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Strong Intuitionistic Fuzzy Euler and Strong Intuitionistic Fuzzy Hamiltonian Graph

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Abstract

This paper is based on Intuitionistic fuzzy (IFG) Euler line, IFG Euler graph, effective IFG Euler graph, IFG unicursal graph, effective IFG unicursal graph, IFG Hamilton cycle, effective IFG Hamilton cycle and effective Hamiltonian path. Some properties of effective IFG. Euler graph, effective IFG unicursal graph, and effective IFG Hamilton cycle are discussed for some standard graphs.

Index Terms IFG Euler line, IFG Euler graph, IFG unicursal graph, IFG Hamilton cycle, IFG Hamiltonian path.

Introduction

During 18th century, many people working on a famous puzzle called Konigsberg bridge problem. Konigsberg which is located in East Prussia, is now called Kaliningrad and is in Lithuania, which has recently separated from USSR. The city divided into four separate landmasses by the river. These four regions were linked by seven bridges so walking over the each bridge once and return to the starting point. This longstanding problem was solved by Swiss Mathematician Leonhard Euler in 1736.

Euler posed and then solved a more general problem: A closed walk in a graph G containing all the edges of G is called an Euler line in G. A graph containing an Euler line is called an Euler graph. In 1856, Sir William R. Hamilton, an Irish Mathematician develop the concept of Hamilton cycle and Hamilton path. A Hamilton cycle in a connected graph defined as a closed walk that transversely vertex of G exactly once, except the starting vertex, at which the walk also terminates. Using IFG graph, let us develop the concept of Euler and Hamiltonian graph and its characteristics of the graph.

Preliminaries:

A summary of basic definitions is given, which are presented in [1, 2, 6, 7, 11, 12, 13].

A IFG subset of a nonempty set V is a mapping σ: V→ [0,1]. An IFG relation on V is a IFG subset of V □□V. A IFG graph is a pair of functions International Review of IFG Mathematics Volume 3 No.1 (June 2008) pp 55-68.σ: V→[0,1] and μ:V □□V →[0,1], where for all *u, v* in V we have min(σ(*u*),σ(*v*)) ≥ μ(*u, v*). A fuzzy graph H:(τ, υ) is called IFG sub graph of G:(σ, μ) if σ(*u*) ≥ τ(*u*) for all *u* ϵ V and μ(*u, v*) ≥ υ (*u, v*) for all *u, v* ϵ V. Further H is a spanning sub graph if τ(*u*) = σ (*u*) for all *u* ϵ V. For any IFG subset τ of σ, such that σ(*u*) ≥ τ(*u*) for all *u*, the IFG sub graph of (σ, μ) induced by τ is the maximal IFG sub graph of (σ, μ)that has IFG node set τ. Evidently this is just the IFG (τ, υ), where υ(*u, v*) = τ(*u*) □□τ(*v*) □□μ(*u, v*) for all *u, v* ϵ V.

A IFG G = (V,□,□) is called a strong IFG if □(x, y) = min(□(x),□(y)) □□x, y □□V. A fuzzy graph G = (V,□,□) is called a complete IFG if □(x, y) = min(□(x),□(y)) □x, y □□E and is denoted by K□. Let G = (□, □) be a fuzzy graph on V and S □□V then the IFG cardinality of S is defined as □□□□(v).

An edge e = xy of a IFG is called an effective edge If □□(x, y) = min (□(x),□(y)). N(x) = {y□V / □(x, y) = □(x) □□□(y)} is called the neighborhood of x and N[x] = N(x) □{x} is the closed neighborhood of x. Let G = (□, □) be a IFG on V. Let x,y□V, we say that x dominates y in G if □(x, y) = min (□(x),□(y)).

A vertex u of a IFG is said to be an isolated vertex if □□(u) □□□(v) > □□(uv) for all v□V \ {u}. That is N(u) = □. A path P in a IFG G = (□,□) is a sequence of distinct vertices x_0, x_1…. x_n such that 0 < □(x_{i-1}, x_i) , n ≥□i ≥□1. Here 1 ≤□n is called the length of a path. A path P is called a cycle if $x_0 = x_n$ and 3≤ n.

