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On Generalizd A Regular-Closed Set in Nano Topoloical Spaces

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Abstract

In this article, we putforth the concepts of generalized α regular-closed sets in nano topological spaces. Furthermore, we study basic properties of generalized α regularcs and its relations with other nano sets. Also we discuss about generalized α regular-open sets and generalized α regular-neighbourhoods in nano topological spaces.

Keywords: Ngαrcs , Ngcs , Ng*cs, Sgbcs, Ngαros.

Introduction

Lellis Thivagar putforth the concept of Nano topological spaces which is defined in terms of lower approximation, Upper approximation and boundary region. Bhuvaneswari introduced and investigated nano g-closed sets in nano topological spaces. S.Sekar and G.Kumar putforth the concept of *gαr* closed sets in topological spaces. The main aim of this paper is to introduce the concept of new form of generalized closed set called generalized *α*- regular closed set in nano topological spaces. Its relationship with other generalized closed sets is also discussed.

Preliminaries

Definition 2.1: Let P $\subseteq (U,\tau_R(x))$, *then it* is said to be

I. *Nα open set* if P$\subseteq$*Nint(Ncl(Nint (P)))*.
II. Nano generalized closed set (briefly *Ngc*) if *Ncl(P)*$\subseteq$ *Q* wherever P$\subseteq$*Q* and Q is nano open.
III. Nano weakly closed set (briefly *Nwc*) if *N(P)*$\subseteq$*Q* wherever P$\subseteq$ *Q* and Q is nano semi open.
IV. Nano generalized*cs (briefly *Ng*c*) if *Ncl(P)*$\subseteq$*Q* wherever P$\subseteq$*Q* and Q is *Ng* open.
V. Nano generalized *α*cs (briefly *Ngαc*) if *Nαcl(P)*$\subseteq$*Q* wherever P$\subseteq$*Q* and Q is *Nα* open.
VI. Nano generalized bcs (briefly *Ngbc*) if *Nbcl(P)*$\subseteq$*Q* wherever P$\subseteq$*Q* and Q is nano open in $(U,\tau_R(x))$.
VII. An Nano *α* generalized closed set (briefly *Nαgc*) if *Nαcl(P)*$\subseteq$*Q* wherever *A*$\subseteq$*Q* and Q is nano open in $(U,\tau_R(x))$.
VIII. Nano semi generalized *b* closed set (briefly *Nsgbc)* if *Nbcl(P)*$\subseteq$*Q* wherever P$\subseteq$*Q* and Q is nano semi open in $(U,\tau_R(x))$.
IX. Nano generalized *αb*cs (briefly *Ngαbc*) if *Nbcl(P)*$\subseteq$*Q* wherever P$\subseteq$*Q* and Q is *Nα* open in $(U,\tau_R(x))$.
X. Nano regular generalized *b*cs (briefly *Nrgbc*) if *Nbcl(P)*$\subseteq$*Q* wherever P$\subseteq$*Q* and Q is nano regular open in $(U,\tau_R(x))$.
XI. Nano generalized pre regular closed set (briefly *Ngprc*) if *Npcl(P)*$\subseteq$*Q* wherever P$\subseteq$*Q* and Q is nano regular open in $(U,\tau_R(x))$.

Nano Generalized α Regular-Closed Sets

Here, nano generalized α regular closed set introduced and its properties are discussed.

Definition 3.1: Let P $\subseteq(U,\tau_R(x))$ *,then it is said to be* Nano generalized α regular closed set (briefly *Ngαr*cs)if *Nαcl(P)*$\subseteq$*Q* wherever P$\subseteq$*Q* and Q is nano regular open in $(U,\tau_R(x))$.

Theorem 3.1: Each Ncs is *Ngαr*cs.

Proof. Suppose P is any closed set in $(U,\tau_R(x))$ such that P$\subseteq$*Q*, where G is Nro.Since *Nαcl(P)* $\subseteq$*Ncl(P)= H*. Hence *Nαcl(P)*$\subseteq$*Q*. Therefore P is *Ngαr*cs in $(U,\tau_R(x))$.

The reverse implication does not imply,

Example 3.1.

