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Characteristics Of Hyper Ideals in Ternary Semi Hyper Rings

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Abstract: Ternary semi hyper ring is an algebraic structure with one binary hyper operation and ternary multiplication. Here this research article we give an important properties of hyper ideals in ternary semi hyper ring. We will now introduce the notion of simple, (zero-) simple ternary semi hyper ring and segregate the minimality and maximality of hyper ideals in ternary semi hyper ring.Here in this paper we will give the relation between the minimality and maximality were invented in ternary semi hyper ring extending and generalizing the results for ternary semi rings.

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Introduction:

In Mathematics Algebraic structures play an vital role with many of real time applications in many of the fields such as theoretical physics, control engineering, coding theory, information sciences etc. In the 19th century ternary algebraic operations were considered by several mathematicians. In 1934 the Hyper structure theory was introduced by F. Marty [46] in that article he discussed the concept of hyper groups based on the notion of hyper operation. Later he begin to analyze the properties and applied them to the groups. After that In year 2020, D. Madhusudhana Rao was investigated the concept of hyper ideals in ternary semi hyper rings. One can go through references for preliminaries.

"(zero)-simple ternary semi hyper rings":

Here In this part we will be developed the concept and "characterize" the "(zero-) simple ternary semi hyper rings". Some of the properties are invented in terms of "hyper ideals".

Definition 3.1: Let H_1 be a ternary semi hyper ring with zero element , then H_1 is said to be a simple if H_1 does not contain any proper hyper ideals.

Example 3.2: 3. Let $H_1 = \{a_1, b_1, c_1, d_1, e_1, f_1\}$ and $[x_1, y_1, z_1]=(x_1 * y_1) * z_1$ for all $x_1, y_1, z_1 \in H_1$, where $\oplus$, $*$ is defined as follows:

$\oplus$	a_1	b_1	c_1	d_1	e_1	f_1
a_1	$\{b_1, c_1\}$	$\{b_1, c_1\}$	$\{b_1, c_1\}$	$\{b_1, c_1\}$	$\{b_1, c_1\}$	$\{b_1, c_1\}$
b_1	$\{a_1, c_1\}$	$\{a_1, c_1\}$	$\{a_1, c_1\}$	$\{a_1, c_1\}$	$\{a_1, c_1\}$	$\{a_1, c_1\}$
c_1	$\{a_1, b_1\}$	$\{a_1, b_1\}$	$\{a_1, b_1\}$	$\{a_1, b_1\}$	$\{a_1, b_1\}$	$\{a_1, b_1\}$
d_1	H_1-d_1	H_1-d_1	H_1-d_1	H_1-d_1	H_1-d_1	H_1-d_1
e_1	H_1-e_1	H_1-e_1	H_1-e_1	H_1-e_1	H_1-e_1	H_1-e_1
f_1	H_1-f_1	H_1-f_1	H_1-f_1	H_1-f_1	H_1-f_1	H_1-f_1

Then (H_1, $\oplus$, [ ]) is a ternary semi hyper ring. Here does not contains any proper hyper ideal of H_1 therefore H_1 is simple.

It is well known that if H_1 is a ternary semi hyper ring with zero, then we know that every hyper ideal of H_1 contains a zero element.

Definition 3.3: Let (H_1, $\oplus$, [ ]) be a ternary semi hyper ring with zero. Then H_1 is called zero-simple if it does not contain no non zero proper hyper ideal and $[H_1 H_1 H_1] \neq \{0\}$.

Remark 3.4: Let (H_1, $\oplus$, [ ]) be a " ternary semi hyper ring" for any element $h_1 \in H_1$ then the "hyper ideal generated" by h_1 are respectively shown by

$$J(h_1) = \langle h_1 \rangle = \{ h_1\} \cup [H_1 H_1 h_1] \cup [H_1 h_1 H_1] \cup [H_1 [H_1 h_1 H_1] H_1] \cup [h_1 H_1 H_1]$$

Lemma 3.5: Assume that (H_1**,** $\oplus$ **, [ ]) be a ternary semi hyper ring. And let A ≠ ∅ be non empty subset of** H_1**,** $[H_1 H_1 A] \cup [H_1 H_1 A H_1 H_1] \cup [H_1 A H_1] \cup [A H_1 H_1] \cup A$ **is the smallest hyper ideal of** H_1 **containing A.**

