

## **A Multi – Item Inventory Pricing Model With A Rate Of Production Proportional To The Rate Of Power Demand**

**I. Antonitte Vinoline , W. Ritha , J. Merline Vinotha**

Department of Mathematics, Holy Cross College (Autonomous), Tiruchirappalli – 620 002, Tamilnadu, India.

Email : [arulavanto@gmail.com](mailto:arulavanto@gmail.com), [ritha\\_prakash@yahoo.co.in](mailto:ritha_prakash@yahoo.co.in), [merlinevinotha@gmail.com](mailto:merlinevinotha@gmail.com)

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### **ABSTRACT:**

Customers are more likely to buy more in real life if the product is more readily available. We develop an inventory model for inventory-dependent demand with various holding cost functions in this paper. When demand corresponds with the law of power and varies with time, this paper provides a monetary lot size model, which is suited for many real-life situations. The rate of output is considered to be proportionate to the demand rate. We further suppose that, because demand is price-sensitive, demand declines linearly with price. The goal is to maximize the total profit characteristic while accumulating the highest quality values of schedule period, reorder point, and price. The most appropriate choice has been proven, and it is to raise overall stock revenue and choose the best variables. The suggested model is determined through numerical calculation.

**KEYWORDS:** Inventory Model, Multi – item, Holding Cost, Demand Rate.

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### **I. INTRODUCTION:**

Multi-product valuation works by making small, scientific changes in the value of all of the products in a portfolio. The portfolio's average value should remain relatively stable – this will ensure that customers do not notice a change in the value-for-price supplied by our entire or shop. Maintaining the average price could also ease suspicious competitors who are worried about disrupting the market. This research attempts to construct a production inventory model by assuming that the finite production rate is proportionate to both the demand rate and the inventory level when the demand rate follows a time function. The predicted price function's mathematical formula appears. Inventory difficulties are mostly linked with proper inventory management, which can lead to a reduction in inventory value. In general, we have two types of materials in our everyday needs in terms of wastage, degradation, or decay. Radioactive chemicals, food grains, modern products, prescription pharmaceuticals, and other products with finite life spans, such as electronic goods,

steels, and woods, are examples. The stock level or inventory is always decreasing due to the limited shelf-life and market demand, and the items within the inventory exhaust or decay. The inventory will suffer as a result of this deterioration, and the value of the inventory will rise.

An acceptable inventory model must be established in order to create the inventory value at the optimum level, i.e. to induce the least inventory value. This study has planned a list model with linear demand, a little of decay, and a constant production rate to reduce inventory value. The assembly is supposed to begin with the buffer stock as a reserve, and to end when certain periods of inventory are reached. We included a steady production rate in conjunction with deterioration in our model, unlike traditional inventory models and many studies employ instant renewal. A producing system's production cost is determined by the combination of several production parameters. These variables include (a) raw materials, (b) technical information, (c) manufacturing technique, (d) firm size, (e) product quality, and so on. In most cases, the pricing of raw materials is inexact. The value of technical information, i.e. labor cost, has been believed to be constant in the past. However, because organizations and employees perform the same activity over and over, they learn how to consistently deliver a consistent level of performance. As a result, the process value per unit product drops with each cycle. In each cycle, a portion of the ordering value may also drop. Various types of learning effects are investigated in a variety of fields.

In order to overcome the aforementioned problems, this study proposes a model of a degrading object with a random coming up with horizon, in which the lifespan of the item is believed to be random in nature and follows an exponential distribution with a well-known mean. Every production cycle, the unit cost lowers due to the learning impacts of the personnel on productivity. Similarly, thanks to the learning effects of the workers, the setup value in each cycle is part constant and part decreasing. The model is designed and solved to maximize the projected benefit of the full coming up with horizon. It's accompanied by some numerical data and examples. Within the most real situations demand of shoppers varies with time. Therefore, several researchers have worked on time-varying inventory models. Gholamian and Heydari [1] developed a mixed number random programming model by random relations in a problem. Escuín et al. [2] mentioned inventory models victimization random and time variable demand for a paper manufacturer. San-José et al. [3] analyzed a inventory system with demand following an power law and partial backordering. Demand of customers is value sensitive, in order that valuation is in the foremost necessary selections in a company. Tripathi et al. [4] investigated inventory dependent demand with an influence rate and holding price functions for two things. Alfares and Ghaithan [5] worked on a valuation and inventory system considering value dependent demand and time-varying holding price together with amount discounts. Chiu et al. [6] investigated the impact of delayed