Let U={x, y, z, w}, U/R={{x, z},{y},{w}}, X={x,y} and τ_R(x)={φ,{y},{x, y, z},{x, z},}.Then **{x, y, z} is a *Ngαr* closed but not a Ncs.**

Theorem 3.2: Each *Ngα*cs is *Ngαr*cs.

Proof. Suppose P is any *Ngα*cs in *(U,τ_R(x))*. Let P*⊆Q* and Q is Nros. Then Q is *Ngα* open. Hence *Nαcl(H)⊆*G. Hence P is *Ngαr* cs.

The reverse implication does not imply,

Example 3.2.

Let U={x, y, z, w}, U/R={{x, z},{y},{w}},X={x, y} and τ_R(x)={φ,{y},{x, y, z},{x, z},U}.Then **{x, y} is a *Ngαr*cs but not a *Ngα*cs.**

Theorem 3.3: Each *Nαg*cs is *Ngαr*cs.

Proof. Suppose *H* is any *Nαg*cs in *(U,τ_R(x))*. Let *H⊆G* and G is Nros. Then G is open. Hence *Nαcl(H)⊆*G*.* Hence *H* is *Ngαr*cs.

The reverse implication does not imply,

Example 3.3.

Let U={x, y, z, w}, U/R={{x, z},{y},{w}},X={x, y} and τ_R(x)={φ,{y},{x, y, z},{x, z},U}.Then **{y, z} is a *Ngαr*cs but not a *Nαg*cs.**

Theorem 3.4: Each *Ngαr*cs is *Ngpr*cs.

Proof. Suppose P is *Ngαr*cs in *(U,τ_R(x))* and Q be any Nros containing P.Then *Npcl(P) ⊆Nαcl(P)⊆Q*.Hence *Npcl(P)⊆Q.* Hence P is *Ngpr*cs.

The reverse implication does not imply,

Example 3.4.

Let U={x, y, z, w}, U/R={{x, z},{y},{w}},X={x, y} and τ_R(x)={φ,{y},{x, y, z},{x, z},U}. Then **{x} is a *Ngprcs* but not a *Ngαrc.***

Theorem 3.5. Each *Ngc*s is *Ngαr*cs.

Proof. Suppose *P* is any *Ng*cs in *(U,τ_R(x))* and Q be any Nros containing P. Since every Nros is nano open, *Nαcl(P) ⊆Ncl(P)⊆Q*. Hence *Nαcl(P)⊆Q.* Hence Q is *Ngαr*cs.

The reverse implication does not imply,

Example 3.5.

Let U={x, y, z, w}, U/R={{x, z},{y},{w}},X={x, y} and τ_R(x)={φ,{y},{x, y, z},{x, z},U}. Then **{x, y} is a Ngαrcs but not a Ngcs**.

Theorem 3.6. Each *Nw*cs is *Ngαr*cs.

Proof. Supose P is any *Nw*cs in *(U,τ_R(x))* and Q be any Nros containing P. As every Nros is nano semi open, *Nαcl(P)⊆Ncl(P) ⊆Q*. Therefore *Nαcl(P)⊆Q*. Hence P is *Ngαr*cs.

The reverse implication does not imply,

Example 3.6.

Let U={x, y, z, w}, U/R={{x, z},{y},{w}},X={x, y} and $\tau_R(x)$={φ,{y},{x, y, z},{x, z},U}.Then **{y, w} is a *Ngαrc* but not a *Nwc*.**

Theorem 3.7. Each *Ng**cs is *Ngαr*cs.

Proof. Suppose P is any *Ng**cs in $(U,\tau_R(x))$ and Q be any Nros containing H. Since every regular open is *Ng* open, *Nαcl(P)⊆Ncl(P)⊆Q*. Therefore *Nαcl (P) ⊆Q*. Hence P is *Ngαr*cs.

The reverse implication does not imply,

Example 3.7.

