Implementation of Lane Change Trajectory Planning Model

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Abstract

The Embedded controllers will become onboard in vehicles as part of highway automation. This controller will take control of vehicle in hand, superseding the actions of driver so as to avoid mishap due to manmade errors. However the controller being a computer needs logic to execute the above referred task. Primary objective of this project is to develop such logic i.e. trajectory model. The Lane change concept is discussed in detail along with steps involved in Lane change process. Intended Lane change model will be useful to calculate the desired optimal trajectory. The model is developed in MATLAB and same results are verified in Simulink software.

Keywords: Embedded Controllers, Trajectory of vehicle, Lane Change, Optimal Trajectory, Lane Changing in Theory.

INTRODUCTION

In the course of Lane change, if the initial position of the vehicle is fixed and the final position can be arbitrarily set, this is called lane changing in theory (LCT). Considering intelligent vehicle changing constraints and the feature of lane changing on curved road, the constraints of 'Lane changing on curved road in practice (LCCP)' are proposed Trajectory of vehicle, velocity and acceleration curves should be smooth and continuous. An abrupt change of trajectory and state of curve is not permitted. The examples of LCT and LCCP are shown in Figure 1.
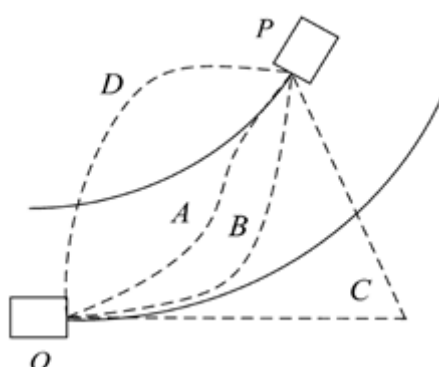

Figure 1. Lane Charge with alternative means

O is the initial position and P is the ending position. There are number of routes from O to P. O → C → P is a route which has sharp curvature change in the route. It does not fulfil the LCCP Constraint 1. From O → D → P, where sharp curvature appears in the initial position. It does not meet the LCCP Constraint 2. O → A → P and O → B → P are the paths that can be attained in

practice. These two routes can fulfill the LCCP requirements. In addition, there are several other paths which can fulfill the LCCP constraints.
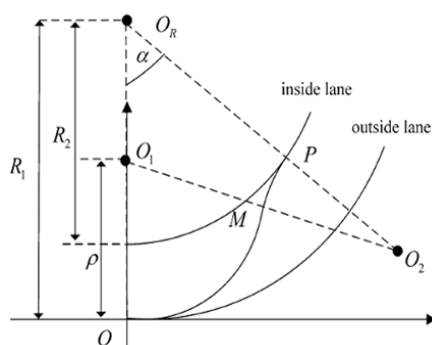

Figure 2. Desired Lane Changing Trajectory

(a) Lane Change Model

In this work, we mainly focus on the paths starting from 'O', consist of two segments arcs like O → A → P. The expected lane changing trajectory is shown in Figure 2.

The location of the vehicle at the starting time is O and the final reaching location is P. The inside and outside lane have the same instant center O_R. Considering that the preferred trajectory consists of two segment arcs which have equal curvature radius ρ. These arcs instantaneous center is O_1 and O_2 respectively. These two arcs are tangent to M, α is the rotation angle linking OOR and POR. Suppose that the centroid of the vehicle reaches the point C at time t along desired trajectory. If C is in OM segment, the trajectory is shown in Figure 3 where xr(t) and yr(t) represent the longitudinal and lateral displacement respectively, (t) is the angel of the vehicle Centroid around instant center OR, ψ(t) denotes the angel of vehicle centroid around instantaneous center O_1, ψ(t) satisfies

$$\psi(t) = \theta(t) + \arcsin\left(\frac{(R_1 - \rho)\sin\theta(t)}{\rho}\right)$$

(1)

Where

θ(t): Angle of the vehicle centroid around instantaneous center O_R

R_1: Curvature radius of outside lane

ρ: Curvature radius of two segments arcs (Equal)

