

Step **20**. (P_2) *suff-prest* A^2 A^1 - elimination. **C**

Step **21**.[$V_{conj\text{-}present}$ =$_{def}$ (P_2) *suff-prest*] - definition. $V_{conj\text{-}present}$

Step **22**. $V_{conj\text{-}present}$ A^2 A^1 - replacement 21, 20. (Normal form)

This natural deduction calculus takes up the previous deduction result of step **7**. The temporal constraints are connected to the aspectual conditions with the connector operator "&." Steps **8** to **11** group the operators together to bring out the predicative relation using combinators, by revealing a complex operator that integrates the aspectual operator and the constraints on the boundaries.The temporal relation [$\delta(J^1)=\delta(J^0)$], which expresses a condition on the boundaries, is propositional, which means that it is susceptible to being true or false. The introduction of combinator $\mathbf{B^2}$ allows to determine the predicative relation (step **11**). Step **12** defines the grammaticalized aspecto-temporal operator, "UNAC-PRST" (Unaccomplished Present). However, this operator depends on the intervals "J^1" and "J^0." To avoid this problem, the existence of intervals that follow certain temporal conditions is assumed.

Hence, in step **13** the existential quantification operator[4]"∃"is introduced (in [1]), and its argument is the entire predicative relation (in step **14**). In step **15**, the compositional combinator $\mathbf{B^2}$ is introduced, which allows to obtain the atemporal (timeless) predicative relation. The definition of the morphologized aspectual operator "prest–process," in step **16**, is the trace of the "UNAC-PRST" operator and has the lexical predicate "P_2" as operand, hence the aspectualized and temporalized predicate "prest–process (P_2)" is carried out at the level of morphological configurations by a conjugated transitive verb. Step **17** is the result of a substitution between steps **16** and **15**. Step **18** is the definition of the morphological operator "prest-process," where the metalinguistic operator "suff-prest" means the present verbal suffix. Moreover, combinator **C*** allows permutation between the two arguments "suff-prest" and "P_2." Steps **16** to **22** refer to the morphology of English, indicating that these steps are all carried out at the level of morphological configurations specific to Indo-European languages. Step **19** shows the replacement of **18** by **17**, and with the elimination of the combinator **C*** (in step **20**), "(P_2) *suff-prest* A^2 A^1" represents the form of the conjugated transitive verb. It thus defines the aspectual operator "$V_{conj\text{-}present}$" which designates the present conjugated verb (infinitive verb + present verbal suffix) in step **21**. After the replacement in step **22**, the final expression (normal form) is obtained.

The definitive expression of the grammaticalized aspectual operator "UNAC-PRST" introduces a "reunitarisation" in the form of a new unit, which is expressed using a complex grammatical operator that constructs the enunciative operator, elementary aspectual operator, and temporal conditions of boundaries (step **13** of the preceding deduction). This operator aims to aspectualizelinguistically the entire predicative relation. As for the definitional expression of the morphologized operator "prest-process" (step **16**), it has the lexical predicate "P_2" as operand. Repeat that this operator is closer to the morphological constraints of a language (such as French or English, for the analogue analyses in French, [20]), where the aspect and the tense are manifested mainly by the morphological markers of a "grammatical tense" of the verb, even if other linguistic processes (adverbs, prepositions, determination of nominal terms, and so on) can complete the aspectual and temporal information. Meanwhile, "UNAC-PRST" is an operator of the logico-grammatical representational level in the theoretical framework of GRACE; therefore, it is more abstract than "prest-process." These representations are more general because they release grammatical roles that are more easily generalized to a typological group of languages than morphologized operators specific to the morpho-syntactic constraints of each language.