Let U={x, y, z, w}, U/R={{x, z},{y},{w}},X={x, y} and $\tau_R(x)$={φ,{y},{x, y, z},{x, z},U}.Then **{x, y, z} is a *Ngαr* closed but not a *Ng** closed.**

Characteristics of Ngαr Closed Sets

Theorem 4.1. If P and *K* are *Ngαr*cs in $(U,\tau_R(x))$ then P ∪*K* is *Ngαr*cs in $(U,\tau_R(x))$.

Proof. Suppose *H* and *K* are *Ngαr*cs in $(U,\tau_R(x))$ and G be any Nros such that P∪*K⊆Q*. Therefore *Nαcl(P)⊆Q, Nαcl(K)⊆Q.* Thus *Nαcl(P∪K)=Nαcl(P)∪Nαcl(K)⊆Q.* Therefore P∪*K* is *Ngαr*cs in $(U,\tau_R(x))$.

Theorem 4.2. If a set P is *Ngα*cs then *Nαcl(P)−P* contains no non empty Nrcs.

Proof. Suppose K is a Nrcs in $(U,\tau_R(x))$ such that K⊆Nαcl(P)−P. Then P⊆$(U,\tau_R(x))$−K. Since P is Ngαrcs and $(U,\tau_R(x))$−K is Nro then Nαcl(P)⊆$(U,\tau_R(x))$−K. (i.e.) K⊆$(U,\tau_R(x))$–Nαcl(P). So K ⊆ (($U,\tau_R(x)$) – Nαcl(P))∩(Nαcl(P) −P) . Therefore K=ϕ

Theorem 4.3. If P*⊆Y⊆* $(U,\tau_R(x))$ and For, if P is *Ngαr*cs in $(U,\tau_R(x))$ then P is *Ngαr*cs relative to *Y*.

Proof. Given that P*⊆Y⊆* $(U, \tau_R (x))$ and P is *Ngαrc*s in $(U, \tau_R(x))$.To prove that P is *Ngαrc*s relative to *Y*. Let us assume that P*⊆Y*∩*Q*, where Q is Nro in $(U, \tau_R (x))$. Since P is *Ngαr* cs, P*⊆Q* implies *Nαcl (P) ⊆Q*. It follows that *Y*∩*Nαcl (P) ⊆Y*∩*Q*. That is P is *gαr*cs relative to*Y*.

Theorem 4.4. For *x*∈$(U,\tau_R(x))$, then the set $(U,\tau_R(x))$*–{x}* is a *Ngαr*cs or Nro.

Proof. Suppose that $(U,\tau_R(x))$ *– {x}* is not Nro, then $(U,\tau_R(x))$ is the only Nros containing $(U,\tau_R(x))$ *- {x}*. (i.e*.) Nαcl((U,τ_R(x))–{x})⊆(U,τ_R(x))*. Then $(U,\tau_R(x))$*–{x}* is *Ngαrc* in $(U,\tau_R(x))$.

Remark 4.1. The intersection of any two subsets of *Ngαr*cs in $(U,\tau_R(x))$ is not *Ngαr*cs in $(U,\tau_R(x))$.

Theorem 4.5. If P is both Nro and *Ngαr*cs in $(U,\tau_R(x))$, then P is *Nα*cs.

Proof. As P is Nro and *Ngαr* closed in $(U,\tau_R(x))$, *Nαcl(P)⊆P.* But always P*⊆Nαcl(P).* Therefore P=*Nαcl(P).* Thus *P is Nα*cs.

Note 4.1. *Ngs*cs and *Ngαrc*s are independent to each other as seen from the following examples.

Example 4.1

Let U={x, y, z, w}, U/R={{x, z},{y},{w}},X={x, y} and $\tau_R(x)$={φ,{y},{x, y, z},{x, z},U}.Then **{x} is a *Ngs*cs but not *Ngαr*cs.** Also **{x, y, z} is a *Ngαr* closed but not a *Ngs* closed.**

Note 4.9. *Ngb*cs and *Ngαrc*s are independent to each other as seen from the following examples.

Example 4.2.

Let U={x, y, z, w}, U/R={{x, z},{y},{w}},X={x, y} and $\tau_R(x)$={φ,{y},{x, y, z},{x, z},U}.Then the set **{x} is a *Ngb* closed but not a *Ngαr* closed.** Similarly the set **{x, y, z} is a *Ngαr* closed but not a *Ngb* closed.**

Note 4.2. *Nsgb*cs and *Ngαr*cs are independent to each other as seen from the following examples.