Lemma 3.6: Let (H_1**,** $\oplus$ **, [ ]) be a ternary semi hyper ring. And let A ≠ ∅ be non empty subset of** H_1, H_1**,** $[H_1, H_1 A] \cup [H_1 H_1 A H_1 H_1] \cup [H_1 A H_1] \cup [A H_1 H_1]$ **is the hyper ideal of H.**

Theorem 3.7: Let (H_1**,** $\oplus$**, [ ]) be a ternary semi hyper ring without zero. Then the following conditions are equivalent.**

1. H_1 **is simple**
2. $\forall a_1 \in H_1$, $[H_1 H_1 h_1] \cup [H_1 h_1 H_1] \cup [H_1 [H_1 h_1 H_1] H_1] \cup [h_1 H_1 H_1] = H_1$
3. $\forall a_1 \in H_1$, $\langle h_1 \rangle = H_1$.

Proof: Now we will show that (1) implies (2):

Assume that H_1 be a simple, by the known result lemma 3.6, we will have

$\forall a_1 \in H_1$, $[H_1 H_1 h_1] \cup [H_1 h_1 H_1] \cup [H_1 [H_1 h_1 H_1] H_1] \cup [h_1 H_1 H_1] = H_1$.

Now to show that (2) implies (3) :

By the known result lemma 3.5,

$\langle h_1 \rangle = [H_1 H_1 h_1] \cup [H_1 H_1 h_1 H_1 H_1] \cup [H_1 h_1 H_1] \cup [h_1 H_1 H_1] \cup \{ h_1\} = H_1 \cup \{ h_1\} = H_1$.

Now we have to show that (3) implies (1) :

Let A be a hyper ideal of H_1 as well as $h_1 \in A$. Then $H_1 = \langle h_1 \rangle \subseteq A \subseteq H_1$ implies that $A = H_1$. Therefore, $H_1$is simple.

*	a_1	b_1	c_1	d_1	e_1	f_1
a_1	a_1	b_1	c_1	c_1	e_1	e_1
b_1	b_1	b_1	d_1	d_1	f_1	f_1
c_1	c_1	d_1	c_1	d_1	c_1	c_1
d_1	d_1	d_1	d_1	d_1	d_1	d_1
e_1	e_1	f_1	c_1	c_1	e_1	f_1
f_1	f_1	f_1	d_1	d_1	f_1	f_1

Theorem 3.8: Let (H_1**,** $\oplus$**, [ ]) be a ternary semi hyper ring with zero. Then the below conditions are true.**

1. **If** H_1 **is a (zero)-simple. Then** $\forall h_1 \in H_1\backslash\{0\}$, $\langle h_1 \rangle = H_1$.
2. **If** $\forall h_1 \in H_1\backslash\{0\}$, $\langle h_1 \rangle = H_1$. **Then either** $[H_1 H_1 H_1] = \{0\}$ **or** H_1 **is (zero)-simple.**

Proof:

(1): Given that H_1 be a (zero)-simple. Then $\forall\, h_1 \in H_1\backslash\{0\}$, $< h_1 >$ is non-zero hyper ideal of H_1. Therefore, $\forall\, h_1 \in H_1\backslash\{0\}$, $< h_1 > = H_1$.

(2) : Given that $\forall\, h_1 \in H_1\backslash\{0\}$, $< h_1 > = H_1$. Then either $[H_1\, H_1\, H_1] \neq \{0\}$.
Let us assume that A be a non zero hyper ideal of H_1. Let $h_1 \in A\backslash\{0\} \Longrightarrow < h_1 > = H_1 \subseteq A \subseteq H_1$.
Therefore $A = H_1$. Hence H_1 is a (zero)-simple.

Theorem 3.9: Any non empty intersection of a the family of hyper-filters of a ternary semi hyper ring $H_1$is also a hyper-filter of H_1.

Theorem 3.10: The Union of any family of "hyper ideals of a ternary semi hyper ring" H_1 is also a "hyper ideal" of H_1.