The example (a) *The postman traverses his route* analyzed by several steps, first, aspectual value 'process' is intuitively derived because it means a gradual progress in the present tense. Second, the structure is changed from the natural language syntax in the form of the enunciator uttering to the applicative expression in which operators and operands are combined: <*traverse*$_2$*his_routeThe_postman*>. Third, this predicative relation in the form of an applicative expression is substituted into the enunciative schema: $ENONC_{J0}$ (& ($PROC_{J1}$ (P_2 A^2 A^1)) ([$\delta(J^1)=\delta(J^0)$])). At this time, the expression of the temporal relation between the enunciative act and the

[4] The existential quantification introduces intervals which are specifiable by each utterance. Illative quantifiers are the analogues in combinatorial logic of the universal and existential quantifiers formulated, with the help of linked variables by the quantifiers. For that, combinators release the arguments (the intervals) "J^0" and "J^1," then abstract from these arguments by introducing two existential illative quantifiers.

utterance is also inserted into the schema. Fourth, the grammaticalized and morphologized operators are defined by using the combinators of the combinatory logic in the applicative expression assigned to the enunciative schema: [UNAC-PRST $=_{def} \exists$ J^0 J^1 {$\mathbf{B^2}$ ($\mathbf{C_2}$ $\mathbf{B}$) $\mathbf{B^2}$ $ENONC_{J0}$& $PROC_{J1}$ ([$\delta(J^1) = \delta(J^0)$]) }], [*prest-*$_{process}$ $=_{def}\mathbf{B^2}$ UNAC-PRST]. Finally, the result derived using the natural deduction method (presented in the next section)can be converted into a machine-readable form as a formalized expression of natural language syntax example (a): $V_{conj\text{-}present}$ A^2 A^1 (which can be redefined by (P_2) suff-prest A^2 A^1, also *traverse*$_2$(-*s*) *his_route*, and at last*The_postman*:= *The postman traverses his route*).Each calculative step gradually moves from an intuitive representation of the organization of aspecto-temporal values to a theorization based on the formalization of this aspecto-temporal concept.

4.2. Application to a Natural Language

The example (a) *The postman traverses his route* refers to a predicative analysis expressed by the applicative expression: (a') *traverse*$_2$*his_routeThe_postman*. The conjugated verb *traverses* can be analyzed as the result of the application of the morphological operator "*present*," on the binary lexical predicate "*traverse*$_2$," thus constructing a conjugated verb (in the present tense). Setting the definition [*traverses* $=_{def}$ prest-process (*traverse*$_2$)], the relation between two applicative expressions is deduced as follow:

(a") prest-process (*traverse*$_2$) *his_routeThe_postman* =*traverses his_routeThe_postman*

By "going back" the previous deductions, carried out according to a rather onomasiological approach by means of successive β-expansions, the main stages of the successive β-reductions are obtained (this time by a semasiological approach), that is, the expressions obtained by the successive eliminations of combinators or quantifiers. A β-reduction consists of analyzing an operator (for example, an operator like "prest-process," associated with a morphological form), constructing a "normal form" which is used to express the meaning of the analyzed operator. A β-expansion consists of introducing combinators to integrate, by functional compositions expressed by combinators, different elementary operators in a single integrative and synthetic operator that aims to unite this composition. While β-reduction describes an analytical operational process that starts with expressions closer to the surface to construct an interpretive "normal form," β-expansion starts with interpretative expressions to synthesize operators associated with surface morpho-syntactic forms. By the β-reduction relation between the applicative expressions, the following expression is obtained:

$(prest_{-process}\ (P_2)\ )\ A^2\ A^1 \rightarrow_{\beta} \exists J^0\ J^1 \{\ PROC_{J0}\ (I\text{-}SAY\ (\&\ (PROC_{J1}\ (P_2\ A^2\ A^1))\ ([\delta(J^1)=\delta(J^0)])\ )\ )\ \}$

Insubstituting by the respective lexemes,the terms of the predicative relation ([P_2: = *traverse*$_2$]; [A^2: = *his-route*]; [A^1: = *The-postman*]), the instantiated β-reduction is obtained:

$(prest_{-process}$ (*traverse*$_2$)) *the_lake the_vapor*$\rightarrow_{\beta}$

$\exists J^0\ J^1\{$ $PROC_{J0}$ (I-SAY (& ($PROC_{J1}$ (*traverse*$_2$*his_route The_postman*)) ([$\delta(J^1)=\delta(J^0)$]))) }

The right part of '$\rightarrow_\beta$' is the normal form of the expression that is located at its left, the latter having a structure closer to the morpho-syntactic organizations of English. This normal form gives the interpretation of the morphologized operator "prest-process" which concerns the binary lexical predicate "$traverse_2$."