differentiation on a vendor-buyer system with process and multiple things. Panda et al. [7] given a replenishment and valuation in a supply chain. Liu et al.[8] studied a joint investment and valuation downside for perishable products considering value and quality dependent demand . Sicilia et al. [9] studied lot size models wherever demand follows an influence law and also the and also the rate is uniform. . Sarkar et al. [10] studied a production system with demand looking on time and worth alongside the influence of inflation and responsible.

## II. NOTATIONS AND ASSUMPTIONS

To develop the integrated model, the following notations and assumptions are defined throughout this paper.

**P**-Period of Schedule.

**M** -Number of Products.

**P<sub>i</sub>(i)** -Production level for item.

**S<sub>i</sub>** -Reorder Point.

**d<sub>i</sub>**- Quantity demanded during the inventory cycle for i<sup>th</sup> item .

**t<sub>pi</sub>** - Period of Production.

**Ad<sub>i</sub>**-Average number of items required.

**sp<sub>i</sub>**- Set up Cost for Producing the i<sup>th</sup> item.

**s<sub>c<sub>i</sub></sub>**- Selling Price.

**pc<sub>i</sub>**- Unit production cost.

**Ch<sub>i</sub>** - Cost of holding an i<sup>th</sup> item.

**b<sub>i</sub>** - Cost of Backlogging an i<sup>th</sup> item.

**P<sub>r<sub>i</sub></sub>(t)** - Rate of producing an item at time t.

**N<sub>s1</sub>(t)** - Lot size level at time t (0≤t≤t<sub>pi</sub> ).

**N<sub>s2</sub>(t)** - Lot size level at time t(0≤t≤P).

**BC<sub>i</sub>**-Item backordering cost.

**HC<sub>i</sub>** - Holding item cost.

**SC<sub>i</sub>**- Setup cost.

**PC<sub>i</sub>**- Production Cost.

**RS<sub>i</sub>**- Revenue item for sales.

**d<sub>i</sub>(t)** -Rate of demand at time t for i<sup>th</sup> item.

**TP<sub>i</sub>(SC<sub>i</sub> , S<sub>i</sub> , P )** - Total Profit for item i.

**Π (  $\vec{p}$ ,  $\vec{s}$ , P )** - Total Profit of the system.

### Assumptions:

1. The planning horizon is infinite.
2. Shortages are totally backordered.
3. Demand rate changes with time with an influence pattern and reduces linearly with worth.

So, the demand rate is assumed to be  $d_i(t) = (c_i - e_i s_{c_i}) \frac{A_{d_i}}{r_i} \left(\frac{t}{P}\right)^{\frac{1}{r_i}-1}$  with  $0 \leq t \leq P$ , where  $0 < r_i < \infty, c_i > 0, e_i > 0$

4. Multiple things are assumed for the inventory system.
5. The demand rate is a smaller amount than the assembly rate for every item.
6. The production rate  $P_{r_i}(t)$  is proportional to demand rate  $d_i(t)$  for every item  $i$  at any time  $t(0 \leq t \leq t_{p_i})$  and is outlined by  $P_{r_i}(t) = \beta_i d_i(t)$  with  $\beta_i > 1$ .

