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New Notions In Fuzzy Ideal Topological Spaces

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Abstract

The aim of the present paper is to introduce a fuzzy αAB□-set, fuzzy □Δ□ set, fuzzy □□□ set, fuzzy pre* - □ open set, fuzzy strongly pre* - □ open set, fuzzy βI* open set. Also, introduce the class of fuzzy stronglypre* - I – open sets which is strictly placed between the class of all fuzzy pre – I – open and the class of all fuzzy pre* - I – open subsets of X. The Concepts of weakly fuzzy γ - I –open sets, weakly fuzzy semi – I open sets are used via idealization. Furthermore, some properties, characterizations and implications of several generalizations of fuzzy open sets are discussed. New Decomposition of fuzzy α-□-open set,fuzzy α□□□ -set and fuzzy β-□-closed set are obtained by using the above sets

Index Terms: fuzzy topological ideal, F□Δ□- set, fuzzy pre* □ open set, fuzzy strongly pre* I open set, FαAB□-set, FβI* open set, F□□□set.

Introduction and Preliminaries

Johann Benedict Listing coined the term **topology** in the nineteenth century, but the concept of a **topological space** did not emerge until the first decades of the twentieth century. In current scientific investigations, **fuzzy set** is one of the most essential and helpful terms. Zadeh[2] was the first to establish the concept of fuzzy sets in 1965. Vaidyanathaswamy[5] first suggested the method of **ideal topological spaces** in 1945. Mahmoud[3] and Sarkar[1] proposed many of the ideal notions in the fuzzy trend separately in 1990, also looked into a variety of other factors. One of the numerous issues in the field is the decomposition of fuzzy continuity in fuzzy Topology. A nonempty collection I of fuzzy subsets of Y is called a **fuzzy ideal** if and only if

1. Q ∈I and P ≤ Q, then P ∈I (**heredity**),
2. P ∈ I and Q ∈I then P ∨ Q ∈I (**finite additivity**).

The triplet (Y, τ, I) means a fuzzy space with a **fuzzy ideal** I and fuzzy topology τ**(short, fits)**.

Definition 1.1.A fuzzy set A of a fuzzy ideal topological space(Y, τ,I) is called

1. A fuzzy -I – open set[1], if A < Int(A*);
2. A fuzzy α-I-open[6] , if A ≤Int(Cl* (Int(A)));
3. A fuzzy semi-I-open[8],if A ≤Cl* (Int(A));
4. A fuzzy pre-I-open[7] ,if A ≤Int(Cl* (A));
5. A fuzzy t-I-set[7], if Int(Cl* (A)) = Int(A);
6. A fuzzy ∗ − perfect set[11], if A = A*;
7. A fuzzy semi -I-regular set[4], if A is both a fuzzy t-I-set and a fuzzy semi I -open set;
8. A fuzzyα* -I-set[6], if Int(A)=Int(Cl* (Int(A)));
9. A fuzzy regular-I-closed[4], if A=(Int(A))*;
10. A fuzzy AB-I -set[4] , if A∈ABI(X) ={U^V : U is fuzzy open and v is fuzzy semi-I-regular};
11. A fuzzy SβI -open set[10],if A ≤Cl* (Int(Cl*(A)));

12. A fuzzy β-I-open[6] , if A ≤Cl(Int(Cl* (A)));
13. A almost fuzzy strongly – I – open set[10], if A < Cl*(Int(A*));

Definition 1.2. [12]. A subset A of a fuzzy ideal topological space (Y, τ, I) is called Weakly fuzzy γ – I - Open set if A < Cl* (Int(Cl(A)))∨Cl(Int(Cl* (A))) .

Proposition 1.1.[4]. Let (Y, τ, I) be a fuzzy ideal topological space. Then

(1) Every fuzzy semi-I-regular set is a fuzzy t-I-set.

(2) Every fuzzy semi-I-regular set is a fuzzy semi-I-open set.

Definition 1.3.[11]In ideal topological space (Y,τ,I), I is said to be codense if τ^ I = φ.