Two vertices are said to be strong adjacent if they are the end vertices of the same strong edge. Let G = (σ, μ) be a IFG on V. Then the strong incident degree of a IFG is defined as number of strong incident edges on a vertex vi. It is denoted as d_{EI}(vi).The minimum strong incident degree of a IFG G is defined by $\delta_{EI}(G)$ = Λ{d_{EI}(G): v□V} The maximum strong incident degree of a IFG G is defined by $\Delta_{EI}(G)$ = v{d_{EI}(G): v□V}.

A path P is called strong path if each edge in a path P is an strong edge. An effective path P is called an strong cycle if $x_0 = x_n$ and□3 ≤ n. A IFG G = (□,□) is said to be strong connected if there exists an strong path between every pair of vertices. A IFG G = (V, □, □) is said to be a complete if each vertex has an (n-1) strong incident degree. v□s International Review of Fuzzy Mathematics Volume 3 No.1 (June 2008) pp 55-68.

Intuitionistic Fuzzy Graphs

3.1 Defınıtıon

Definition 3.1.1 Minmax IFG is of the form G = (V, E), where

(i) V = {v_1, v_2, . . . v_n} such that $\mu_1 : V \rightarrow [0, 1]$ and $\nu_1 : V \rightarrow [0, 1]$ denote the degrees of membership and non - membership of the element $v_i \in V$ respectively and $1 \geq \mu_1(v_i) + \nu_1(v_i) \geq 0$, for every vi ∈ V (i = 1, 2, . . . , n).

(ii) E ⊂ V × V where $\mu_2 : V \times V \rightarrow [0, 1]$ and $\nu_2 : V \times V \rightarrow [0, 1]$ are such that

$\min[\mu_1(v_i), \mu_1(v_j)] \geq \mu_2(v_i , v_j)$

$\max[\nu_1(v_i), \nu_1(v_j)] \geq \nu_2(v_i , v_j)$

and $1 \geq \mu_2(v_i , v_j) + \nu_2(v_i , v_j) \geq 0$ for every $(v_i , v_j) \in E$.

Here the triple (v_i , μ_{1i} , ν_{1i}) denotes the degree of membership and degree of non - membership of the vertex.

The triple (e_{ij} , μ_{2ij} , ν_{2ij}) denotes the degree of membership and degree of non - membership of the edge relation $e_{ij} = (v_i , v_j)$ on V × V.

Definition 3.1.2 An IFG, G = (V, E) is said to be a semi-μ strong IFG if $\mu_{2ij} = \min(\mu_{1i} , \mu_{1j})$ for every i and j.

Definition 3.1.3 An IFG, G = (V, E) is said to be a semi-v strong IFG if $\nu_{2ij} = \max(\nu_{1i} , \nu_{1j})$ for every i and j.

Definition 3.1.4 An IFG, G = (V, E) is said to be strong IFG if $\mu_{2ij} = \min(\mu_{1i} , \mu_{1j})$ and $\nu_{2ij} = \max(\nu_{1i}, \nu_{1j})$ for all $(v_i , v_j) \in E$.

3.2 Strong Intuıtıonıstıc Fuzzy Euler Graphs

Definition 3.2.1 A walk w in a IFG (σ, μ) is a sequence of vertices x_0 , x_1…. x_n such that 0 < μ (x_{i-1},x_i) , n ≥ i ≥ 1. A vertex may appear more than once.

Definition 3.2.2 A walk w is called a closed walk if $x_0 = x_n$ and 3≤ n.

Definition 3.2.3 A walk w is called an strong walk if each edge in a walk w is an strong edge.