Examples 4.3.

Let U={x, y, z, w}, U/R={{x, z},{y},{w}},X={x, y} and τ_R(x)={φ,{y},{x, y, z},{x, z},U}.Then **{z} is a *Nsgb* closed but not *Ngαr* closed.** Similarly **{y, z} is a *Ngαr* closed but not *Nsgb* closed.**

Nano Generalised α Regular Open Sets and Nano Generalized α Regular Neighbourhoods

Here, we putforth Nano generalized α regular open sets(briefly *Ngαro*) and nano generalized α regular nbd (briefly *Ngαr* nbd) in topological spaces by using the notions of *Ngαr* open sets and study some of their properties.

Definition 5.1. Let P $\subseteq(U,\tau_R(x))$ *th*en it is said to be Nano generalized α regular open set (briefly *Ngαris)* if A^c is *Ngαr* closed in $(U,\tau_R(x))$. We denote the family of all *Ngαros* in $(U,\tau_R(x))$,by *Ngαr* $O(U,\tau_R(x))$.

Theorem 5.1. Let P and *K* be *Ngαro*s in $(U,\tau_R(x))$. Thus P∩*K* is *Ngαros* in $(U,\tau_R(x))$.

Proof. Let P and *K* are *Ngαros* in $(U,\tau_R(x))$. Then P^c and K^c are *Ngαr*cs in $(U,\tau_R(x))$. $P^c \cup K^c$ is also *Ngαr*cs in $(U,\tau_R(x))$.(i.e.) $H^c \cup K^c = (P\cap K)^c$ is a *Ngαrc*s in $(U,\tau_R(x))$. Hence P∩*K, Ngαros* in $(U,\tau_R(x))$.

Theorem 5.2. Let *Nint* $(P)\subseteq K\subseteq P$ and if P is *Ngαro* in $(U,\tau_R(x))$, then *K* is *Ngαro* in $(U,\tau_R(x))$.

Proof. For, if *Nint* $(P) \subseteq K\subseteq P$ and if P is *Ngαro* in $(U,\tau_R(x))$, then $P^c\subseteq K^c\subseteq Ncl(P^c)$Since P^c is *Ngαr* closed in $(U,\tau_R(x))$. Hence *K* is *Ngαr* open in $(U,\tau_R(x))$.

Definition 5.2. Let *x* be a point in $(U,\tau_R(x))$. A subset *N* of $(U,\tau_R(x))$is said to be *Ngαr*nbd of *x* iff there exists a *Ngαros* Q such that *x in QN.*

Definition 5.3. Let N$\subseteq(U,\tau_R(x))$ is called a *Ngαr*nbd of P$\subset(U,\tau_R(x))$ iff there exists a *Ngαros* Q such that $P\subset Q\subset N$.

Theorem 5.3. Each nbd *N* of *x* in $(U,\tau_R(x))$ is called a *Ngαr*nbd of *x*.

Proof. Suppose *N* be a nbd of a point *x* in $(U,\tau_R(x))$. To prove that *N* is a *Ngαr*nbd of *x*.By definition of nbd, there exists a Nos Q such that *x* in $Q\subset N$. Thus *N* is a *Ngαr*nbd of $(U,\tau_R(x))$.

Remark 5.1. In particular, a *Ngαr*nbd of *x* in $(U,\tau_R(x))$ need not be a nbd of *x* in $(U,\tau_R(x))$.

Remark 5.2. The *Ngαr*nbd *N* of *x* in $(U,\tau_R(x))$ need not be a *Ngαr* open in $(U,\tau_R(x))$.

Theorem 5.4. Let $N \subseteq(U,\tau_R(x))$ is *Ngαr* open, then *N* is *Ngαr*nbd of each of its points.

Proof. Let *N* is *Ngαr* open. Suppose $x \in N$. We prove that *N* is *Ngαr*nbd of *x*. For *N* is a *Ngαr* open set such that *x* in $N \subset N$. Since *x* is an arbitrary point of *N*, it follows that *N* is a *Ngαr*nbd of each of its points.