Theorem 3.11: Let us assume that $(H_1, \oplus, [\ ])$ be a ternary semi hyper ring and let A be a hyper ideal of H_1. Let S is a ternary sub semi hyper ring. Then the below conditions are to be true.
1.If S is a simple so that $S \cap A \neq \emptyset$, then $S \subseteq A$.
2.If S is a (zero)-simple so that $S\backslash\{0\} \cap A \neq \emptyset$, then $S \subseteq A$.

***Proof*: (1):** Given that S is simple so that $S \cap A \neq \emptyset$. Let $h_1 \in S \cap A$. By the known result lemma 3.6, $[H_1\, H_1\, h_1] \cup [H_1\, H_1\, h_1\, H_1\, H_1] \cup [H_1\, h_1\, H_1] \cup [h_1 H_1\, H_1] \cap S$ is a hyper ideal of S**.** Then we have $[H_1\, H_1\, h_1] \cup [H_1\, H_1\, h_1\, H_1\, H_1] \cup [H_1\, h_1 H_1] \cup [h_1\, H_1\, H_1] \cap S = S$

$\Longrightarrow S \subseteq [H_1\, H_1\, h_1] \cup [H_1\, H_1\, h_1\, H_1\, H_1] \cup [H_1\, h_1 H_1] \cup [h_1\, H_1\, H_1] \subseteq [H_1\, H_1 A] \cup [H_1\, H_1 A\, H_1\, H_1] \cup [H_1 A\, H_1] \cup [A\, H_1\, H_1] \subseteq A$. Therefore, $S \subseteq A$.

(2): Let us assume that S is (zero)-simple such that $S\backslash\{0\} \cap A \neq \emptyset$. Let $h_1 \in S\backslash\{0\} \cap A$. By lemma 3.5, and theorem 3.8 (1),

we will get $S = < h_1 > = \{ h_1\} \cup [H_1\, H_1\, h_1] \cup [H_1\, h_1\, H_1] \cup [H_1\, [H_1\, h_1 H_1]\, H_1] \cup [h_1\, H_1\, H_1] \cap S \subseteq <h> = \{ h \} \cup [H_1\, H_1\, h_1] \cup [H_1\, h_1\, H_1] \cup [H_1\, [H_1\, h_1 H_1]\, H_1] \cup [h_1\, H_1\, H_1] \subseteq < h > \subseteq A$.
Therefore $S \subseteq A$.

Theorem 3.12 :
Let H_1,be a ternary semi hyper ring then the below conditions are equivalent.
(1) Principal hyper ideals of H_1 form a chain.
(2) Hyper ideals of H_1 form a chain.
***Proof* :**
Now we will show that (1) implies (2) :
Suppose that principal hyper ideals of H_1 form a chain.
Let us assume that A_1, B_1 be two hyper ideals of H_1. Suppose if possible $A_1 \not\subseteq B_1$, $B_1 \not\subseteq A_1$.
Then there exists $a \in A_1\backslash B_1$ and $b \in B_1\backslash A_1$.
Since $a_1 \in A_1 \Rightarrow < a_1 > \subseteq A_1$ and since $b_1 \in B_1 \Rightarrow < b_1 > \subseteq B_1$.
Since principal hyperideals form a chain, either $< a_1 > \subseteq < b_1 >$ or $< b_1 > \subseteq < a_1 >$.
If $< a_1 > \subseteq < b_1 >$, then $a_1 \in < b_1 > \subseteq B_1$. Which is a contradiction.
If $< b_1 > \subseteq < a_1 >$, then $b_1 \in < a_1 > \subseteq A_1$. Which is also a contradiction.
Therefore either $A_1 \subseteq B_1$ or $B_1 \subseteq A_1$ and hence hyperideals from a chain.

Now we have to show that (2)implies (1) :
Suppose that hyper ideals of H_1 form a chain.
Therefore principal hyper ideal of H_1form a chain.

Minimal & Maximal hyperideals of ternary semi hyperrings:

In the below section, we will give some of the important properties of (zero-) minimal hyper ideals and (zero)-maximal hyper ideals of ternary semi hyper rings and explained about the relationship between the (zero-) minimal hyper ideals, (zero)-maximal hyper ideals and the (zero)-simple ternary semi hyper rings.

Definition 4.1: Let a ternary semi hyper ring (H_1, $\oplus$, [ ]) having without zero. And let a hyper ideal A of H_1 is said to be as ***minimal hyper ideal*** of H if there does not having hyper ideal B of H_1 such that B ⊆ A.