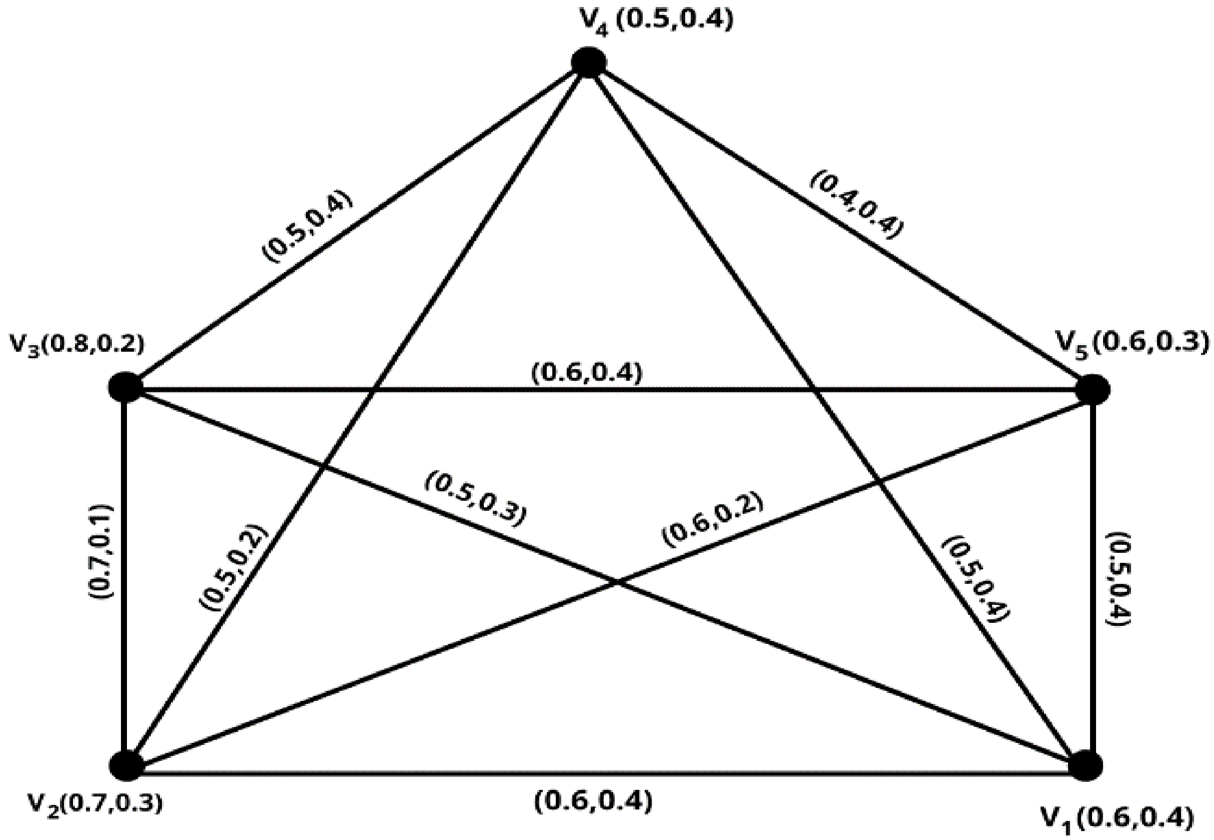
Definition 3.2.4 A strong walk w is called an strong closed walk if $x_0 = x_n$ and 3≤ n.

Definition 3.2.5 If some closed walk in a IFG contains all the edges of the IFG then the walk is called IFG Euler line and the IFG is called IFG Euler graph.

Definition 3.2.6 If some strong closed walk in a IFG contains all the edges of the IFG then the walk is called an strong IFG Euler line and the graph is called an strong IFG Euler graph.

Definition 3.2.7 If some open walk in a IFG contains all the strong edges of the IFG then the walk is called IFG unicursal line and the graph is called IFG unicursal graph.

Definition 3.2.8 In a IFG unicursal graph, each edge is an strong edge then the graph is called strong IFG unicursal graph.



Here $v_1v_2v_3v_4v_5v_1v_4v_2v_5v_3v_1$ is an Strong if Euler line, so the graph is an Strong if Euler Graph.