Definition 1.4.A subset A of a fuzzy topological space (Y, τ) is called

(a) A fuzzy pre – open set, if A < Int(Cl(A));
(b) A fuzzy β – open set, if A < Cl(Int(Cl(A)));

Definition 1.4.A subset H of an ideal topological space (Y, τ, I) is called a B – I set[9] if if H ∈B-I (Y)={A^B: A ∈τ and B is a t-I-set};

Lemma 1.1: [1] Let (Y, τ, I) be a fits and A, B subsets of Y. The following properties hold:

(a) If A ≤ B, then A* ≤ B*,

(b) (A ∨ B)*= A* ∨ B*,

(c) A* = Cl(A*) ≤ Cl(A),

(d) if U ∈ τ, then U ^ A* ≤ (U ^ A)*,

(e) if U ∈ τ, then U ^ Cl*(A) ≤ Cl*(U ^ A).

Lemma 1.2.[11]. Let (Y, τ, I) be an ideal space, where I is codense, then the following hold:

1. Cl(A) = Cl* (A), for every ∗– open set A;

2. Int(A) = Int* (A), for every∗– closed set A.

Note: Throughout this article, we use the following notation

- **Intr** denotes Interior of a set.
- **Clr** denotes Closure of a set.
- **Clr*** denotes Kuratowski Operator
- **(Y, τ, I)** denotes fuzzy ideal topological space**(short, fits)**

FAABI-SET, FIΔI SET, FΦXI SET

Definition 2.1. Let (Y, τ, I) be a fits. A fuzzy set P of Y is said to be

(1) A fuzzy αABI -set if P ∈ αABI (Y) ={U∧ V: U is fuzzy α-I-open and V is fuzzy semi-I- regular}.

(2) A fuzzy IΔI -set if Clr*(Intr(P))=Y.

Definition 2.2.A fuzzy set P of a space (Y, τ, I) is said to be

1.A fuzzy semi* -I-open set, if P≤Clr(Intr* (P));A fuzzy semi* -I-closed set, if its complement is fuzzy semi*-I-open;

2.A fuzzy X-I -set , if P∈XI(Y)={U ∨ V:U ∈ τ and V is a fuzzy α* -I-set};

Remark 2.1

Assume that (Y, τ, I) be a fits.

(1) If P is fuzzy α-I-open set then P is a fuzzy αABI -set but the converse is not possible.

(2) IfP is fuzzy semi-I-regular set then P is fuzzy αABI -set. But the reverse is not possible.

Proposition 2.1.Let (Y,τ,I) be a fits. If P isfuzzy αABI - set then P is fuzzy semi- I -open.

Proof. Let P be a fuzzy αABI - set. Then P = U∧V where U∈FαIO(Y) and V is a fuzzy semi- I-regular set. By Proposition 1.1, V∈FSIO(Y). If V is a fuzzy semi-I-open and P is a fuzzy α-I-open, then P=V∧U is a fuzzy semi-I-open. Therefore, P ∈FSIO(Y).

Proposition 2.2. Let (Y, τ, I) be a fits and P < Y.

(1) If P is fuzzy semi-I-regular then P is both fuzzy strong β-I-open and fuzzy semi* -I-closed.

(2) If P is both fuzzy strong β-I-open and fuzzy semi* -I-closed then P is fuzzy semi- I-regular set.

Theorem 2.1. Let P < Y be a fuzzy set of a fuzzy I- submaximal space (Y, τ, I). Then the following are equivalent.

(1) P is a fuzzy t-I-set.

(2) P is a fuzzy semi*-I-closed set.

(3) P is bothfuzzy α*-I-set andFαBI-set.

Theorem 2.2. For a fuzzy set P < Y of a fuzzy I -sub maximal ideal topological space(Y, τ,I), the following are equivalent.

(1) P is fuzzy semi-I-regular set.

(2) P is fuzzy semi*-I-closed set and a FαABI-set.

(3) P is a Fα*-I-set and FαABI -set.

Proof.