Definition 4.2: Let a ternary semi hyper ring (H_1, $\oplus$, [ ]) having with zero. And a hyper ideal A of H is known as zero -***minimal hyper ideal*** of H_1 if there does not having non zero hyper ideal B of H such that B ⊆ A.

OR

Let a ternary semi hyper ring (H_1, $\oplus$, [ ]) having with zero. And a hyper ideal A of H_1 is known as zero-***minimal hyper ideal*** of H_1 if for any hyper ideal B of H_1such that B ⊆ A, we get B = {0}.

Theorem 4.3: Let a ternary semi hyper ring (H_1, $\oplus$, [ ]) without zero and A be a hyper ideal of H_1. Then the below conditions are true.

1. **A is simple if and only if A is a minimal hyper ideal without zero of H_1.**
2. **If A is a minimal hyper ideal of H_1 with zero, then A is a (zero-) simple.**

***Proof*: (1):**

Let A be a minimal hyper ideal of H without zero. Let B be a hyper ideal of A. Then we get [AAB] ⋃ [*AABAA*] ∪ [*ABA*] ∪ [*BAA*] ⋂ B is a hyper ideal of B**.** Then we have [AAB] ⋃ [*AABAA*] ∪ [*ABA*] ∪ [*BAA*] ⋂ B = B ⟹A⊆ [AAB] ⋃ [*AABAA*] ∪ [*ABA*] ∪ [*BAA*] ⊆ B. Therefore, A⊆ B and hence A is equal to B. Therefore, A is a simple.

Conversely, assume that A is a simple and B is hyper ideal of H_1 so that B ⊆ A. Then we will get B ⋂ A ≠ ∅. By known theorem 3.11(i) we will have A ⊆ B ⟹ A = B. Therefore A is the minimal hyper ideal of H_1.

(2) The proof of (2) is similar to above (1).

Theorem 4.4: Let a ternary semi hyper ring (H_1, $\oplus$, [ ]) with zero and A be a non zero hyper ideal of H. Then the below conditions are to be true.

1. **If A is a (zero)-minimal hyper ideal of H. Then either there exist a nonzero hyper ideal B of A such that [AAB] ⋃ [*AABAA*] ∪ [*ABA*] ∪ [*BAA*] = {0} or A is a (zero)-simple.**
2. **If A is a (zero)-simple, then A is a (zero)-minimal hyper ideal of H_1.**

Proof: The proof is same as the proof of theorem 4.3(1) and the theorem 3.11(2).

Theorem 4.5: Let a ternary semi hyper ring (H_1, $\oplus$, [ ]) without zero having proper hyper ideals. H contains exactly one proper hyperideal of H_1 or H_1 contains exactly two proper hyperideal A_1, A_2 such that $A_1 \bigcup A_2 = H_1$ and $A_1 \bigcap A_2 = \emptyset$. Then if and only if every proper hyper ideal of H_1 is minimal

Proof: Assume that any proper hyper ideal of H_1 is minimal and A be a proper hyper ideal of H_1. Then A will be have a minimal hyper ideal of H_1. Then we get the following cases.

Case-1: ∀ $a \in H_1$\A, H_1 = < a >. Let B is any proper hyper ideal of H_1 and B ≠ A, then since A is minimal hyperideal, we get B\A ≠ ∅. Hence ∃ a ∈ B\A ⊆ H_1\A. Therefore H_1 = < a > ⊆ B ⊆ H_1, so B = H_1. This is a contradiction and hence A = B. Therefore in this case A is unique proper hyperideal of H.

Case-2: ∃ $a \in H_1$\A, H_1 ≠ < a >. We have < a > ≠ A and < a > is a minimal hyper ideal of H_1. By known theorem3.10, < a > ∪ A is a hyper ideal of H_1. Since A ⊂ < a > ∪ A. Hence by hypothesis of our theorem we will get < a > ∪ A = H_1. Here < a > ⋂ A ⊆ < a > and < a > is the minimal hyperideal of H. Therefore < a > ⋂ A = ∅. Let B be the any other proper hyper ideal of H_1, then B is a minimal hyper ideal of H_1. We obtain B = B ∩ H = P ∩ (< a > ∪ A) = (P ∩ < a >) ∪ (P ∩ A). If P ∩

A ≠ ∅. Since B and < a > are minimal hyperideals of H. We get B = < a >. In this case H contains exactly two proper hyperideals A and < a > such that < a > ∪ A = H and < a > ∩ A = ∅.