When the vehicle is in OM segment, the expected displacement can be estimated as

Longitudinal displacement,

$$x_r(t) = \rho \sin\left[\theta(t) + \arcsin\left(\frac{(R_1 - \rho)\sin\theta(t)}{\rho}\right)\right]$$

(2)

Lateral displacement,

$$y_r(t) = \rho - \rho\cos\left[\theta(t) + \arcsin\left(\frac{(R_1 - \rho)\sin\theta(t)}{\rho}\right)\right]$$

(3)

In this work, polynomial method is adopted to solve the unknown functions in above equations. Assuming that θ(t) satisfies quantic polynomial

$$\theta(t) = C_5 t^5 + C_4 t^4 + C_3 t^3 + C_2 t^2 + C_1 t + C_0$$

(4)

Where C_i, i = 0, …, 5 represents unresolved coefficients. Considering the derivative of θ(t) with respect to time, the angular velocity and angular acceleration can be determined as

$$\dot{\theta}(t) = 5C_5 t^4 + 4C_4 t^3 + 3C_3 t^2 + 2C_2 t + C_1$$

(5)

$$\ddot{\theta}(t) = 20C_5 t^3 + 12C_4 t^2 + 6C_3 + 2C_2$$

(6)

Above mentioned θ(t), $\dot{\theta}(t)$ and $\ddot{\theta}(t)$ in starting and finishing position satisfy the equations

$$\theta(t_0) = 0$$

(7)

$$\theta(T) = \alpha$$

(8)

$$\dot{\theta}(t_0) = \frac{\sqrt{V_x^2(t_o) + V_y^2(t_o)}}{R_1}$$

(9)

$$\dot{\theta}(T) = \frac{\sqrt{V_x^2(T) + V_y^2(T)}}{R_2}$$

(10)

$$\ddot{\theta}(t_0) = \frac{\sqrt{a_x^2(t_o) + a_y^2(t_o)}}{R_1}$$

(11)

$$\ddot{\theta}(T) = \frac{\sqrt{a_x^2(T) + a_y^2(T)}}{R_2}$$

(12)

Where

t: Instantaneous time t_0: Starting time   T: Finishing time α: Angular acceleration

$V_x(t_0)$: Longitudinal velocity at time t_0 $V_y(t_0)$: Lateral velocity at time t_0

$V_x(T)$: Longitudinal velocity at time T $V_y(T)$: Lateral velocity at time T

R_2: Curvature radius of inside lane $a_x(t_0)$: Longitudinal acceleration at time t_0

$a_y(t_0)$: Lateral acceleration at time t_0 $a_x(T)$: Longitudinal acceleration at time T

$a_y(T)$: Lateral acceleration at time T.

Lane change model of a vehicle when it negotiates a turn so as to implement lane change is shown in Figure 3.
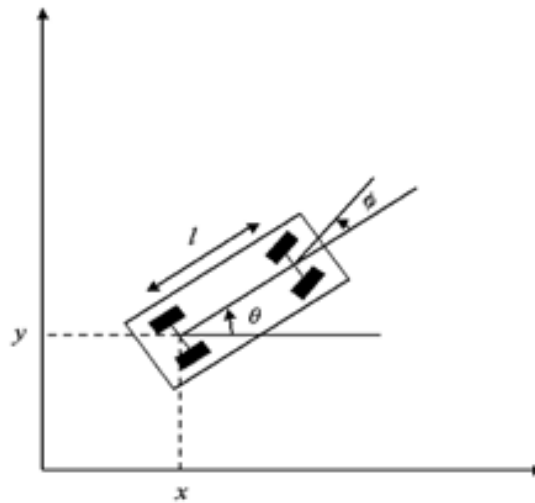

Figure 3. **Lane change model of a vehicle when it negotiates a turn**

Where

θ: Angle of car with X axis   l: Inter distance between axles   Φ: Angle of front axle

Lane change model constitutes the following three mathematical models

- Optimum track model
- Link model
- Controller model

(b) Optimum Track Model

Relation between various state variables associated with this model are given below.