(1) ⇒(2): It is obvious. [Proposition 2.2]

(2) ⇒(3): It is obvious.[Theorem 2.1]

(3) ⇒(1): If P is a fuzzy αABI-set then P is both a fuzzy semi-I-open and a fuzzy αBI - set. Again, if P is a fuzzy α*-I-set and fuzzy αB- I - set, then P is a fuzzy semi-I- regular set.

Definition 2.3. A fits (Y, τ, I) is said to be fuzzy I-extremely disconnected if Clr* (P) ∈τ for each P ∈τ.

Theorem 2.3. Assume that (Y, τ,I) be an * - extremely disconnected fits. Then FαIO(Y) = FαABI(Y), where FαIO(Y) denotes the family of fuzzy α-I-open subsets of Y and FαABI(Y) denotes family of fuzzy αABI sets.

Proof. We know that, If P is fuzzy α-I-open set then P is FαABI-set. Thus, FαIO(Y) ≤ FαABI(Y). Assume that P ∈ FαABI(Y). Then P = U∧V where U∈FαIO(Y) and V is a fuzzy semi-I-regular. Now V is fuzzy semi- I-regular implies that V is a fuzzy t-I-set and also V∈ FSIO(Y). Hence Intr(V) = Intr(Clr*(V)) and V≤Clr*(Intr(V)) which implies that Intr(V) = Intr(Clr* (V)) and Clr* (V) = Clr* (Intr(V)). Since Y is * - Extremely disconnected, Intr(Clr*(Intr(V))) = Clr*(V). Thus, Intr(V) = Intr(Clr*(V)) = Intr(Clr*(Intr(V)))= Clr*(V) ≥V and so V ∈ τ(Y). We have U∈FαIO(X) and V ∈ τ(Y). Thus, P = U∧V is fuzzy α-I-open. Hence FαABI(Y) ≤ FαIO(Y). Thus FαIO(Y)= FαABI(Y).

Definition 2.4.A fuzzy set P < Y of a fits (Y, τ,I) is called aFΦXI-set if P = U∧V, where U ∈ τ(Y) and V is Fβ-I-closed. The family of all FΦXI-sets of a fits(Y,τ,I) will be denoted by FΦXI(Y).

Theorem 2.4.Assume that P < Y be a fuzzy set of a fuzzy ideal space(Y,τ,I). Then P∈FΦXI(Y) if and only if P=U∧ *F*β-I-Clr(P), U ∈ τ(Y).

Proof.

(⇐):Assume that P=U∧β-I-Clr(A), U ∈ τ(Y). Since β-I-Clr(P) is fuzzy β-I-closed, P∈FΦXI(Y).

(⇒): Assume that P∈FΦXI(Y). Then P=U∧V where U ∈ τ(Y) and V is fuzzy β-I- closed. Since P ≤ V, Fβ-I-Clr(P) ≤ *F*β-I-Clr(V) = V. Thus U ∧ *F*β-I-Clr(P) ≤ U∧V = P ≤U∧ *F*β-I-Clr(P) and hence P = U ∧ *F*β-I-Clr(P).

Theorem 2.5. Let P < Y be a fuzzy set of a fits (Y,τ,I). If P ∈FΦXI(Y),then

(1) Fβ-I-Clr(P)|P ∈ FβIC(Y) (short, fuzzy β-I-closed is FβIC(Y)).

(2) P ∨ (Y|Fβ-I-Clr(P)) ∈ FβIO(Y) (short, fuzzy β-I-Open is FβIO(Y)).

Proof.

(1)Assume that P ∈FΦXI(Y). By Theorem 2.4, P=U∧Fβ-I-Clr(P) U ∈ τ(Y). Hence Fβ-I- Clr(P)\P = Fβ-I-Clr(P)\(U∧β-I-Clr(P)) = Fβ-I-Clr(P) ∧ (Y\(U∧Fβ-I-Clr(P))) = β-I-Clr(P) ∧ ((Y\U) ∨ (Y\Fβ- I-Clr(P))) = (Fβ-I-Clr (P) ∧ (Y\U)) ∨ (Fβ-I-Clr(P) ∧ (Y\Fβ-I-Clr (P))) = (Fβ-I-Clr (P)∧ (Y\U))∨ ∅ = Fβ-I-Clr(P) ∧ (Y\U). Thus, Fβ-I-Clr(P)\P ∈ FβIC(Y).