3.2. Some characteristics of IFG Euler Graphs

Theorem 3.2.1. A given strong connected IFG G is a strong IF Euler graph iff the strong incident degree of every vertex is even.

Proof: Let us give that G is a strong IF Euler graph. It therefore contains a strong IFG Euler line.

To trace this strong walk let us know that every time when the strong walk meeting vertex v it passes through two new edges incident on v with one we entered v and with the other exited. This is true for all intermediate vertices of a strong walk and the terminal vertex, because we exited and entered the same vertex at the beginning and end of the strong walk. Therefore the strong incident degree of every vertex is not odd.

To prove the sufficiency of the condition let the strong incident degree of every vertex of G is even. Now we construct a strong walk starting at an arbitrary vertex v and going through the edges of G such that all the edges are traced once. We will continue tracing to our possibility. Since all the strong incident degree of every vertex is of even degree, we can exit every vertex we enter; the tracing cannot stop at any vertex but v. Since v is also of even degree, we shall reach v when the tracing comes to an end. If this closed strong walk W includes all the strong edges of IFG G, G is IFG Euler graph. If not, exclude from G all the edges in W and obtain a sub graph G' formed by the remaining edges of G. Since both G and W have all their vertices of even strong incident degree, the strong incident degree of the vertices G' are also even. Moreover G' must touch W at least at one vertex 'a' because G is strong connected. Starting from 'a' we can again construct a new strong walk in graph G' since all the vertices of G' are of even strong incident degree this walk in G' must terminate at vertex 'a' but this strong walk in G' can be combined with W to form a new strong walk which starts and ends at vertex v and has more edges than W. This process can be repeated until we obtain a closed strong walk that traverses all the edges of G. Thus G is a strong IF Euler graph.

Theorem 3.2.2. In a IFG strong connected graph G with 2k odd vertices, there exist k edge-disjoint IF sub graphs such that they together contain all edges of G and that each is an strong IFG unicursal graph.

Proof: Let us suppose the odd vertices of the given IFG G be v1, v2, ……vk, w1, w2 ….wk in a arbitrary order . Adding k strong edges to G in the vertices pairs (v1, w1) (v2, w2)… (vk, wk) to form a new strong IFG G` .

Since the strong incident degree of every vertex of G` is even, G` consists of an strong IFG Euler line ρ. Now if we remove from ρ the k strong edges we just added ρ will be split into k walks, each of which is IFG unicursal line: the first removal will leave a single IFG unicursal line; the second removal will split that into two IFG unicursal lines and each successive removal will split a IFG unicursal line into two IFG unicursal lines, until there are k of them.

Theorem 3.2.3. A strong connected graph G is a strong IFG Euler graph if and only if it can be decomposed into strong cycles.

Proof: Suppose G can be decomposed into strong cycles. Since the strong incident degree of every vertex in a strong cycle is two, the strong incident degree of every vertex in G is even. Hence G is a strong IFG Euler graph.

Conversely, suppose G is a strong IFG Euler graph. Consider a vertex v1. There are at least two strong edges incident at v1. Let one of these edges is between v1 and v2. Since vertex v2 is also even, it must have at least another edge say between v2 and v3. Proceeding in this way, we arrive at a vertex that has previously been traversed, thus forming a strong cycle C. Let us remove C from G. All vertices in the remaining IFG must also be of even strong incident degree. From the remaining IFG remove another strong cycle in exactly the same way as we removed C from G. Continue this process until no edges are left.

Strong if graphs:

4.1. Some Definitions:

Definition4.1.1. A path P in a IFG G = (σ, μ) that contains every vertex of a IFG is called a IFG Hamiltonian path of G.