Converse part is obvious.

Theorem 4.6: Let (H, ⨁, [ ]) be a ternary semihyperring with zero having nonzero proper hyperideals. Then every nonzero proper hyperideal of H is 0-minimal if and only if H contains exactly one nonzero proper hyperideal of H or H contains exactly two nonzero proper hyperideal A_1, A_2 such that $A_1 \bigcup A_2$ = H and $A_1 \bigcap A_2$ = {0}.

Proof: Proof is similar to the proof of theorem 4.5.

Definition 4.7: Let (H, ⨁, [ ]) be a ternary semihyperring. A hyper ideal A of H is known as ***maximal hyperideal*** of H if for every hyperideal B of H such that A ⊆ B we have B = H.

Theorem 4.8: (H, ⨁, [ ]) be a ternary semihyperring with zero having proper hyperideals. Then every proper hyperideal of H is maximal if and only if H contains exactly one proper hyperideal of H or H contains exactly two proper hyperideal A_1, A_2 such that $A_1 \bigcup A_2$ = H and $A_1 \bigcap A_2$ = ∅.

Proof: Suppose that every proper hyperideal of H is maximal and A be a proper hyperideal of H. Then A be a maximal hyperideal of H. Then we get the following cases.

Case-1: ∀ a ∈ H\A, H = < a >. If B is also proper hyperideal of H and B ≠ A, then since A is maximal hyperideal, we get B\A ≠ ∅. Hence ∃ a ∈ B\A ⊆ H\A. Therefore H = < a > ⊆ B ⊆ H, so B = H. This is a contradiction and hence A = B. Therefore in this case A is unique proper hyperideal of H.

Case-2: ∃ a ∈ H\A, H ≠ < a >. We have < a > ≠ A and < a > is a maximal hyperideal of H. By theorem 3.10, < a > ∪ A is a hyperideal of H. Since A ⊂ < a > ∪ A and A is a maximal hyperideal of H. Hence we get < a > ∪ A = H. Here < a > ⋂ A ⊆ < a > and by hypothesis we have < a > ⋂ A = ∅. Let B be the any arbitrary proper hyperideal of H, then B is a maximal hyperideal of H. We obtain B = B ∩ H = P ∩ (< a > ∪ A) = (P ∩ < a >) ∪ (P ∩ A). If P ∩ A ≠ ∅. Since B ∩ < a > and < a > are maximal hyperideals of H. We get B = < a >. In this case H contains exactly two proper hyperideals A and < a > such that < a > ∪ A = H and < a > ∩ A = ∅.

Converse part is obvious.

Theorem 4.9: Let a ternary semi hyper ring (H_1, ⨁, [ ]) with zero having non zero proper hyper ideals. Then H_1 contains exactly one non zero proper hyper ideal of H_1 or H_1 contains exactly two nonzero proper hyper ideal A_1, A_2 such that $A_1 \bigcup A_2 = H_1$ and $A_1 \bigcap A_2$ = {0} if and only if any non zero proper hyper ideal of H_1 is maximal.

Proof: Proof is same as the proof of theorem 4.8.

Theorem 4.10: Let a ternary semi hyper ring (H_1, ⨁, [ ]). A proper hyper ideal A of H_1 is maximal if and only if

1. **H_1\A = {h} and [H_1 H_1h] ⋃ [H_1 H_1Hh H_1 H_1] ∪ [H_1h H_1] ∪ [h H_1 H_1] ⊆ A for some $h \in H_1$ or**
2. **H_1\A ⊆ [H_1 H_1h] ⋃ [H_1 H_1h H_1 H_1] ∪ [H_1h H_1] ∪ [h H_1 H_1] for all $h \in H_1$\A.**

Proof: Suppose A is a maximal hyper ideal of H_1. Then the following two cases are arising.