### III. MATHEMATICAL MODELLING:

Let us consider a factory that produces  $M$  completely different things (where  $i = 1, 2, \dots, M$ ). Every item has average demand  $A_{d_i}$  that has got to be glad. The demand varies with time with an influence pattern and reduces linearly with worth. Additionally the assembly rate changes proportionately with demand rate. The manager wishes to satisfy the client demand and optimize the entire profit of the system, at the same time. The inventory cycle starts with  $s$  units of internet stock at time zero. At identical time  $t = \text{zero}$ , production begins with the assembly rate  $P_{r_i}(t)$ , at time  $t_{1i}$  reaches zero and continues till  $t = t_{p_i}$  for every product  $i$ , consequently the filling amount energy are going to be made. Additionally throughout the interval  $[0, t_{p_i}]$ , the inventory level of product  $i$  will increase at a rate  $P_{r_i}(t) - d_i(t)$ . Then the stock level decreases up to  $t = t_{2i}$  in line with demand. Finally, throughout the interval  $[t_{2i}, P]$ , demand is backlogged. Assume that  $N_{s1}(t)$  and  $N_{s2}(t)$  are a unit the on-hand inventory levels of item  $i$  at time  $t$  within the intervals  $[0, A_{d_i}]$  and  $[A_{d_i}, P]$ , severally. The programming amount, the backorder size and also the asking price are three call variables of the system. Within the following, an approach is conferred to seek out the optimum values.

Rate of demand is taken as

$$d_i(t) = (c_i - e_i s_{c_i}) \frac{A_{d_i}}{r_i} \left(\frac{t}{P}\right)^{\frac{1}{r_i}-1} \text{ where } 0 < r_i < \infty, \\ c_i > 0, e_i > 0$$

Demand during the period of schedule  $[0, P]$

$$\int_0^P d_i(t) dt = \int_0^P (c_i - e_i s_{c_i}) \frac{A_{d_i}}{r_i} \left(\frac{t}{P}\right)^{\frac{1}{r_i}-1} dt \\ d_i(t) dt = (c_i - e_i s_{c_i}) A_{d_i} P \dots(1)$$

Differential equations that describes the system

$$\begin{aligned}\frac{dN_{s1}(t)}{dt} &= Pr_i(t) - d_i(t) \quad \text{where } Pr_i(t) = \beta_i d_i(t) \\ &= (\beta_i - 1) (c_i - e_i s_{c_i}) \frac{A_{d_i}}{r_i} \left(\frac{t}{P}\right)^{\frac{1}{r_i}-1} \quad \dots(2)\end{aligned}$$

$$N_{s1}(0) = s_i \quad \text{where } 0 \leq t \leq tp_i$$

$$\begin{aligned}\frac{dN_{s2}(t)}{dt} &= -d_i(t) \\ &= -(c_i - e_i s_{c_i}) \frac{A_{d_i}}{r_i} \left(\frac{t}{P}\right)^{\frac{1}{r_i}-1} \quad \dots(3)\end{aligned}$$

$$N_{s2}(P) = s_i, \quad \text{where } tp_i \leq t \leq P$$

Let the boundary conditions are

$$N_{s1}(0) = N_{s2}(P) = s_i,$$

The solutions of equation (1) and equation (2) are ,

Integrate equation (2) on both sides

$$\begin{aligned}\int \frac{dN_{s1}(t)}{dt} dt &= \int (\beta_i - 1) (c_i - e_i s_{c_i}) \frac{A_{d_i}}{r_i} \left(\frac{t}{P}\right)^{\frac{1}{r_i}-1} dt + c \\ N_{s1}(t) &= (\beta_i - 1) (c_i - e_i s_{c_i}) \frac{A_{d_i}}{r_i} P \left(\frac{t}{P}\right)^{\frac{1}{r_i}} + c \quad \dots(*)\end{aligned}$$

$$\text{Put } t=0, \therefore c = s_i$$

Substitute  $c = s_i$  in equation (\*)

$$N_{s1}(t) = (\beta_i - 1) (c_i - e_i s_{c_i}) \frac{A_{d_i}}{r_i} P \left(\frac{t}{P}\right)^{\frac{1}{r_i}} + s_i \quad 0 \leq t \leq tp_i \quad \dots(4)$$