Definition4.1.2. A IFG cycle that contains every vertex of a IFG G is called a IFG Hamilton cycle of G

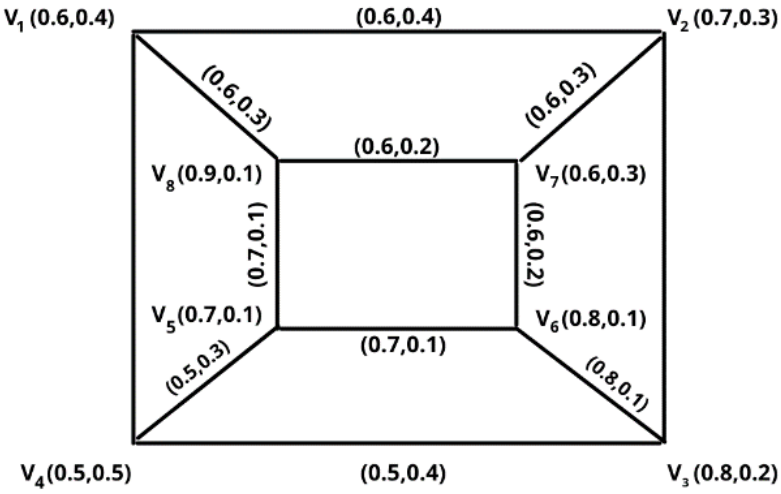
Definition4.1.3. A IFG G is called IFG Hamiltonian if it has IFG Hamilton cycle.

Definition4.1.4. A strong path P in a IFG that contains every vertex of a IFG G is called an strong Hamiltonian path of G.

Definition4.1.5. A strong IFG cycle that contains every vertex of an IFG G is called strong IFG Hamilton cycle of G.

Definition4.1.6. A IFG G is called strong IFG Hamiltonian if it has strong IFG Hamiltonian cycle.

Example 4.1.7



Here v1v2v7v6v3v4v5v8v1 is an strong Intuitionistic Fuzzy Hamilton cycle and v1v2v7v6v3v4v5v8 is strong Intuitionistic Fuzzy Hamiltonian path.

4.2. Some Characterıstıcs of Intuıtıonıstıc Fuzzy Hamıltonıan Graphs:

Theorem 4.2.1. In a IFG full complete graph where has n vertices they have (n-1)/2 edge disjoint IFG Hamilton cycle, It is even number when n is less than equal to 2.

Proof: A Intuitionistic fuzzy complete graph G of n vertices they have n (n-1) /2 strong edges and a Intuitionistic fuzzy Hamilton cycle in G consists of n strong edges. The number of edge disjoint Hamilton cycle in G cannot exceed (n-1) /2. That there are (n-1) /2 edge disjoint Intuitionistic fuzzy Hamilton cycles, when n is odd, it can be used as follows: The Intuitionistic fuzzy subgraph in figure 4.1 is a Intuitionistic fuzzy Hamilton cycle. Let keep the circle where the vertices fixed on circle, polygonal pattern can be rotated by 360 /(n-1), 2.360/ (n-1), 3.360/ (n-1) … (n-3) /2 · 360 / (n-1) strong incident degrees. Observe the each rotation produces a Intuitionistic fuzzy Hamilton cycle where the cycle has no strong edges in common with any of the previous ones. Thus we have (n-1) /2 new Intuitionistic fuzzy Hamilton cycles and each one is disjoint.

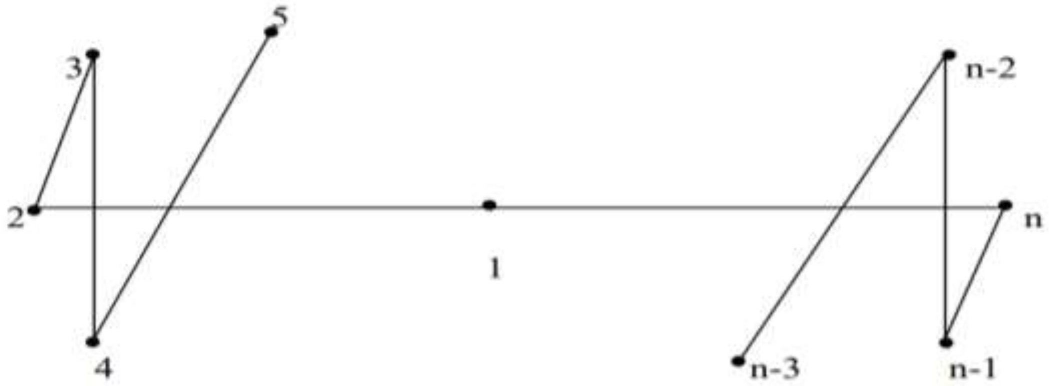


Figure 4.1.