Case 1: ∃ $h \in H_1$\A ∋ [H_1 H_1h] ⋃ [H_1 H_1h H_1 H_1] ∪ [H_1h H_1] ∪ [h H_1 H_1] ⊆ A. By lemma 3.5,

A ∪ {h} = A ∪ [H_1 H_1h] ⋃ [H_1 H_1h H_1 H_1] ∪ [H_1h H_1] ∪ [h H_1 H_1] ∪ {h}

= A ∪ {[H_1 H_1h] ⋃ [H_1 H_1h H_1 H_1] ∪ [H_1h H_1] ∪ [h H_1 H_1] ∪ {h}}= A ∪ < h >.

Since A ∪ < h > is a hyperideal of H_1, A ∪ {h} is a hyperideal of H_1. Here A is a maximal hyperideal of H as well as A ⊆ A ∪ {h}. We get A ∪ {h} = H. Therefore H\A = {h}.

Case 2: For any $h \in H_1$\A, [H_1 H_1h] ⋃ [H_1 H_1h H_1 H_1] ∪ [H_1h H_1] ∪ [h H_1 H_1] ⊈ A. Since

$[H_1\, H_1h] \cup [H_1\, H_1h\, H_1\, H_1] \cup [H_1h\, H_1] \cup [h\, H_1\, H_1]$ is a hyper ideal of H_1. By known result lemma 3.6, and theorem 3.10, $A \cup [H_1\, H_1h] \cup [H_1\, H_1h\, H_1\, H_1] \cup [H_1h\, H_1] \cup [h\, H_1\, H_1]$ is a hyper ideal of H_1 as well as $A \subseteq A \cup [H_1\, H_1h] \cup [H_1\, H_1h\, H_1\, H_1] \cup [H_1h\, H_1] \cup [h\, H_1\, H_1]$. Since A is maximal hyper ideal of H_1 and hence

$A \cup [H_1\, H_1h] \cup [H_1\, H_1h\, H_1\, H_1] \cup [H_1h\, H_1] \cup [h\, H_1\, H_1] = H_1$. Therefore, for all $h \in H_1\backslash A$ we get $H_1\backslash A \subseteq [H_1\, H_1h] \cup [H_1\, H_1h\, H_1\, H_1] \cup [H_1h\, H_1] \cup [h\, H_1\, H_1]$.

Conversely, suppose that B is a hyper ideal of H_1 such that $A \subset B$. Then $B\backslash A \neq \emptyset$. If $H_1\backslash A = \{h\}$ and $[H_1\, H_1h] \cup [H_1\, H_1h\, H_1\, H_1] \cup [H_1h\, H_1] \cup [h\, H_1\, H_1] \subseteq A$ for some $h \in H_1$. Then $B\backslash A \subseteq H_1\backslash A = \{h\}$. Thus $B\backslash A = \{h\}$ and hence $B = A \cup \{h\} = H_1$. Therefore, A is a maximal hyper ideal of H_1. If $H_1\backslash A \subseteq [H_1\, H_1h] \cup [H_1\, H_1h\, H_1\, H_1] \cup [H_1h\, H_1] \cup [h\, H_1\, H_1]$ for all $h \in H_1\backslash A$. Then $H_1\backslash A \subseteq [H_1\, H_1x] \cup [H_1\, H_1x\, H_1\, H_1] \cup [H_1x\, H_1] \cup [x\, H_1\, H_1] \subseteq [H_1\, H_1B] \cup [H_1\, H_1B\, H_1\, H_1] \cup [H_1B\, H_1] \cup [B\, H_1\, H_1] \subseteq B$ for all $x \in B\backslash A$. Therefore, $H_1 = H_1\backslash A \cup A \subseteq B \subseteq H_1$ and hence $B = H_1$. Hence A is a maximal hyper ideal of H_1.

Note 4.11: Let a ternary semi hyper ring $(H_1, \oplus, [\ ])$. Let $\mathfrak{U}$ indicate union of all proper hyper ideals of H_1.

Lemma 4.12: Let a ternary semi hyper ring $(H_1, \oplus, [\ ])$. Then $\mathfrak{U} = H_1$ iff $< h > \neq H_1\ \forall\ h \in H_1$.

Theorem 4.13: Let a ternary semi hyper ring $(H_1, \oplus, [\ ])$ without zero. Then any one of the below statements are satisfied.