(2)We know that Fβ-I-Clr(P)\P is fuzzy β-I-closed, Y\(Fβ-I-Clr(P)\P) is fuzzy β-I-open. Therefore Y\(Fβ - I - Clr(P)\P) = Y\ (Fβ-I-Clr(P) ∧ (Y\P)) = (Y\Fβ-I-Clr(P)) ∨ P. Therefore, P∨ (Y\Fβ-I-Clr(P)) ∈ FβIO(Y)

Fuzzy pre*- I- open set, Fuzzy strongly pre*- I- open set, FβI* open set.

Definition 3.1. A fuzzy set P < Y of (Y, τ, I) is called **fuzzy pre*–I– open** (briefly FP*IO) set, if P < Intr* (Clr(P)).

Definition 3.2.

(a) A fuzzy set P < Y of (Y, τ,I) is said to be a **fuzzy strongly pre*–I–open** set (briefly FS.P*–I–open set) if P < Intr* (Clr* (P)). We denote that all fuzzy S.P*–I–open set by FS.P*IO(Y).

(b) A fuzzy set P < Y of (Y, τ, I) is said to be a **fuzzy strongly semi*–I–open** set (briefly FS.S*–I–open set) if P < Clr* (Intr* (P)). We denote that all fuzzy S.S*–I–open set by FS.S*IO(Y).

Lemma 3.1. Let (Y, τ, I) be a fits.

1. If a fuzzy set P < Y is fuzzy pre – I – open set, then P is a fuzzy S.P*–I– open set.

2. If a fuzzy set P < Y is fuzzy S.P*–I– open set, then P is a fuzzy pre*–I– open set.

Theorem 3.1.Let (Y, τ, I) be a fits. Then P is a fuzzy S.P*–I– open set if and only if there exists a fuzzy S.P*–I–open Q such that P < Q < Clr* (P).

Proof. Assume that P < Y be a fuzzy S.P*–I– open set, then P < Intr* (Clr* (P)). We put Q = Intr* (Clr* (Q)), which is a fuzzy- ∗– open set. Therefore Q = Intr*(Q) < Intr* (Clr* (Q)) be a fuzzy S.P*–I– open set such that P < Q = Intr* (Clr* (Q)) < Clr* (P).

Conversely, Let Q ∈ FS.P*IO(Y). We have P < Q < Clr* (P), By taking fuzzy ∗– closure,Clr* (P) < Clr* (Q). Furthermore P< Q < Intr* (Clr* (Q)) < Intr* (Clr* (P)). Therefore P ∈ FS.P*IO(Y)

Corollary 3.1. Suppose (Y, τ, I) be a fits, then P is a fuzzy S.P*–I– open set if and only if there exists a fuzzy open set P < Q < Clr* (P).

Corollary 3.2. Let (Y, τ, I) be a fits. If P is a fuzzy S.P*−I− open set, then Clr* (P) is afuzzyS.S*−I− open set.

Proof. Assume that P ∈ FS.P*IO(Y). Then P < Intr* (Clr* (P)) and Clr* (P) < Clr* (Intr* (Clr* (P))). Therefore Clr*(P) ∈FS.S*IO(Y).

Corollary 3.3. Let (Y, τ, I) be a fits. If P is a fuzzy strongly semi*−I− open set, then Intr* (P) is a fuzzy S.P*−I− open set.

Proof. Assume that P∈FS.S*IO(Y). Then P < Clr* (Intr* (P)) ⇒ Intr* (P) < Intr* (Clr* (Intr* (P))). Therefore, Intr* (P) ∈ FS.P*IO(Y).

Theorem 3.2. Let (Y, τ, I) be a fits, P < Yand Q< Y.