Applıcatıons of strong ıntuıtıonıstıc fuzzy euler ıntuıtıonıstıc fuzzy hamıltonıan graphs:

Konıgsberg Brıdge Problem

The Pregel River in Konigsberg has formed Two islands A and B connected with seven bridges to one another and to the banks C and D as shown in figure 5.1.

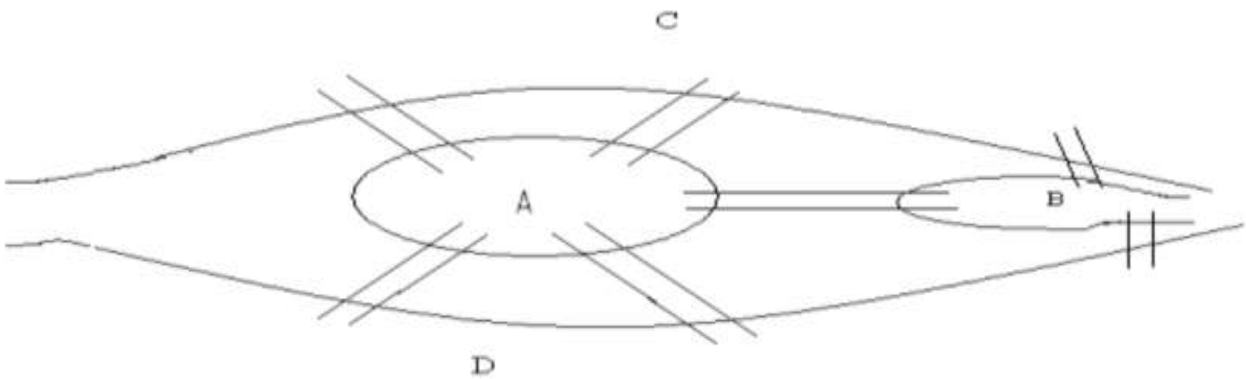


Figure 5.1.

Euler represented the problem as to start from any of the four land areas of the city A, B, C, or D, to walk over each of the seven bridges only once and return to the initial position by means of a graph as shown in figure 5.2. The land areas represent the vertices and the bridges represent the edges.

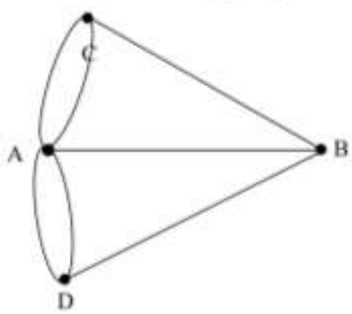


Figure 5.2.

Here we replace edge as an strong edge because each bridge and each city is equally important to us. So the new strong IFG is as shown in figure 5.3.