1. **H_1 is simple**
2. **$\forall\ h \in H_1, < h > \neq H_1$.**
3. **$\exists\ h \in H_1 \ni < h > = H_1$, $h \notin [H_1\, H_1h] \cup [H_1\, H_1h\, H_1\, H_1] \cup [H_1h\, H_1] \cup [h\, H_1\, H_1] \subseteq \mathfrak{U} = H_1\backslash\{h\}$ and $\mathfrak{U}$ is the maximal unique hyper ideal of H_1.**
4. **$H_1\backslash\mathfrak{U} = \{h \in H_1 : [H_1\, H_1h] \cup [H_1\, H_1h\, H_1\, H_1] \cup [H_1h\, H_1] \cup [h\, H_1\, H_1] = H_1\}$ and $\mathfrak{U}$ is the maximal unique hyper ideal of H_1.**

Proof:Let us assume that H_1 is not simple. Then ∃ a proper hyper ideal of H_1 which implies that $\mathfrak{U}$ is a hyper ideal of H_1, then we get following cases.

Case -1: Let us assume that $\mathfrak{U} = H_1$. by the known result

Lemma 4.12, implies that $\forall\ h \in H_1, < h > \neq H_1$ and hence condition (2) is satisfied.

Case- 2: Let us assume that $\mathfrak{U} \neq H_1$.

We get $\mathfrak{U}$ is the maximal hyper ideal of H_1. Suppose A is the maximal hyper ideal of H_1. Then since A is a proper hyper ideal of H_1, we will have $A \subseteq \mathfrak{U} \subseteq H_1$. But A is a maximal hyper ideal of H_1, we obtained $A = \mathfrak{U}$. Therefore $\mathfrak{U}$ is the maximal unique hyper ideal of H_1. By the known theorem 4.10, gives

(a) $H_1\backslash\mathfrak{U} = \{h\}$ and $[H_1\, H_1h] \cup [H_1\, H_1h\, H_1\, H_1] \cup [H_1h\, H_1] \cup [h\, H_1\, H_1] \subseteq \mathfrak{U}$ for some $h \in H_1$ or
(b) $\forall\ h \in H_1\backslash\mathfrak{U}$, $H_1\backslash\mathfrak{U} \subseteq [H_1\, H_1h] \cup [H_1\, H_1h\, H_1\, H_1] \cup [H_1h\, H_1] \cup [h\, H_1\, H_1]$.

Suppose that, $H_1\backslash\mathfrak{U} = \{ h \}$ and $[H_1\, H_1h] \cup [H_1\, H_1h\, H_1\, H_1] \cup [H_1h\, H_1] \cup [h\, H_1\, H_1] \subseteq \mathfrak{U}$ for some $h \in H_1$. Then $[H_1\, H_1h] \cup [H_1\, H_1h\, H_1\, H_1] \cup [H_1h\, H_1] \cup [h\, H_1\, H_1] \subseteq \mathfrak{U} = H_1\backslash\{h\}$. Since $h \notin \mathfrak{U}$, we get $< h > = H_1$. If $h \in [H_1\, H_1h] \cup [H_1\, H_1h\, H_1\, H_1] \cup [H_1h\, H_1] \cup [h\, H_1\, H_1]$, then $\{h\} \subseteq [H_1\, H_1h] \cup [H_1\, H_1h\, H_1\, H_1] \cup [H_1h\, H_1] \cup [h\, H_1\, H_1]$. By known theorem lemma 3.5, implies $H_1 = < h > = [H_1\, H_1h] \cup [H_1\, H_1h\, H_1\, H_1] \cup [H_1h\, H_1] \cup [h\, H_1\, H_1] \cup \{h\} \subseteq [H_1\, H_1h] \cup [H_1\, H_1h\, H_1\, H_1] \cup [H_1h\, H_1] \cup [h\, H_1\, H_1] \subseteq [H_1\, H_1\mathcal{U}] \cup [H_1\, H_1\mathcal{U}\, H_1\, H_1] \cup [H_1\mathcal{U}\, H_1] \cup [\mathcal{U}\, H_1\, H_1] \cup \mathfrak{U} = \mathfrak{U} \subseteq H_1$. Therefore we get $H_1 = \mathfrak{U}$. Which is impossible and hence $h \notin [H_1\, H_1h] \cup [H_1\, H_1h\, H_1\, H_1] \cup [H_1h\, H_1] \cup [h\, H_1\, H_1]$ and so the condition (3) is satisfied.