Integrate equation (3) on both sides

$$\begin{aligned}\int \frac{dN_{s2}(t)}{dt} dt &= \int -(c_i - e_i s_{c_i}) \frac{A_{d_i}}{r_i} P \left(\frac{t}{P}\right)^{\frac{1}{r_i}-1} dt + c \\ N_{s2}(t) &= -(c_i - e_i s_{c_i}) A_{d_i} P \left(\frac{t}{P}\right)^{\frac{1}{r_i}} + c\end{aligned}$$

$$\text{Let } t=P, \quad c = s_i + (c_i - e_i s_{c_i}) A_{d_i} P$$

Substituting  $c$  in  $N_{s2}(t)$

$$N_{s2}(t) = s_i + (c_i - e_i s_{c_i}) A_{d_i} P - (c_i - e_i s_{c_i}) A_{d_i} P \left(\frac{t}{P}\right)^{\frac{1}{r_i}} \quad \dots(5)$$

$$\text{where } tp_i \leq t \leq P$$

At time  $Ad_i$  the assembly cycle of heap size is finished and therefore the most inventory level will be calculated by each relative Equation (4) and Equation (5). In order that  $Ad_i$  and stock level at  $Ad_i$  are respectively:

$$N_{s1}(tp_i) = s_i + (\beta_i - 1) (c_i - e_i s_{c_i}) A_{d_i} P \left(\frac{tp_i}{P}\right)^{\frac{1}{r_i}}$$

$$N_{s2}(tp_i) = s_i + (c_i - e_i s_{c_i}) Ad_i P - (c_i - e_i s_{c_i}) Ad_i P \left( \frac{tp_i}{P} \right)^{\frac{1}{r_i}}$$

$$\text{Since } N_{s1}(tp_i) = N_{s2}(tp_i)$$

$$\left[ (\beta_i - 1) + 1 \right] (c_i - e_i s_{c_i}) Ad_i P \left( \frac{tp_i}{P} \right)^{\frac{1}{r_i}} = (c_i - e_i s_{c_i}) Ad_i P$$

$$tp_i = \frac{P}{\beta_i r_i} \quad \dots(6)$$

$$\begin{aligned} N_{s1}(tp_i) &= s_i + (\beta_i - 1)(c_i - e_i s_{c_i}) Ad_i P \left( \frac{tp_i}{P} \right)^{\frac{1}{r_i}} \\ &= s_i + \left( \frac{\beta_i - 1}{\beta_i} \right) (c_i - e_i s_{c_i}) Ad_i P \quad \dots(7) \end{aligned}$$

Production level for product i is calculated by

$$P_i(i) = \int_0^{tp_i} Pr_i(t) dt$$

$$P_i(i) = (c_i - e_i s_{c_i}) Ad_i P \quad \dots(8)$$

As it was expected, the leap amount is adequate the demand of planning amount. we have a tendency to assume that  $l(tp_i) \geq 0$  and  $s_i \leq 0$ , in order that  $(c_i - e_i s_{c_i}) Ad_i P \left( \frac{\beta_i - 1}{\beta_i} \right) \leq s_i \leq 0$ .

Suppose that the stock level reaches zero within the production amount at time  $t_{1i}$ .

Since  $N_{s1}(t_{1i}) = 0$ , from Equation (4) we have a tendency to get  $t_{1i}$  for item i

According to the variables  $s_i$  and  $P$ :

$$\begin{aligned} N_{s1}(t_{1i}) &= \\ s_i + (\beta_i - 1)(c_i - e_i s_{c_i}) Ad_i P \left( \frac{t_{1i}}{P} \right)^{\frac{1}{r_i}} &= 0 \\ t_{1i} &= \left( \frac{-s_i}{(\beta_i - 1)(c_i - e_i s_{c_i}) Ad_i P} \right)^{r_i} P \quad \dots(9) \end{aligned}$$

The net stock level of interval  $[tp_i, P]$  reaches zero at time  $t_{2i}$ . Resolution equation  $N_{s1}(t_{1i}) = 0$ ,  $t_{2i}$  can be obtained for item i in step with variables  $s_i$  and  $P$

Solving  $N_{s2}(t_{2i}) = 0$ ,  $t_{2i}$  can be obtained

$$\begin{aligned} N_{s2}(t_{2i}) &= s_i + (c_i - e_i s_{c_i}) Ad_i P \left( 1 - \left( \frac{t_{2i}}{P} \right)^{\frac{1}{r_i}} \right) = 0 \\ t_{2i} &= \left( \left[ 1 + \frac{-s_i}{(c_i - e_i s_{c_i}) Ad_i P} \right]^{r_i} \right) P \quad \dots(10) \end{aligned}$$

We take into account four varied value within the inventory system for every product i as follows.

