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Pseudo Integral near Subtraction Semi groups

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Abstract: In this research article we introduce the concept of pseudo integral near_ subtraction semi group and studied about A-divisor ideals and A-potent near subtraction semi groups, relation between prime, completely prime ideals A-Potent ideals in near subtraction semi group.

Key Words: Ideal, A-divisor, A-Potent, Rees quotient, near subtraction semigroup.

1. Introduction:

Abbott introduced subtraction algebra in 1967. By using the notion of subtraction algebra in 1992 introduced subtraction semi group by schein in the year 1992. In 2007 Deena introduced near subtraction algebra. Jun et al., studied about ideals in near subtraction algebra and developed some basic properties. The main theme of this paper is to study about pseudo integral near subtraction semi groups.

2. Preliminaries:

DEFINITION 2.1

Let S be a non empty set along with a binary operation ‘ – ‘ is known as a *subtraction algebra* if it satisfies the following axioms

1) a - (b - c) =a ,
2) a – (a –b) = b – (b –a)
3) (a – b) –c = (a – c) – b ∀ a, b, c ∈ S.

In a subtraction algebra:

1) a – 0 = a and 0 – a = 0, 2) (a – b) – a = 0, 3) (a – b) – b = a – b , 4) (a –b) – (b – a) = a – b.

DEFINITION 2.2: Let S be a nonempty set along with two binary operations ‘-‘ and ‘.’ is known as to be a ***near_subtraction semi group***if the below axioms are to be satisfied :

1) S forms a subtraction algebra with the binary operation ‘-‘
2) S forms is a semi group with the binary operation ‘.’and
3) (a – b)c = a.c – b.c∀ a*, b, c*∈ S.

REMARK 2.3: Assume that Γ is the subtraction algebra. Then all the mappings of Γ into Γ of the set M(Γ) is a near _subtraction semi group under the compositions of mappings and point wise subtraction . M(Γ) is not a subtraction semi group.

3. Pseudo Integral Near_Subtraction Semi group:

DEFINITION 3.1 : Let the algebraic structure (S, -, .) be a near_ subtraction semi group. And let *I* be a non empty subset of S such that a –b ∈*I* and for every a ∈*I*, b ∈S is known as a ***left ideal*** of S if a*i* – a(b – *i*) ∈*I* for all a, b ∈*I* and *i*∈*I*.

DEFINITION 3.2 : LetA be an ideal of a near_subtraction semi group S is known as to be a P***seudo symmetric*** then a*,b*∈S, ab∈A which implies that axb∈A ∀ x∈S.

EXAMPLE 3.3 : Let S = { x*, y,z*} in which ‘-‘ and ‘.’ are defined as follows:

.	x	y	z
x	x	x	x
y	x	x	x
z	x	y	z

-	x	y	z
x	x	x	x
y	y	x	y
z	z	z	x

Then (S, -, .) is a near_subtraction normal semi group. All the ideals of S are {x}, {x, y}, {x, y, *z*} which are Pseudo symmetric.

DEFINITION 3.4: All the intersection of ideals of a near_ subtraction semi group S is known as to be a ***kernel*** of S. significantly it is represented by*Ker*.

EXAMPLE 3.5: In example 3.3. Let X = {x}, Y = {x, y}, Z = {x, y, z}. Then the sets X, Y, Z are ideals of S. Here X = X ∩ Y ∩ Z = {x} is the ideal which is the kernel of S.

DEFINITION 3.6 :A near_subtraction semi group S with nonempty kernel *Ker* is known as to be a***Pseudo integral near_ subtraction semi group*** if *Ker* is a Pseudo symmetric ideal of S.

EXAMPLE 3.7 : In example 3.5, the ideal X is the kernel of S which is the pseudo symmetric ideal of S. Therefore the near_subtraction semi group S is a Pseudo integral near_subtraction semi group.

LEMMA 3.8 : Every Pseudo symmetric near_subtraction semi group with nonempty kernel is a Pseudo integral near_subtraction semi group.
THEOREM 3.9: If S is a near_subtraction semi group with the kernel is empty then S^0 is a Pseudo integral near_subtraction semi group.
Proof **:**Since S has empty kernel, then the kernel of S^0 is {0}. Suppose xy= 0.
Then x = 0 or y= 0 and hence xS^0y= 0. Thus {0} is a Pseudo symmetric ideal.
Then the near_subtraction semi group S^0 is a Pseudo integral near_ subtraction semi group.