1. If P ∈FSP*IO(Y, τ,I), for each α ∈Δ, then ∨ {P_α: α ∈Δ} ∈FSP*IO(Y, τ, I)

2. If P ∈FSP*IO(Y, τ, I), and Q ∈ τ, then P∧Q ∈FSP*IO(Y, τ, I).

Proof.(1) Since P_α∈FSP*IO(Y, τ, I), we have Pα < Inrt* (Clr* (Pα)), for each α ∈Δ. Then

$\vee_{\alpha\in\Delta}P_\alpha < \cup_{\alpha\in\Delta}Intr^* (Clr^* (P_\alpha))$

$< Intr^* (\cup_{\alpha\in\Delta} Clr^* (P_\alpha))$

$= Intr^* (\vee_{\alpha\in\Delta}(P^*_\alpha \vee P_\alpha))$
$=Intr^*(\vee_{\alpha\in\Delta}P^*_\alpha \vee \cup_{\alpha\in\Delta}P_\alpha)$
$< Intr^*((\vee_{\alpha\in\Delta}P_\alpha) * \vee \cup_{\alpha\in\Delta}P_\alpha)$
$= Intr^*(Clr^* (\vee_{\alpha\in\Delta}P_\alpha))$

This shows that $\vee_{\alpha\in\Delta}P_\alpha$∈FSP*IO(Y, τ, I).

2) Assume that P∈FSP*IO(Y, τ, I) and Q ∈τ. Then P < Intr* (Clr* (P)) and Q = Intr(Q) < Intr* (Q). Thus,

P∧Q < Intr* (Clr* (P)) ∧ Intr* (Q) = Intr* (Clr* (P) ∧ Q) = Intr* ((P*∨ P) ∧ Q) = Intr* ((P*∧ Q) ∨ (P ∧ Q)) < Intr* ((P ∧ Q)*∨ (P ∧ Q)) = Intr* (Clr* (P ∧ Q))

Definition 3.3. A fuzzy set P < Y of a fuzzy fits (Y, τ, I) is called Fβ_I^* open set if P <Clr(Intr* (Clr(P))).

Definition 3.4. A fuzzy set P < Y of a fits(Y, τ, I) is called a fuzzy weakly semi − I – open set, if P < Clr* (Intr (Clr(P)));

Theorem 3.3. Assume that (Y, τ, I) be a fits, where I is condense. Then the subset P < Y satisfies the following statements.

1. If P is fuzzy S.P*−I− open set then P is a fuzzy strong β−I− open set.

2. If P is fuzzy S.P*−I− open set then P is a fuzzy β− open set.

3. If P is fuzzy S.P*−I− open set then P is a fuzzy weakly semi − I – open set.

4. If P is fuzzy S.P*−I− open set then P is a fuzzy weakly γ − I – open set.

5. If P is fuzzy S.P*−I− open set then P is a fuzzy pre − open set.

Proof. It is obvious.

Theorem 3.4.Let (Y, τ, I) be a fits. If every fuzzy open set is fuzzy ∗− closed, then every fuzzy strongly β − I – open set is fuzzy S.P* −I− open set.

Proof. Assume that P ∈ FSβIO(Y). Then P < Clr* (Intr(Clr* (P))). Since Intr(Clr* (P)) ∈ τ, by hypothesis Intr(Clr* (P)) = Clr* (Intr(Clr* (P))). Therefore P < Clr* (Intr(Clr* (P))) = Intr(Clr* (P) < Intr* (Clr* (P)). Hence P∈FS.P*IO(Y).

Theorem 3.5. Let (Y, τ, I) be a fits. Assume that P is fuzzy ∗− perfect. Then P satisfies the following statements.

1. If P is fuzzy S.P* −I− open set then P is almost fuzzy strong −I− open set.

2. P is a fuzzy S.P* −I− open set if and only if it is fuzzy I− open set.

Proof.(1) Assume that P∈FS.P*IO(Y).Then P < Intr* (Clr* (P)) = Intr (Clr* (P)) < Clr* (Intr(Clr* (P))) = Clr* (Intr (P*)). This implies P is almost fuzzy strong − I – open set.