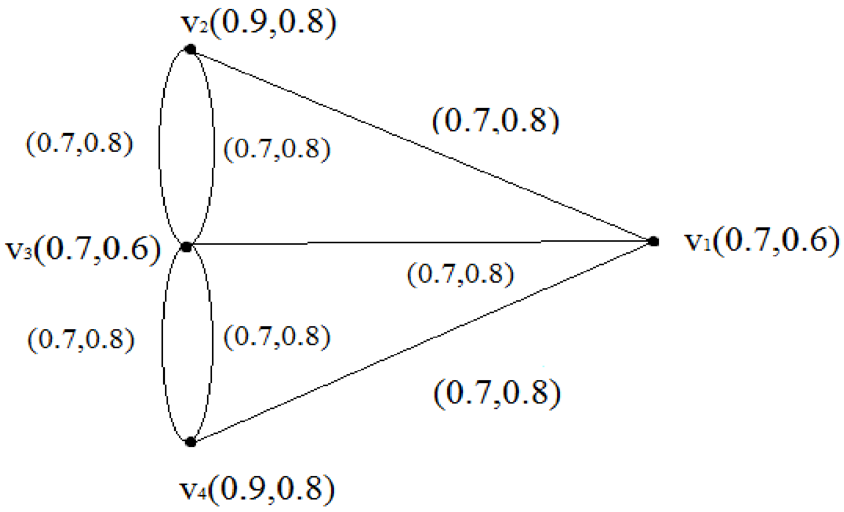


Figure 5.3.

Now looking at the graph we find that the strong incident degree of every vertex is not even. Hence it is not possible to walk over every seven bridges only once and return to the intial point. Thus, the graph is not an strong intuitionistic fuzzy Euler graph.

The Knıght's Tour Puzzle:

In the Euler Period a famous recreational problem was The Knight's tour puzzle. The only piece which makes two moves horizontally or vertically from its place followed by one move in opposite direction.

The problem can be stated as follows: Is it possible for a knight to look in on every square in a chessboard quite once and fall back to its initial square? We model this problem at IFG G, (i.e) If the knight can move from s_i to s_j in a single move if and only if the vertices of graph v_i corresponds to the squares s_i in the chessboard then two vertices vi and v_j are effective adjacent. So, the problem has been reduced in finding a Strong fuzzy Hamilton cycle in the corresponding graph.

First let us consider this problem on a 4x4 chessboard. We show that this is not possible on a 4x4 chessboard. In order to prove this we concentrate on the left side upper square and the right side lower square of the 4x4 chessboard The only way to include left side upper square in the knight's tour is to make the two moves as shown in figure 5.4.

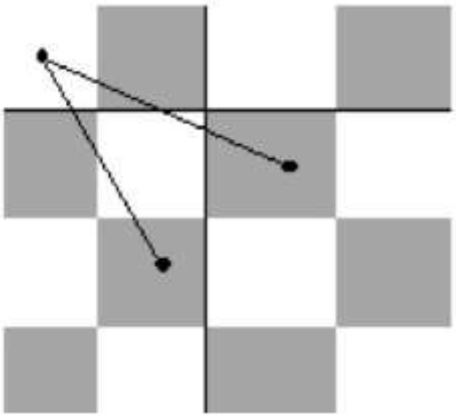


Figure 5.4

Similarly, Only by including the two moves we can insert the right side lower square as shown in figure 5.5.

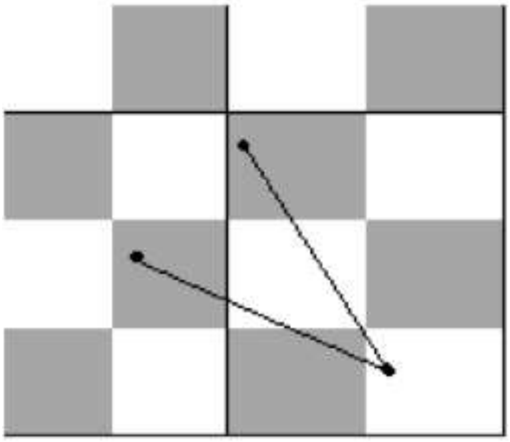


Figure 5.5

Thus, the four moves has to be included by the Knight's tour as shown in figure 5.6. Since these already form a Intuitionistic fuzzy cycle, a full tour is not possible to be included as a part. Hence, On a 4x4 chessboard knight's tour is not possible