DEFINITION 3.10 : Assume that *I* be an ideal of a near_subtraction semi group S. And let $s \in S$ is known as to be a ***left*(*right*) *I-divisor*** if there is any other element $y \in S\backslash I$ such that s*y*(*ys*)∈*I*. if *s* is a left *I*-divisor and a right *I*-divisor element, then it is known as ***I-divisor***. Let *I*is an ideal of a near_subtraction semi group S. And the ideal*J* in S is said to be a ***left*(*right*) *I-divisor ideal***provided every element of *J* is a left(right) *I*-divisor element and *J* is ***I-divisor ideal***provided if it is both a right *I*-divisor ideal and a leftt*I*-divisor ideal of a near_subtraction semi group S.

THEOREM 3.11 : If S is a near_subtraction semi group with non empty kernel *Ker* then S has no non-trivial *Ker*-divisor elements, such that S is a Pseudo integral near_subtraction semi group.

Proof **:** Let *xy*∈*Ker* and *s* ∈S. Now suppose if possible *x*∉*Ker*, *y* ∉*Ker*. Since *xy*∈ *Ker*, we know that *x* is a non-trivial *Ker*-divisor in S. This is not true which contradicts to our supposition. Our supposition is wrong .Then either *x*∈ *Ker* or *y*∈ *Ker.* which implies that
xsy∈ *Ker* . Then *Ker* is a Pseudo symmetric ideal. Hence S is pseudo integral near_subtraction semi group.

DEFINITION 3.12: Assume that *I* be any ideal of a near_subtraction semi group S. Put S/*I* = S*I*∪ {*I*}. Now we define a function **.** from S/*I* × S/*I* into S/*I* as follows. Let *a*, *b*∈ S/*I*. (1) if *a* = I or *b* = I then we define *a . b* = *I*, (2) if *a*, *b*∈ S*I*, *ab*∈*I* then we define *a*.*b* = I, (3) if *a*, *b*∈ S*I*, *ab*∉*I* then define *a.b* = *ab*. Then S/*I*is a near_subtraction semi group. The near_subtraction semi group S/*I* is known as ***Rees Quotient near_subtraction semi group*** of S over an ideal *I.*

LEMMA 3.13 : Assume that S be a near_subtraction semi group. An ideal *I* of S is a Pseudo symmetric ideal if and only if the Rees Quotient near_subtraction semi group S/*I* is a Pseudo integral near_subtraction semi group.

DEFINITION 3.14: Let *I* be an ideal of a near_subtraction semi group S is known as to be a ***completely prime ideal*** if *x, y*∈ S, *xy*∈*I*, implies either *x*∈ *I*, or *y*∈ *I*,.

DEFINITION 3.15:: Let *I* be an ideal of a near_subtraction semi group S is known as to be a ***prime ideal*** of S if X,Y are ideals of S, XY ⊆*I* implies either X⊆*I*orY⊆ *I*

THEOREM 3.16 : Let S be a near_subtraction semi group and every prime ideal P which is minimal relative to containing a Pseudo symmetric ideal *I* in a near_ subtraction semi group S is a completely prime.

THEOREM 3.17: Let S be a near_subtraction semi group every minimal prime ideal in a Pseudo integral near_subtraction semi group is completely prime.
Proof **:**Let S be a Pseudo integral near_ subtraction semi group then kernel *Ker* is Pseudo symmetric ideal. Assume that P be a minimal prime ideal in S. Clearly *Ker*⊆ P. Therefore P is a minimal ideal relative to containing a Pseudo symmetric ideal *Ker*. By the theorem 3.16, P is a completely prime.

DEFINITION 3.18 : Let us assume that *I* be an ideal of a near_subtraction semi group S. Let s be an element of S is said to be an ***I-potent*** if then there exists a natural number *n* such that $s^n \in I$.

DEFINITION 3.19 : An ideal B of S is said to be an ***I-potent ideal*** provided that there exists a natural number *n* such that $B^n \subseteq I$.

REMARK 3.20 : We find the following notation is more useful:
$N_o(A)$ = The set of all *I*-potent elements in X.
$N_1(A)$ = The largest ideal contained in $N_o(A)$.
$N_2(A)$ = The union of all *I* -potent ideals.

THEOREM 3.21: Let S be a Pseudo integral near_subtraction semi group every prime ideal contains all K-potent elements and hence $N_0(K) \subseteq P^*$. Where P^* is the intersection of all prime ideals.

THEOREM 3.22 : If S is a near_subtraction semi group and N is a maximal ideal in S containing a pseudo symmetric ideal *I*, then N contains all *I*-potent elements in S or S\\N is a singleton which is *I*-potent.