Average number of production run is  $\frac{1}{P}$

**Holding Item Cost:**

$$HC_i = \left( \left[ 1 + \frac{-s_i}{(c_i - e_i s_{c_i}) Ad_i P} \right]^{r_i} \right) P - \left( \left[ \frac{-s_i}{(\beta_i - 1)(c_i - e_i s_{c_i}) Ad_i P} \right]^{r_i} \right) P$$

$$= P \left[ \frac{[(c_i - e_i s_{c_i}) Ad_i P + s_i]^{r_i}}{[(c_i - e_i s_{c_i}) Ad_i P]^{r_i}} - \frac{(-s_i)^{r_i}}{((\beta_i - 1)^{r_i} (c_i - e_i s_{c_i})^{r_i} Ad_i^{r_i} P^{r_i})} \right]$$

**Cost of holding an i<sup>th</sup> item:**

$$HC_i = \frac{C_{h_i} (-s_i)^{r_i+1}}{((\beta_i - 1)^{r_i+1} (c_i - e_i s_{c_i})^{r_i} Ad_i^{r_i} P^{r_i+1})} - \frac{C_{h_i} (c_i - e_i s_{c_i}) Ad_i P}{\beta_i^{r_i} (r_i+1)} \quad \dots(11)$$

**Item Backordering Cost:**

$$BC_i = \left( \frac{[s_i (c_i - e_i s_{c_i}) Ad_i P]^{r_i+1}}{(r_i+1) (c_i - e_i s_{c_i})^{r_i} Ad_i^{r_i} P^{r_i}} + \frac{(-s_i)^{r_i+1}}{((r_i+1)(\beta_i - 1)^{r_i} (c_i - e_i s_{c_i})^{r_i} Ad_i^{r_i} P^{r_i})} - \frac{(c_i - e_i s_{c_i}) Ad_i P}{(r_i+1)} - s_i \right) b_i \quad \dots(12)$$

**Lost Sale Cost:**

$$\begin{aligned} LSC &= S \int_{t_1}^P (1 - \gamma) d(t) dt \\ &= S(1 - \gamma) \left[ (c_i - e_i s_{c_i}) Ad_i \left( \frac{1}{P^{r_i-1}} \right) [(P)^{\frac{1}{r_i}} - (t_1)^{\frac{1}{r_i}}] \right] \quad \dots(13) \end{aligned}$$

**The Production Cost:**

$$\begin{aligned} PC_i &= p_{c_i} \frac{P_i(i)}{P} \\ &= (c_i - e_i s_{c_i}) p_{c_i} Ad_i \quad \dots(14) \end{aligned}$$

**The Setup Cost :**

$$SC_i = \frac{S_{p_i}}{P} \quad \dots(15)$$

**Revenue Item For Sales :**

$$\begin{aligned} RS_i &= S_{c_i} \frac{P_i(i)}{P} \\ &= (c_i - e_i s_{c_i}) S_{c_i} Ad_i \quad \dots(16) \end{aligned}$$

**Total Profit :**

$$\begin{aligned} TP_i(S_{c_i}, S_i, P) &= RS_i - HC_i - BC_i - PC_i - SC_i - LSC \\ &= (c_i - e_i s_{c_i}) S_{c_i} Ad_i - \frac{(Ch_i + b_i) [s_i + (c_i - e_i s_{c_i}) Ad_i P]^{r_i+1}}{(r_i+1) (c_i - e_i s_{c_i})^{r_i} Ad_i^{r_i} P^{r_i}} + \\ &\quad \frac{(Ch_i + b_i) (-s_i)^{r_i+1}}{((r_i+1)(\beta_i - 1)^{r_i} (c_i - e_i s_{c_i})^{r_i} Ad_i^{r_i} P^{r_i})} + \frac{(c_i - e_i s_{c_i}) Ad_i P}{(r_i+1)} \left( \frac{Ch_i}{(\beta_i)^{r_i}} \right) + b_i S_i - \frac{S_{p_i}}{P} - (c_i - e_i s_{c_i}) p_{c_i} Ad_i - S(1 - \gamma) \end{aligned}$$