(2) Assume that P∈FS.S*IO(Y).Then P < Intr* (Clr* (P)) < Intr* (Clr(P)) = Intr(P*). Hence P ∈FIO(Y). **Conversely,** if P ∈FIO(Y), then P < Intr(P*) = Intr* (Clr* (P)). Hence P∈FS.P*IO(Y).

Corollary 3.4. Let (Y, τ, I) be a fits. If P is fuzzy ∗−perfect, then every fuzzy pre* −I− open set is fuzzy S.P* −I− open set.

Proof. Assume that P ∈FP*IO(Y). Since it is fuzzy ∗− perfect, then P < Intr* (Clr(P)) = Intr* (Clr* (P)). Hence P∈FS.P*IO(Y).

Corollary 3.5. If P is fuzzy I − open set then P is fuzzy S.P* −I− open set.

Proof. If P ∈FIO(Y)., then P < Intr (P*) <Intr(P* ∨ P) < Intr* (Clr* (P)). Hence P∈FS.P*IO(Y).

Theorem 3.6. Let (Y, τ, I) be a fits, Where I is codense. If P is fuzzy pre* −I− open set then P is fuzzy S.P* −I− open set

Proof. It is obvious.

Theorem 3.7. Let (Y, τ, I) be a fits and P < Y be a fuzzy pre – open set and fuzzy semi – closed set. Then P is fuzzy S.P* −I– open set.

Proof. If P is fuzzy pre−open set, then P< Intr(Clr(P)). Since P is fuzzy semi−closed set then Intr(Clr(P)) = Intr(P), then P < Intr(P) < Intr* (Clr* (P)). Hence P ∈ FS.P*IO(Y).

Theorem 3.8. Let (Y, τ, I) be a fits. Let P < Y be a fuzzy S.P* −I− open set and fuzzy- ∗ − closed set, then A is fuzzy S.S* −I− open set.

Proof. Let P is fuzzy S.P* −I− open set, then P < Intr* (Clr* (P)). Since P is fuzzy ∗− closed set then Intr* (Clr* (P)) = Intr* (P). Now P < Intr* (P) < Clr* (Intr* (P)) Hence P∈FS.S*IO(Y).

Theorem 3.9. Let (Y, τ, I) be a fits. Let P be fuzzy Pre* −I− open set and fuzzy closed set. Then P is a fuzzy S.P* −I− open set

Proof. Let P∈FP*IO(Y). Then P <Intr* (Clr(P)). Since P is fuzzy closed set, then P < Intr* (Clr(P)) = Intr* (P) < Intr* (Clr* (P)). Therefore P∈FS.P*IO(Y).

Theorem 3.10. Let (Y, τ, I) be a fuzzy I − extremely disconnected space and A < Y. If P is fuzzy semi − I − open set then P is a fuzzy S.P* −I− open set.

Proof. Let P be a fuzzy semi − I – open set, then P<Clr* (Intr(P)). By Lemma 1.3, we obtain P <Intr(Clr* (P)) < Intr* (Clr* (P)). Hence P∈FS.P*IO(Y).

Lemma 3.2. A fits (Y, τ, I) is fuzzy I − extremely disconnected set if and only if Clr* (Intr* (P)) < Intr* (Clr* (P)), for every fuzzy set P of Y.

Proof. By Def 2.3., we obtain Clr* (P) ∈ τ. Thus Clr* (Intr* (P)) < Clr* (P) = Intr(Clr* (P)) < Intr* (Clr* (P)). Thus Clr* (Intr* (P)) < Intr* (Clr* (P)).

Conversely, since Clr* (Intr(P)) < Clr* (Intr* (P)) < Intr* (Clr* (P)) < Intr* (Clr(P)). Then Y is fuzzy I − extremely disconnected set.

Corollary 3.6. Let (Y, τ, I) be a fuzzy -I − extremely disconnected space and A < Y. If P is fuzzy strongly semi* −I− open set then P is fuzzy S.P* −I− open set.