$$\dot{x}(t) = u_1 \cos(\theta(t))$$

$$(13)$$

$$\dot{y}(t) = u_1 \sin(\theta(t))$$

$$(14)$$

$$\dot{\theta}(t) = \frac{u_1 \tan(\phi(t))}{l}$$

$$(15)$$

$$\dot{\phi}(t) = -\frac{p_4(t)}{1389}$$

$$(16)$$

Where

    u_1: Linear velocity of rear wheels

(c)Link Model

Relation among various state variables related with this model are given below.

$$\dot{p}_1(t) = 2x(t))$$

$$(17)$$

$$\dot{p}_2(t) = 2y(t))$$

$$(18)$$

$$\dot{p}_3(t) = u_1 p_1(t) \sin(\theta(t)) - u_1 p_2(t) \cos(\theta(t))$$

$$(19)$$

$$\dot{p}_4(t) = -\frac{u_1 p_3(t)(1 + \tan^2(\phi(t)))}{l}$$

$$(20)$$

    Where $p_{1,2,3,4}$: State variables

(d) Controller Model

Relations between various state variables associated with this model are given below.

$$u_2(t) = -\frac{p_4(t)}{1389}$$

(21)

Where

u$_2$: Angular velocity of front wheels

2. IMPLEMENTATION OF TRAJECTORY MODEL IN MATLAB

As discussed in earlier chapters, embedded controllers will become onboard in vehicles as part of highway automation. This controller will take control of vehicle in hand overriding the actions of driver so as to avoid mishap due to manmade mistakes. However the controller being a computer needs logic to execute the above mentioned task. Primary objective of this project is to develop such logic i.e. trajectory model and demonstrate the utility of same in MATLAB as it will be difficult to demonstrate with the aid of real embedded controller. Following values are chosen as inputs for the sake of demonstration.

1. Radius of curvature, ρ=60 m

2. Radius of curvature of outside lane, R1=100 m

3. Radius of curvature of inside lane, R2=121 m

4. Rotation angle, α=0.7 rad

5. Beginning time, t$_o$ = 0 sec

6. Finishing time, T = 18 sec

7. $\theta(t_0)$: Angular displacement at time t_0= 0

8. $\theta(T)$: Angular displacement at time T = α =0.7 rad

9. $V_x(t_0)$: Longitudinal velocity at time t_0= 5 m/s

10. $V_x(T)$: Longitudinal velocity at time T= 3.6 m/s

11. $V_y(t_0)$: Lateral velocity at time t_0= 0 m/s

12. $V_y(T)$: Lateral velocity at time T= 0.6 m/s

13. $a_x(t_0)$: Longitudinal velocity at time t_0= 0 m/s^2

14. $a_x(T)$: Longitudinal velocity at time T= 0.09 m/s^2

15. $a_y(t_0)$: Lateral velocity at time t_0= 0.4 m/s^2

16. $a_y(T)$: Lateral velocity at time T= 0 m/s^2

Substituting $\theta(t_0)$ =0 at time t= t_0= 0 in eqn 4 we get C_0 = 0

Substituting above inputs in equation 9 we get $\dot{\theta}(t_0) = 0.05$

Substituting same in equation 5 for t = t_0 = 0 we get C_1 = 0.05

Substituting above inputs in equation 11 we get $\ddot{\theta}(t_0) = 0.004$

Substituting same in eqn 6 for t = t_0 = 0 we get C_2 = 0.002

Substituting $\theta(T)$ = 0.7 for t = T = 18 in equation 4 we get

$$-0.848 = C_5\, 18^5 + C_4\, 18^4 + C_3\, 18^3$$
(22)

Substituting above inputs in equation 10 we get $\ddot{\theta}(T) = 0.00074$

Substituting the above value for t = T = 18in equation 6 we get

$$-0.092 = 5\,C_5\, 18^4 + 4\,C_4\, 18^3 + 3\,C_3\, 18^2$$
(23)

Substituting above inputs in equation 10.12 we get

$$\dot{\theta}(T) = 0.03$$

Substituting the above value for t = T = 18in equation 5 we get

$$-0.00326 = 20\,C_5\, 18^3 + 12\,C_4\, 18^2 + 6\,C_3\, 18$$
(24)

Above (22, 23 &24) 3 equations are solved using MATLAB by issuing the following command to obtain unsolved coefficients.