These semantic analyses correspond to three levels of the GRACE model.

- Morpho-syntactic configuration (level 1 of GRACE model) with the value of present: *The vapor traverses the lake.*
- Logico-grammatical representation (level 2): ($prest_{-process}$ *$traverse_2$*) *his_routeThe_postman*.
- Analysis and representation of enunciative conditions (level 4):

$\exists J^0 J^1\{PROC_{J0}$ (I-SAY (& ($PROC_{J1}$ (*$traverse_2$his_routeThe_postman*)) ([$\delta(J^1)=\delta(J^0)$]))) }

With an analysis analogous to the previous one, other types of utterances in the form of the applicative representation can be analyzed.

(b) *The lake is artificial* (descriptive state in the present tense).

The example (b) describes a stable situation without any movements or change of properties. Therefore, its linguistic aspectual value represents 'descriptive state' in the present tense, which means a descriptive static situation. The main operators of this example (b) are analyzed as follows:

[STATE-PRST $=_{def} \exists J^0 O^1$ {$\mathbf{B^2}$ ($\mathbf{C_2}$ $\mathbf{B}$) $\mathbf{B^2}$ $ENONC_{J0}$& $STATE_{O1}$ ([$\delta(O^1) = \delta(J^0)$]) }]

[$prest_{-state} =_{def}$ $\mathbf{B^2}$ STATE-PRST]

In these definitions of operators, the predicative relation is represented by the aspectual metalinguistic operator "STATE," and the symbol "O1" represents the static interval of the aspectual value. Thus, the following β-reduction is deduced:

($prest_{-state}$ (*is_artificial*)) *The-lake*$\rightarrow_\beta$

$\exists J^0 J^1$\{ $PROC_{J0}$ (I-SAY (& ($STATE_{O1}$ (*is_artificial (The_lake)*))) ([$\delta(O^1)=\delta(J^0)$]))) }

These semantic analyses correspond also to three levels of the GRACE model.

- Morpho-syntactic configuration (level 1 of GRACE) with the value of present:*The lake is artificial.*
- Logico-grammatical representation (level 2):($prest_{-state}$ (*is_artificial*)) *The-lake*.
- Analysis and representation of enunciative conditions (level 4):

$\exists J^0 J^1$\{ $PROC_{J0}$ (I-SAY (& ($STATE_{O1}$ (*is_artificial (The_lake)*))) ([$\delta(O^1)=\delta(J^0)$]))) }

Ultimately, six abstract aspectual operators are grammaticalized in languages and six respective morphologized operators, which are the respective linguistic traces of the preceding operators.

Grammaticalized aspectual operators	**Morphologized aspectual operators**
UNAC-PRST(Unaccomplishedpresent)	*prest-process*
STATE-PRST (Static present)	*prest-state*
UNAC-PAST(Unaccomplishedpast)	*past-process*
STATE-PAST (static past)	*past-state*
EVENT(Pastperfect)	*past-event*
RESUL-PRST(Presentresultingstateofapastevent)	*past-resultingstate*