Theorem 5.5. Suppose $(U,\tau_R(x))$ be a nano topological space. If *F* is *Ngαr* closed subset of $(U,\tau_R(x))$and *x* in F^c. Then there exists a *Ngαr* nbd *N* of *x* such that $N \cap F = \varphi$.

Proof. Suppose *F* is *Ngαr* closed subset of $(U,\tau_R(x))$and *x* in F^c. Suppose F^c is *Ngαr* open set of $(U,\tau_R(x))$. Hence by Theorem, F^c contains a *Ngαr*nbd of each of its points. Hence there exists a *Ngαr*nbd *N* of *x* such that $N \subset F^c$. (i.e.) $N \cap F = \varphi$.

Definition 5.4. Let *x* be a point in $(U,\tau_R(x))$. The set of all *Ngαr* nbd of *x* is called the *Ngαr*nbd system at *x*, and is denoted by *Ngαr* − *N(x).*

Theorem 5.6. Suppose x be a point in a nano topological space and each $x \in (U,\tau_R(x))$, Let *Ngαr* − $N(U,\tau_R(x))$ be the collection of all *Ngαr*nbd of *x*. Then the following holds.

1. *∀x ∈ (U,τ_R(x)), Ngαr − N(x) φ.*
2. ***(N ∈ Ngαr − N(x) ⇒ x ∈ N.***
3. ***N ∈ Ngαr − N(x), M ⊃ N ⇒ M ∈ Ngαr − N(x).***
4. ***N ∈ Ngαr -N(x), M ∈ Ngαr- N(x) ⇒ N ∩ M ∈ Ngαr − N(x), if finite intersection of Ngαros is Ngαr open.***
5. ***N ∈ Ngαr − N(x) ⇒ there exists M ∈ Ngαr − N(x) such that M ⊂ N and M ∈ Ngαr − N(y) for every y ∈ M.***

Proof.

1. As *(U,τ_R(x))* is *Ngαros* it is a *Ngαr*nbd of each *x ∈ (U,τ_R(x))*. Hence there exists at least one *Ngαr*nbd (namely *(U,τ_R(x))*) for each *x ∈ (U,τ_R(x))*. Therefore *Ngαr − N(x) ≠ φ* for each *x ∈ (U,τ_R(x))*.
2. Let *N ∈ Ngαr − N(x),* then *N* is *Ngαr*nbd of *x*. By Defn. of *Ngαr*nbd, *x ∈ N*.
3. Suppose *N ∈ Ngαr − N(x)* and *M ⊃ N*. Then there is a *Ngαros Q* such that *x ∈ Q ⊂ N*. Since *N ⊂ M, x ∈ Q ⊂ M* and so *M* is *Ngαr*nbd of *x*. Hence *M ∈ Ngαr − N(x).*
4. (iv) Suppose *N ∈ Ngαr − N(x), M ∈ Ngαr - N(x).* Then by Defn. of *Ngαr*nbd, there exists *Ngαr* open sets Q1 and Q2 such that *x ∈ Q1 ⊂ N* and *x ∈ Q2 ⊂ M*. Hence *x∈Q1∩Q2⊂N∩M* (1) As Q*1 ∩ Q2* is a *Ngαr* open set, it follows from (1) that *N ∩ M* is a *Ngαr*nbd of *x*. Thus *N ∩ M* in *Ngαr − N(x).*
5. (V)Suppose *N* in *Ngαr − N(x).* Then there is a *Ngαr* open set *M* such that *x ∈ M ⊂ N*. As *M* is *Ngαr* open set, it is *Ngαr*nbd of each of its points. Thus *M ∈ Ngαr − N(y)* for each *y ∈ M*.

Conclusion

Here, we give notion of new class of set namely *Ngαr*cs in Nano topological spaces. Basic properties and characteristics of *Ngαrc*s are being discussed. Also the relationship between *Ngαr* closed with other generalized closed sets are investigated further we have putforth the concepts of *Ngαr* open set and *Ngαr*nbd.

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