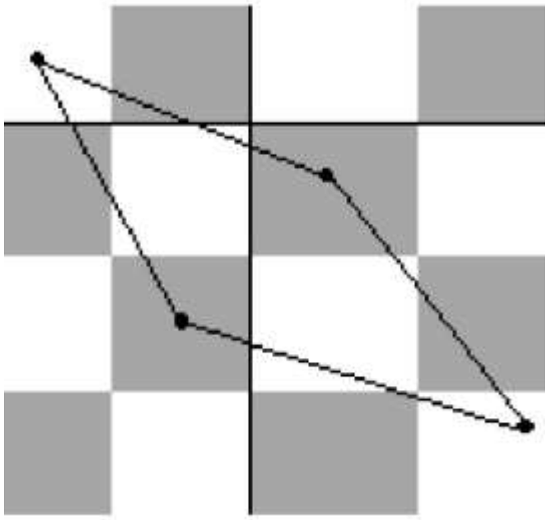


Figure 5.6

Next, let us consider this problem on a 5x5 chessboards. Here also such a move is not possible. A non-fuzzy Hamiltonian is a fuzzy bipartite graph with an odd number of vertices which corresponds to IFG. This can be generalized and we may conclude that a knight's tour with an odd number of squares is not possible on a chessboard. Consider a standard 8x8 chessboard. The corresponding fuzzy graph with order 64 and size 168 contains several Strong fuzzy Hamilton cycles, one of which is shown in figure 5.7.

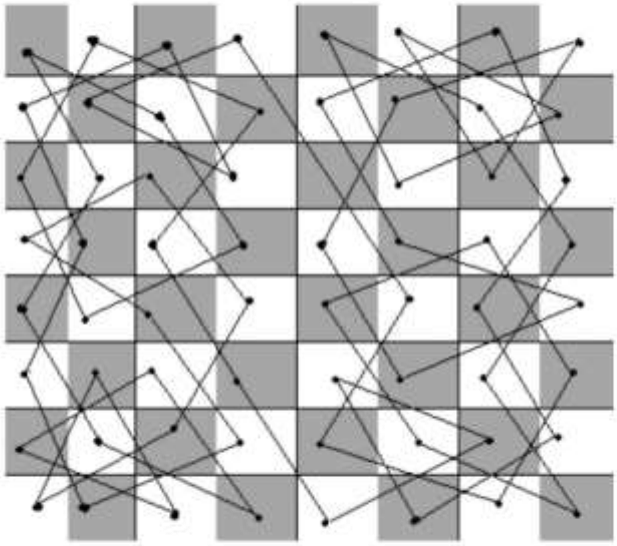


Figure 5.7

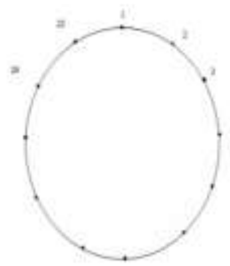
SEATING ARRANGEMENT PROBLEM

Let us assume that a seminar is arranged for a few days, choosing one student representative from each college in Trichy. Let us assume that there are 25 colleges in Trichy. In each meal (breakfast, lunch and dinner) , each member has different neighbours when they decided to sit around a table in such a way. This kind of arrangement can last for how many days?

Now we consider this problem in Intuitionistic fuzzy graph theory. We represent each member of the group by a vertex and if a member m1 is permitted to sit next to another member m2, we introduce an effective edge between m1 and m2. Therefore, the IFG is a complete Intuitionistic fuzzy graph since each member is permitted to sit next to one another.

Here, we use the concept of Strong fuzzy Hamilton cycles. At the first meal, they can sit in any order. Clearly, this is a Strong fuzzy Hamilton cycle of the corresponding graph. Each member will have different neighbours in the second meal if they sit in such a way, then clearly, this is another Strong fuzzy Hamilton cycle of the Strong fuzzy graph with completely different from the previous one with set of edges. (i.e) These two Strong fuzzy Hamilton cycles are edge-disjoint.

Thus, our problem is reduced to finding the number of edge-disjoint Strong fuzzy Hamilton cycle in the intuitionistic fuzzy graph. In our problem, we can find 12 edge-disjoint Strong fuzzy Hamilton cycle and this can be obtained by using the Theorem 4.7.



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