[c3 c4 c5] = solve 'c3*18^3+c4*18^4+c5*18^5 =

 -0.848','c3*3*18^2+c4*4*18^3+c5*5*18^4 =

-0.092','c3*108+c4*3888+c5*116640 = 0.00326')

Solutions obtained from MATLAB command window is furnished below Coefficients thus obtained are summarized in Table 1. Correctness of coefficients is checked with the aid of following code in MATLAB (For t = 0sec).

Table 1. Summary of Coefficients

Coefficient	Value
C_0	0
C_1	0.05
C_2	0.002
C_3	-0.000408799725651577503429355281 20713
C_4	0.0000208070416095107453134449016918
C_5	-0.000000342999034699994919473657 4701399

Upon executing the code the result thus produced is furnished below Correctness of coefficients is also checked for t = T = 18 sec with the aid of following code in MATLAB.

Step-1: To clear the earlier figures created if any in MATLAB window

clf

Step-2: To clear the earlier variables created if any in MATLAB window

Clear

Step-3: Key in the value of radius of curvature

rho =60;

Beginning time, to = 0 sec

Finishing time, T = 18 sec

Step-4: Key in the value of radius of curvature of outside lane

R1=100;

Step-5: Key in the value of radius of curvature of inside lane

R2=121;

Step-6: Key in the value of rotation angle, α =0.7 rad alpha=0.7;

Step-7: Key in the value of coefficients

c0=0;

c1=0.05;

c2=0.002;

c3=-0.00040879972565157750342935528120713;

c4=0.000020807041609510745313214449016918;

c5=-0.00000034299903469999491947365747091399;

Step-8: To create time values with initial value of 0 seconds and final time of 18 seconds with a step of 1 second   t=0:1:18;

Step-9: theta=c5*t.^5+c4*t.^4+c3*t.^3+c2*t.^2+c1.*t+c0;

Step-10: aaa=(theta+asin(((R1-rho)*sin(theta))/rho));

 x=rho*sin(aaa);

Step-11: y=rho-rho*cos(aaa);

Step-12: Plotting x and y to get the ultimate goal i.e. trajectory model with line width of plot as 4

plot(x,y,'LineWidth',4)

Step-13: Imposing grid on the plot

grid on

3. IMPLEMENTATION OF LANE CHANGE MODEL IN SIMULINK

Equating equations (4) and (9) gives

$$\dot{\phi}(t) = u_2(t)$$

(25)

Final expected outcome of lane change model is optimal trajectory to be followed by vehicle which is nothing but a plot between 'y' and 'x' which means final outputs are 'x' and 'y'.