Proof **:**If possible suppose that N which not having all *I*-potent elements.
Let s be an element of S\\N which any *I*-potent element and *t* be any element in S\\N.
But given that N is a maximal ideal, N ∪ < s > = S = N ∪< t >⇒< s > = < t >.
Since *b*∉ N , we have that *t* ∈ < s >. Let us assume that *n* be the least positive integer such that $s^n \in I$. Since *I* is a Pseudo symmetric ideal then *I* is a semi Pseudo symmetric ideal and hence $<s>^n \subseteq I$. Therefore $t^n \in I$ and hence t is*I*-potent.
Similarly we can also show that if *m* is the least positive integer such that $t^m \in I$, then $s^m \in$ I. Therefore there exists a natural number *p* such that $s^p \in I$ and $s^{P-1} \notin I$ for all *s* ∈ S\\N.
Let s, t∈ S\\N. Since N is maximal ideal, we have s = atb for some a*,b*∈ S^1.
Now since *I* is a pseudo symmetric ideal, we have $(st)^{p-1} = (st)^{p-2}st = (st)^{p-2}atbs \in I$
⇒*st*∉ S\\N. ∴st∈ N. Suppose s ≠ t. Then one of a*,b*is not an empty symbol say a.
If a∈N then s∈N. If *a* ∈ S\\N then a t∈ N and hence s∈N.
In all the cases our assumption is wrong. Hence s = t..

THEOREM 3.23 :**If N is a maximal ideal of S where S be a Pseudo integral near_subtraction semi group. then either N contains all *Ker*-potent elements of S or S\\N is singleton which is *Ker*-potent.**
Proof:Let S be a Pseudo integral near _subtraction semi group then which implies The non-empty kernel *Ker* of S is a Pseudo symmetric ideal. Since N is a maximal ideal in Swhich implies that *Ker*⊆N. Therefore

N is a maximal ideal in S containing a Pseudo symmetric ideal Ker. By theorem 3.22, N contains all *Ker*-potent elements in S or S\N is singleton which is *Ker*-potent.

THEOREM 3.24: **If P is a max. near_ subtraction sub semi group of a pseudo integral near _subtraction semi group S such that** $P \cap Ker = \varnothing$**, then S\P is a mini. prime ideal in S.**

Proof **:** Let *y*, *z*∈S\P and let P^* be the near subtraction sub semi group of S generated by P ∪ {*y*, *z*}. Since P^* contains P properly, we have $P^* \cap Ker \neq \varnothing$. So there exists $x_1x_2....x_n \in P$ such that $x_1y^{i_1}x_2y^{i_2}....x_ny^{i_n} \in Ker$. Put *x* = $x_1x_2....x_n$. Clearly *x*∈P.

Since *Ker* is a pseudo symmetric ideal, we obtain $(xy)^{i_1+i_2+....+i_n} \in Ker,$ by suitable intersection of some elements. Thus *xy* is *Ker*-potent. Therefore for s ∈S, *xsy* is *Ker*-potent.
If, *xsy*∈P, then we have P ∩ Ker ≠ ∅. It is a contradiction. So *xsy*∈ S\Ker.
If *y* – *z* ∈ P, *sy*∈ P, thensince *y*, *z* ∈P for all *s*∈S, thus *xsy*∈P. This is a contradiction. Thus for all *s*∈ S, *y* – *z*∈ S\P, *sy*∈S\P for all *s*∈S. Similarly we can show that *ys*∈ S\P for all *s* ∈S. Therefore S\Pis an ideal in S. Since P is a near subtraction sub semi group of S, S\P is a completely prime ideal and hence S\P is a prime ideal.
Now we show that S\P is a maximal ideal. Let P be any prime ideal of S such that P ⊆ S\P. Let *y* ∈ S\P. Then as above there is an element *x*∈P such that *xsy* is *Ker*-potent for all *s*∈S. Since P is a prime ideal, either *x*∈P or *y*∈P. Since *x*∈P, we have *x*∉P and hence *y*∈P. Therefore P = S\P. So S\P is a minimal prime ideal in S.

THEOREM 3.25 : Let S be a pseudo integral near_subtraction semi group. A subset T of S is a maximal near_ subtraction sub semi group of S with $T \cap Ker = \varnothing$ **iff** P = S\T **isa minimal prime ideal of S.**

Proof **:** Let us assume thatT is a maximal near _subtraction sub semi group with $T \cap Ker = \varnothing$, then by theorem 3.24, P = S\T is a minimal prime ideal of S.
Conversely suppose that P = S\T is a minimal prime ideal of S. Since S is a pseudo integral near_ subtraction semi group and P is a minimal prime ideal of S, then corollary 3.18, P is completely prime and hence T is a near _subtraction sub semi group of S. Since P is minimal, we have T is a minimal near_subtraction sub semi group of S with $T \cap Ker = \varnothing$.

Conclusion: Mainly in this research we studied about the notion of *I*-divisor, *I*-potent, Pseudo Symmetric ideals in near subtraction semi groups.

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