Two first grammaticalized and morphologized operators 'UNAC-PRST/STATE-PRST' and '*prest-process*/*prest-state*' describe each an unaccomplished present and a static present, as the examples(a) and (b). The third grammaticalized operator 'UNAC-PAST' describes an unaccomplished past (imperfect past) as the example *He was singing*, and its morphologized operator 'past-process' represents a connection with the morphology of a conjugated verb, and its linguistic aspectual value is process in the past tense. The fourth operator 'STATE-PAST' represents a static past as the example *She was kind*, and 'past-state' is its morphological operator which expresses a conjugated verb with a stable situation in the past. The fifth 'EVENT' operator is for an accomplished situation, that is, the past perfect as*He traversed the whole continent of America*, and its morphologized operator 'past-event' represents an aspectual value of conjugated verb in the perfect tense. The last 'RESUL_PRST' operator represents a static situation which is acquired by a syntactic agent and results from the occurrence of the event immediately preceding it. Therefore, the past perfect focuses attention on the resulting state and not on the event that gives rise to it. In the following example *He got his baccalaureate, he can be enrolled in a university*, the event *He got his baccalaureate*leads to the resulting state *John has (currently) his baccalaureate*, which is a present state, concomitant with the act of enunciation.Hence, the past perfect *got* has the resulting state value. The second clause *he can be enrolled in a university* describes a static situation in the present tense. The last morphologized operator 'past-resulting state' is a linguistic trace of the grammaticalized operator. These operators of present perfect is the opposite of preterit and expresses a past event.

The grammatical meanings of these operators whose operand is the predicative relation (P_2 A^2 $A^1$for a transitive verb, andP_1A^1 for an intransitive verb) are explicitly defined using the following respective application expressions (for the transitive verb case):

$$[\text{UNAC-PRST} =_{def} \exists\ J^0 J^1\ \{\mathbf{B^2}\ (\mathbf{C_2}\ \mathbf{B})\ \mathbf{B^2}\ \text{ENONC}_{J0} \&\ \text{PROC}_{J1}\ ([\delta(J^1) = \delta(J^0)])\ \}\ ]$$

$$[\text{STATE-PRST} =_{def} \exists\ J^0\ O^1\ \{\mathbf{B^2}\ (\mathbf{C_2}\ \mathbf{B})\ \mathbf{B^2}\ \text{ENONC}_{J0} \&\ \text{STATE}_{O1}\ ([\delta(O^1) = \delta(J^0)])\ \}\ ]$$

$$[\text{UNAC-PAST} =_{def} \exists\ J^0 J^1\ \{\mathbf{B^2}\ (\mathbf{C_2}\ \mathbf{B})\ \mathbf{B^2}\ \text{ENONC}_{J0} \&\ \text{PROC}_{J1}\ ([\delta(J^1) < \delta(J^0)])\ \}\ ]$$

[STATE-PAST $=_{def}\exists$ J^0 O^1 {$\mathbf{B^2}$ ($\mathbf{C_2}$ $\mathbf{B}$) $\mathbf{B^2}$ $ENONC_{J0}$& $STATE_{O1}$ ([$\delta(O^1) < \delta(J^0)$]) }]

[EVENT $=_{def}\exists$ J^0F^1 {$\mathbf{B^2}$ ($\mathbf{C_2}$ $\mathbf{B}$) $\mathbf{B^2}$ $ENONC_{J0}$& $EVEN_{F1}$ ([$\delta(F^1) < \delta(J^0)$]) }]

[RESUL-PRST $=_{def}\exists$ J^0 O^1F^1{$\mathbf{B^2}$ ($\mathbf{C_2}$ $\mathbf{B}$) $\mathbf{B^2}$ $ENONC_{J0}$& STATE-$RESUL_{O1}$ ([$\delta(O^1) = \delta(J^0)$]& [$F^1 < J^0$]) }]

The six respective morphologized operators with respect to the grammaticalized operators operate directly on lexical predicates (in the form of pre-lexical operators) and thus come close to morphological configurations. And to pass to the morphological configurations of English, these morphological operators transform into grammatical morphemes which then operate in the form of verbal suffixes.