Proof. It is obvious by Lemma 3.2.

Theorem 3.11. Let (Y, τ, I) be a fits, P < Y and Q < Y. If P is a fuzzy S.P* −I− open set and Q is a fuzzy pre − open set, then P ∨ Q is fuzzy pre* −I− open set.

Proof. Assume that P∈FS.P*IO(Y).We have P < Intr* (Clr* (P)), and Q ∈ FPO(Y) then Q < Intr(Clr(Q)). Now:

P ∨ Q < Intr* (Clr* (P)) ∨ Intr(Clr (Q)) < Intr* (Clr(P)) ∨ Intr* (Clr(Q)) < Intr* (Clr(P ∨ Q)).

Hence P ∨ Q ∈ FP*IO(Y).

Theorem 3.12. Let (Y, τ, I) be a fits, P < Y and Q < Y. If P is a fuzzy S.P* −I− open set and Q is a weakly fuzzy semi −I− open set, then P ∨ Q is fuzzy β_I^* open set.

Proof. Assume that P ∈ FS.P*IO(Y). Then P < Intr* (Clr* (P)), P is weakly fuzzy semi − I − open and Q < Clr* (Intr(Clr (Q))) We have

P ∨ Q < Intr* (Clr* (P)) ∨ Clr* (Intr(Clr(Q)))

< Clr (Intr* (Clr (P))) ∨ Clr(Intr* (Clr (Q)))

= Clr (Intr* (Clr (P)) ∨ Intr* (Clr(Q)))

<Clr(Intr* (Clr(P ∨ Q))).

Thus P ∨ Q ∈ Fβ_I^*O(Y)

Theorem 3.13. Let (Y, τ, I) be a fits, where I is codense then P is fuzzy α −I −open set if and only if it is a fuzzy S.S* −I− open set and fuzzy S.P* −I− open set.

Proof. Necessity, this is obvious.

Conversely, Let P is a fuzzy S.S* −I− open set and FSP*I(Y), we have:

P < Intr* (Clr* (P))<Intr* (Clr* (Clr* (Intr* (P))))= Intr* (Clr* (Intr* (P))) = Intr(Clr* (Intr(P))).

Hence P ∈ FαIO(Y).

Theorem 3.14. Let (Y, τ, I) be a fits. Then P < Y satisfies the following statements.

1. If P is a fuzzy S.P* −I− open set, then SIClr(P) = Intr* (Clr(P)).

2. If P is a fuzzy S.P* −I− closed set, then SIntr(P) = Clr* (Intr(P)).

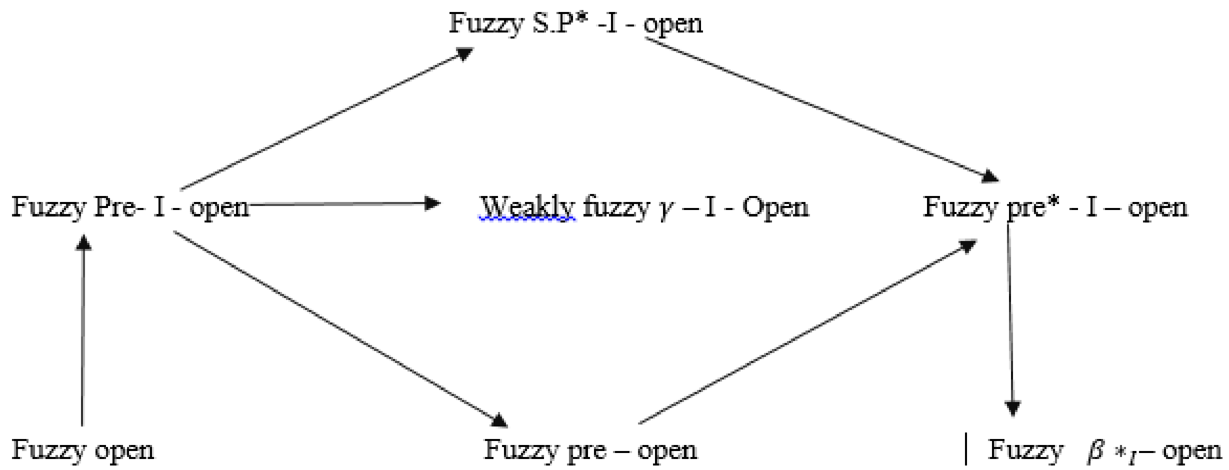
Proof.(1) Let P be a fuzzy S.P* −I− open set in Y. Then we have P < Intr* (Clr* (P)) < Intr* (Clr(P)). Thus we have SIClr(P) = Intr* (Clr(P)).