Following inputs are considered

1. u_1 = 10 m/sec = 36 KMPH

2.   l = 1 m = 1000 mm

3.   Following initial conditions are considered

4.   x = -1 m = -1000 mm

5.   y = 0 m

6.   θ = 0

7.   Φ= 0

8. p_1 = 90000000

9. p_2 = 640

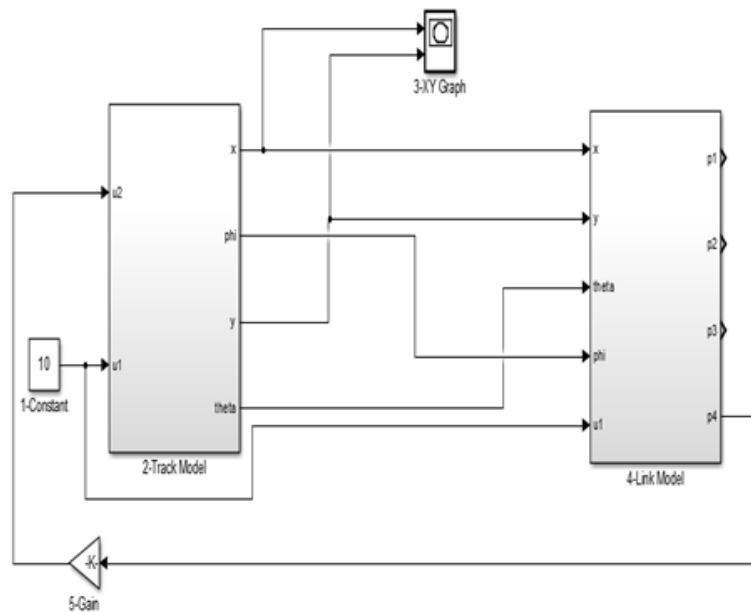10. p_3 = -18000

11. p_4 = -10000

Figure 4. Integrated controller Simulink model

In the above two models except u_1 all the parameters are unknown. However initial conditions are available for other parameters except u2. Initial condition of u2 will be obtained by integrating the initial condition of ϕ according to equation 25. For the constant input value of u1, later values of all the parameters will be obtained from the above mentioned equations by setting up the track model and link models in loop. This philosophy has been practically implemented with the aid of Simulink.
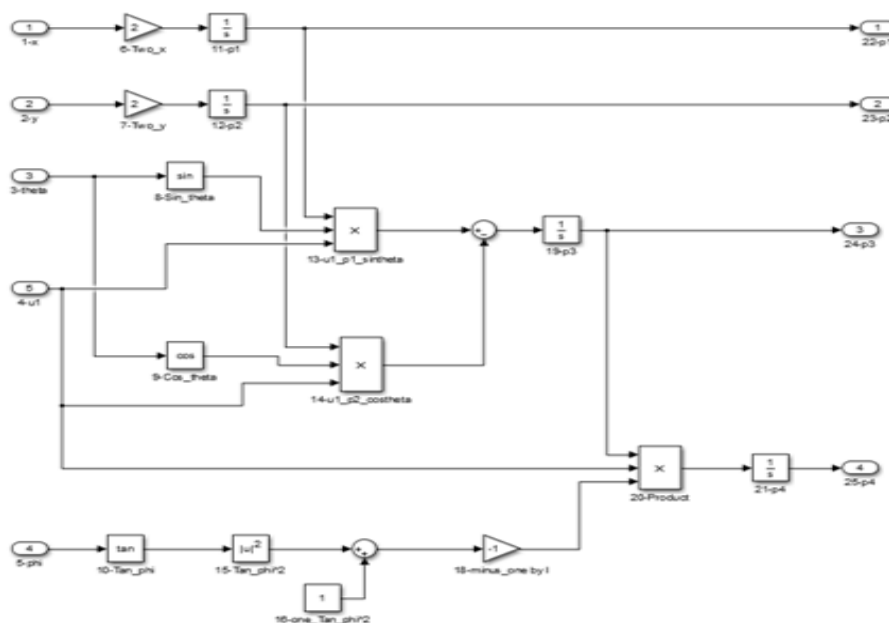
Figure 5. Track Simulink model

Integrated Simulink model for controller model is shown in Figure 4 (Integrated controller Simulink model). Details of blocks used for controller Simulink model are given in Table .2. Separate model has been built for track model connecting to block 2 i.e. 2-Track model and the same is shown in Figure 5. Details of blocks used for track Simulink model are given in Table-3.

Table 2. Details of blocks used for controller Simulink model

Sl. No.	Name in model	Name of block in browser	Source	Properties
1.	1-Constant	Constant	Commonly used blocks	Constant value = 10
2.	2-Track model	Subsystem	Commonly used blocks	----
3.	3-XY Graph	XY Graph	Sinks	x-min=-1, x-max=3 y-min=0, y-max=0.65
4.	4-Link model	Subsystem	Commonly used blocks	----
5.	5-Gain	Gain	Commonly used blocks	Gain = -0.00072