[prest-$_{process}$ $=_{def}$$\mathbf{B^2}$ UNAC-PRST] [prest-$_{process}$ $=_{def}$$\mathbf{C^*}$ suff-present]

[prest-$_{state}$ $=_{def}$$\mathbf{B^2}$STATE-PRST] [prest-$_{state}$ $=_{def}$$\mathbf{C^*}$ suff-present]

[past-$_{process}$ $=_{def}$$\mathbf{B^2}$ UNAC-PAST] [past-$_{process}$ $=_{def}$$\mathbf{C^*}$ suff-imperfect-past]

[past-$_{state}$ $=_{def}$$\mathbf{B^2}$ STATE-PAST] [past-$_{state}$ $=_{def}$$\mathbf{C^*}$ suff-imperfect-past]

[past-$_{event}$ $=_{def}$$\mathbf{B^2}$ EVENT] [past-$_{event}$ $=_{def}$$\mathbf{C^*}$ suff-perfect]

[past-$_{resulting\ state}$ $=_{def}$$\mathbf{B^2}$ RESUL-PRST] [past-$_{resulting\ state}$ $=_{def}$$\mathbf{C^*}$ suff-perfect]

The configuration thus obtained is the observable morphological trace of more abstract grammaticalized aspectual operators. The resolution of the indeterminacy corresponding to these aspect values (particularly event in the past or resulting state present) is carried out, using the contextual exploration strategy, considering the contextual clues.

These analyses par natural deduction show that the applicative formalism of combinatory logic is susceptible to formally represent certain reasonings by grammatical elements, in particular by grammatical units having an aspectual and temporal value, on condition however of being able to represent the meaning of these grammatical units and to operate with these formal representations.These reasonings are presented in this section, in the form of deductions where the temporal and aspectual notions are operators.

5. Conclusions

This paper examines how to implement the aspecto-temporal formalization of utterances. In particular, it shows how certain aspecto-temporal analyses are treated in the same framework GRACE by means of formal calculations. These calculations are expressed in an applicative language accompanied by a composition operator, the combinatory logic combinators. They allow the construction of elementary metalinguistic aspectual operators and linguistic temporal relations. Topological and enunciative conceptualizations resulting from theorization are formulated by sets of operators that relate to specifically predicative relations. From the GRACE model, this work restricts to formal aspect-temporal analyses. Taking lexical meanings into account (verbs, prepositions and preverbs in particular) requires formal representations of these meanings. This study also uses intervals of instants on which the predicative and enunciative relations are realizing or are realized, that is, validated or considered as being “true” at the different instants of these intervals. Therefore, the analytic processes add operators whose meaning is specifically aspecto-temporal, to the applicative formalism of combinatorylogic, particularly in the main section topological operators for considering the open or closed nature of the intervals of tenses are presented. This study shows how the applicative formalism of combinatory logic is capable of formally representing certain reasonings that can be triggered by grammatical elements, especially by grammatical units having an aspectual and temporal value. These reasonings is presented, in sections 3 and 4, in the form of deductions where the temporal and aspectual notions are operators.

The calculations presented above are not yet fully formalized and are not therefore fully automatable. To achieve this objective, it is necessary to specify more formally “the semantics of topological intervals” as well as the rules that manage the relations between topological temporal intervals and temporal zones of realization of states, events, and processes.The nature of the problems encountered revealed a real complexity in the representations and processing of this type of information. The concrete enunciator must therefore resolve and dominatethis operational complexity of the treatments for which this study proposed a first formalized conceptual framework.

The formal analysis which is the fruit of a work bearing on the concept and on the precise definitions associated with topological intervals, implements the mathematical notion of continuity.Computers are based on mathematical and logical formula having a binary system of 0 and 1, and always have a system in which one operator is applied to an operand. This study of converting natural language into a combination of operators and operands is applicable to any language, and inevitably produces a machine-understandable representation. In conclusion, this paper shows a theoretical and logical first step in converting natural language into formal expressions that can be understood by machines.

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