(2) Let P be a fuzzy S.P* −I− closed set in Y, then we have P > Clr* (Intr* (P)) > Clr* (Intr(P)). Hence SIntr(P) = Clr* (Intr(P)).

Theorem 3.15. Let (Y, τ, I) be a fits, then each fuzzy pre − I − regular set in Y is fuzzy S.P* − I – open set and fuzzy S.P* −I− closed set.

Proof. We know that every fuzzy pre−I −regular set is fuzzy pre−I −open set and fuzzy pre−I −closed set. Therefore, it is fuzzy S.P* −I− open set and fuzzy S.P* −I− closed set.

Remark 3.1. The following diagram holds for any fuzzy set P < Y of a fits (Y, τ, I).



Decomposition of Fα-I-open set,FαABI -set and Fβ-I-closed set

Definition 4.1. A fuzzy set P of a fits (Y, τ, I)is said to be

(a) Fgβ-I-closed set if Fβ- I-Clr(P)≤ M whenever P ≤M and M is fuzzy open set in Y.

(b) Fgβ - I - openif Y\P isFgβ-I-closed set.

Theorem 4.1. Let G be a fuzzy set of (Y, τ,I). Then the following are equivalent:

(1) G is a fuzzy α-I-open set.

(2) G is a fuzzy pre-I-open set and a FαAB- I-set.

Proof.

(1) ⇒(2): It is obvious.

(2) ⇒(1): Since G is fuzzy pre-I-open set and it is FαAB- I-set. By Proposition 2.1, G is a fuzzy semi-I- open set. Now G ∈ FSIO(Y) and G ∈ FPIO(Y). Therefore G ∈ Fα IO(Y).

Theorem 4.2. Let G be a fuzzy set of (Y, τ,I). Then the following are equivalent:

(1) G is a FαABI -set.

(2) G=A∧B where A is a FAB-I -set and B is a FIΔI - set.

Proof.

1 (⇒) 2: Let G be a fuzzy αABI -set. Thus G = C ∧ D where C ∈FαIO(Y) and D is fuzzy semi-I-regular set. From Lemma 1.4, we have C = E∧F where E∈ τ and F∈FIΔI(Y). Moreover, we have G= C∧D =E∧F∧D =(E∧D)∧F such that A= E∧D is a fuzzy AB -I -set and B is a FIΔI -set.

1(⇐)2: Let G=A∧B where A is a fuzzy AB- I - set and B is a FIΔI -set. Since A is a fuzzy AB- I -set, there exist a fuzzy open set U and a fuzzy semi-I-regular set V such that A = U∧V. We have G=A∧B=U∧V∧B=(U∧B) ∧V where U∧B is, by Lemma 1.4, a fuzzy α-I- open set. Thus, G is a FαABI -set.

Theorem 4.3. Assume that(Y, τ,I) be a fits. Then P < Y satisfies the following statements.

(1) P is fuzzy β-I-closed set,

(2) P is a FΦXI-set and Fgβ-I-closed set.

Proof.

(1)⇒(2):It is obvious.

(2) ⇒(1): Let P be a FΦXI-set. Then we have P=U ∧ Fβ-I-Clr(G), U ∈ τ in Y. We have P ≤ U. Since P is Fgβ-I-closed set, then Fβ-I-Clr(P) ≤U. Hence Fβ-I-Clr(P) ≤ U ∧ Fβ- I-Clr(P) = P. Hence P ∈ FβIC(Y).

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