Table 3. Track Simulink model

Sl. No.	Name in model	Name of block in browser	Source	Properties
1.	1-u1	In1	Commonly used blocks	Port number = 2
2.	2-u2	In1	Commonly used blocks	Port number = 1
3.	3-Phi	Integrator	Commonly used	Initial condition =

				blocks	0
4.	4-Tan_phi	Trigonometric function	Math operations	Function: tan
5.	5-One by l	Gain	Commonly used blocks	Gain = 1
6.	6-Product	Product	Math operations	Number of inputs = 2
7.	7-Theta	Integrator	Commonly used blocks	Initial condition = 0
8.	8-Cos_theta	Trigonometric function	Math operations	Function: cos
9.	9-Sin_theta	Trigonometric function	Math operations	Function: sin
10.	10-u1_Cos_theta	Product	Math operations	Number of inputs = 2
11.	11-u1_Sin_theta	Product	Math operations	Number of inputs = 2
12.	12-x	Integrator	Commonly used blocks	Initial condition = -1
13.	13-y	Integrator	Commonly used blocks	Initial condition = 0
14.	14-x	Out1	Commonly used blocks	Port number = 1
15.	15-y	Out1	Commonly used blocks	Port number = 3
16.	16-theta	Out1	Commonly used blocks	Port number = 4
17.	17-phi	Out1	Commonly used blocks	Port number = 2

Separate model also has been built for link model connecting to block 4 i.e. 4-Link model and the same is shown in Figure 6. Details of blocks used for link Simulink model are given in Table 4.
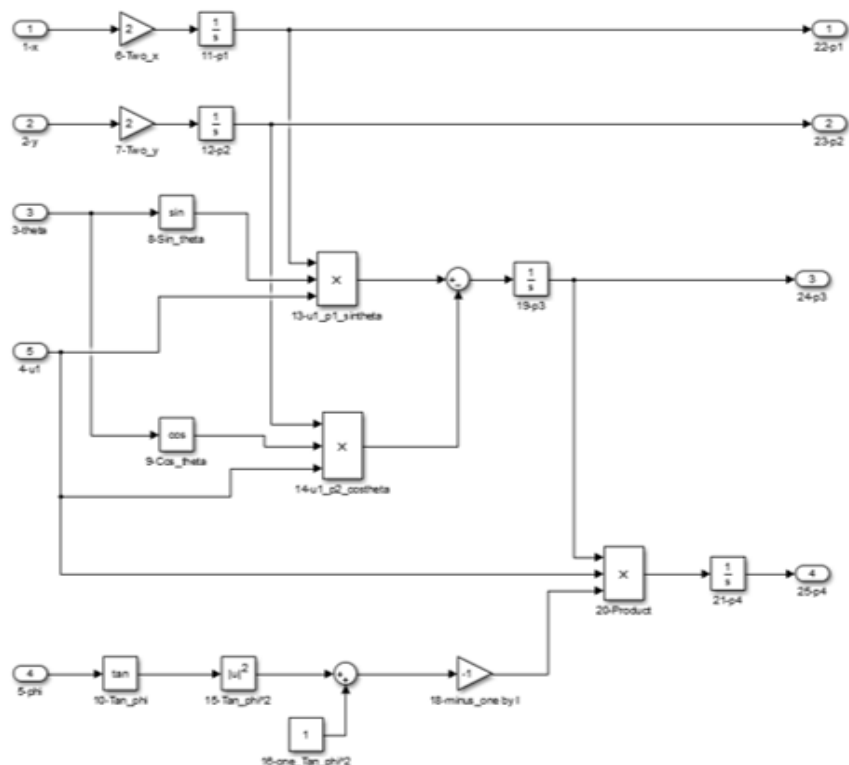
Figure 6. Link Simulink model