Let us suppose that for all $h \in H_1\backslash\mathfrak{U}$, $H_1\backslash\mathfrak{U} \subseteq [H_1\, H_1h] \cup [H_1\, H_1h\, H_1\, H_1] \cup [H_1h\, H_1] \cup [h\, H_1\, H_1]$. Let $h \in H_1\backslash\mathfrak{U}$, then $h \in [H_1\, H_1h] \cup [H_1\, H_1h\, H_1\, H_1] \cup [H_1h\, H_1] \cup [h\, H_1\, H_1]$. So $\{h\} \subseteq [H_1\, H_1h] \cup [H_1\, H_1h\, H_1\, H_1] \cup [H_1h\, H_1] \cup [h\, H_1\, H_1]$. So by Lemma 3.5, implies $< h > = [H_1\, H_1h] \cup [H_1\, H_1h\, H_1\, H_1] \cup [H_1h\, H_1]$

$\cup$ [h H_1 H_1] $\bigcup$ {h} = [H_1 H_1h] $\bigcup$ [H_1 H_1h H_1 H_1] $\cup$ [H_1h H_1] $\cup$ [h H_1 H_1]. Since $h \notin \mathfrak{U}$. We get < h > = H_1. Therefore, H_1 = < h > = [H_1 H_1h] $\bigcup$ [H_1 H_1h H_1 H_1] $\cup$ [H_1h H_1] $\cup$ [h H_1 H_1].

Conversely, let $h \in$ H such that [H_1 H_1h] $\bigcup$ [H_1 H_1h H_1 H_1] $\cup$ [H_1h H_1] $\cup$ [h H_1 H_1] = H_1. Let $h \in \mathfrak{U}$, then < h > $\subseteq \mathfrak{U} \subset H_1$ By the known theorem lemma 3.5, implies < h > = [H_1 H_1h] $\bigcup$ [H_1 H_1h H_1 H_1] $\cup$ [H_1h H_1] $\cup$ [h H_1 H_1] $\bigcup$ {h} = H_1 $\bigcup$ {h} = H_1. Which is a contradiction and hence we have $h \in H_1\backslash\mathfrak{U}$ and implies that $H_1\backslash\mathfrak{U}$ = {$h \in H_1$: [H_1 H_1h] $\bigcup$ [H_1 H_1h H_1 H_1] $\cup$ [H_1h H_1] $\cup$ [h H_1 H_1] = H_1}and hence the condition (4) is to be satisfied. Therefore, this completes the proof.

Theorem 4.14: Let a ternary semi hyper ring (H_1, $\bigoplus$, [ ]) with zero and [H_1 H_1h] $\bigcup$ [H_1 H_1h H_1 H_1] $\cup$ [H_1 H_1h H_1 H_1] $\cup$ [h H_1 H_1] ≠ {0} . Then only below conditional statements are satisfied.

1. **H_1 is (zero)-simple**
2. **$\forall$ $h \in H_1$, < h > ≠ H_1.**
3. **$\exists$ $h \in H_1 \ni$ < h > = H_1, $h \notin$ [H_1 H_1h] $\bigcup$ [H_1 H_1h H_1 H_1] $\cup$ [H_1h H_1] $\cup$ [h H_1 H_1] $\subseteq \mathfrak{U}$ = $H_1\backslash${h} and $\mathfrak{U}$ is the unique maximal hyperideal of H.**
4. **$H_1\backslash\mathfrak{U}$ = {$h \in H_1$: [H_1 H_1h] $\bigcup$ [H_1 H_1h H_1 H_1] $\cup$ [H_1h H_1] $\cup$ [h H_1 H_1] = H_1} and $\mathfrak{U}$ is the maximal unique hyper ideal of H_1.**

Proof: The proof is same the proof of Theorem 4.13.

Conclusion: We will introduce the notion of simple, (zero)- simple and characterize the minimality and maximality of hyper ideals in ternary semi hyper rings. The relation between the minimality and maximality is investigated in ternary semi hyper rings extending and generalizing the analogues results for ternary semi rings.

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