Table 4. Details of blocks used for link Simulink model

Sl.No.	Name in model	Name of block in browser	Source	Properties
1.	1-x	In1	Commonly used blocks	Port number = 2
2.	2-y	In1	Commonly used blocks	Port number = 1
3.	3-theta	Integrator	Commonly used blocks	Initial condition = 0
4.	4-u1	Trigonometric function	Math operations	Function: tan
5.	5-phi	Gain	Commonly used blocks	Gain = 1
6.	6 Two_x	Product	Math operations	Number of inputs = 2
7.	7- Two_y	Integrator	Commonly used blocks	Initial condition = 0
8.	8- Sin _theta	Trigonometric function	Math operations	Function: cos
9.	9- Cos_theta	Trigonometric function	Math operations	Function: sin
10.	10-Tan_phi	Product	Math operations	Number of inputs = 2
11.	11-p1	Product	Math operations	Number of inputs = 2
12.	12-p2	Integrator	Commonly used blocks	Initial condition = -1
13.	13-	Integrator	Commonly used blocks	Initial condition = 0

	u1_p1_sinthet a			
14.	14- u1_p2_costhet a	Out1	Commonly used blocks	Port number = 1
15.	15-Tan_phi^2	Out1	Commonly used blocks	Port number = 3
16.	16- one_Tan_phi^ 2	Out1	Commonly used blocks	Port number = 4
17.	Un named block	Sum	Math operations	List of signs = +-
18.	18- Minus_oneby l	Gain	Commonly used blocks	Gain = -1
19.	19-p3	Integrator	Commonly used blocks	Initial condition = -18000
20.	20-Product	Product	Math operations	Number of inputs = 2
21.	21-p4	Integrator	Commonly used blocks	Initial condition = -10000
22.	22-p1	Out1	Commonly used blocks	Port number = 1
23.	23-p2	Out1	Commonly used blocks	Port number = 2
24.	24-p3	Out1	Commonly used blocks	Port number = 3
25.	25-p4	Out1	Commonly used blocks	Port number = 4

4. RESULTS

The figure 7 shows that the trajectory is so smooth that the change in speed is smooth and driver doesn't feel any fatigue. The figure 8 explains the lane changeover pattern of the vehicle in X-Y Plane viewed from the Top. This graph provides the confidence that the smooth transition takes from one lane to another lane change over which is as expected in the mathematical model. This smooth change over indicates that the driver of the follower vehicle will have enough time to change his position and hence possibility of accident can be avoided.
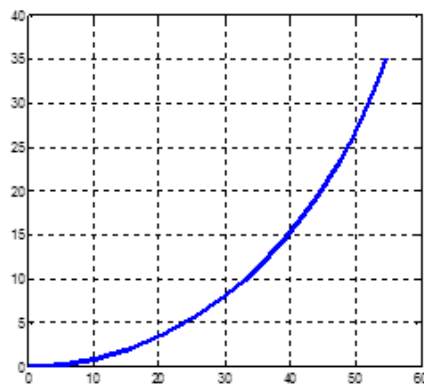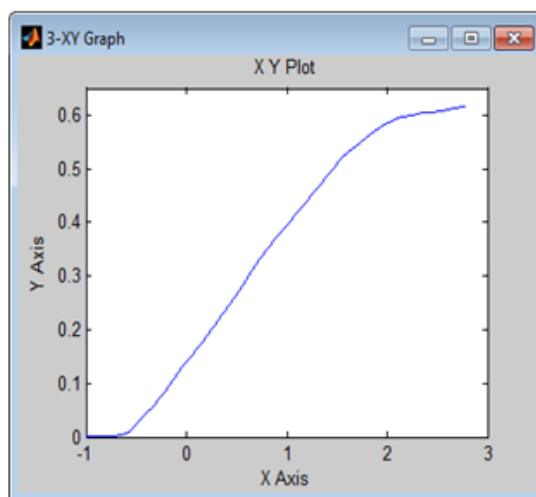

Figure 7. Trajectory output in MATLAB

Figure 8. Optimal trajectory for lane change in Simulink

5. CONCLUSIONS

This controller will take control of vehicle in hand overriding the actions of driver so as to avoid mishap due to driver's mistakes. However the controller being a computer needs logic to execute the above cited task. Primary objective of this project is to develop such logic i.e. trajectory model. The Lane change concept is discussed in detail along with steps involved in Lane change process. Intended Lane change model will be useful to reach the desired optimal trajectory. The model is developed in MATLAB and same results are verified in SIMULINK software vehicle.

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