$$\gamma \left[ (c_i - e_i s_{c_i}) Ad_i \left( \frac{1}{p^{r_i-1}} \right) [(P)^{\frac{1}{r_i}} - (t_1)^{\frac{1}{r_i}}] \right] \quad \dots(17)$$

$$\therefore \pi(\vec{p}, \vec{s}, P) = \sum_{i=1}^N TP_i(p_i, s_i, P) \quad \dots(18)$$

With relevancy the aim of this project that's finding the simplest production policies to maximize the overall profit per unit time for the multi-product inventory system .

$$\frac{\partial \pi(\vec{p}, \vec{s}, P)}{\partial s_i} = \frac{(-s_i)^{r_i} (Ch_i + b_i)}{((\beta_i - 1)^{r_i} (c_i - e_i s_{c_i})^{r_i} Ad_i^{r_i} P^{r_i})} - \frac{(Ch_i + b_i) [s_i + (c_i - e_i s_{c_i}) Ad_i P]^{r_i}}{(c_i - e_i s_{c_i})^{r_i} Ad_i^{r_i} P^{r_i}} + b_i \quad \dots(19)$$

$$\begin{aligned} \frac{\partial \pi(\vec{p}, \vec{s}, P)}{\partial P} &= \sum_{i=1}^M \frac{r_i (Ch_i + b_i) (-s_i)^{r_i+1}}{((r_i+1)(\beta_i - 1)^{r_i} (c_i - e_i s_{c_i})^{r_i} Ad_i^{r_i} P^{r_i})} - \sum_{i=1}^M s_i + (c_i - e_i s_{c_i}) Ad_i P^{r_i} \frac{((c_i - e_i s_{c_i}) Ad_i P - r_i s_i) (Ch_i + b_i)}{(r_i+1)(c_i - e_i s_{c_i})^{r_i} Ad_i^{r_i} P^{r_i+1}} \\ &+ \sum_{i=1}^M \frac{(c_i - e_i s_{c_i}) Ad_i}{(r_i+1)} \left( \frac{Ch_i}{(\beta_i)^{r_i}} + b_i \right) + \frac{\sum_{i=1}^M S_{p_i}}{P^2} = 0 \end{aligned} \quad \dots(20)$$

Assume a new variable  $z_i = \frac{-s_i}{(c_i - e_i s_{c_i}) Ad_i P}$ ,

$$-(c_i - e_i s_{c_i}) \frac{(\beta_i - 1)}{\beta_i} nP \leq s_i \leq 0 \text{ is equivalent to } 0 \leq z_i \leq \frac{(\beta_i - 1)}{\beta_i}$$

Equation (19) and (20) becomes

$$\begin{aligned} \text{Equation (19) implies } & \frac{(-s_i)^{r_i} (Ch_i + b_i)}{((\beta_i - 1)^{r_i} (c_i - e_i s_{c_i})^{r_i} Ad_i^{r_i} P^{r_i})} - \frac{(Ch_i + b_i) [s_i + (c_i - e_i s_{c_i}) Ad_i P]^{r_i}}{(c_i - e_i s_{c_i})^{r_i} Ad_i^{r_i} P^{r_i}} + b_i \\ &= z_i^{r_i} \left( \frac{(Ch_i + b_i)}{(\beta_i - 1)^{r_i}} \right) - \left( -z_i^{r_i} + (c_i - e_i s_{c_i})^{r_i} Ad_i^{r_i} P^{r_i} (Ch_i + b_i) \right) + b_i \\ &= (1 - z_i^{r_i}) - \frac{z_i^{r_i}}{(\beta_i - 1)^{r_i}} - \frac{b_i}{(Ch_i + b_i)} \end{aligned} \quad \dots(21)$$

Equation (20) implies

$$\begin{aligned} \sum_{i=1}^M \frac{r_i z_i^{r_i+1} (c_i - e_i s_{c_i}) Ad_i (Ch_i + b_i)}{(r_i+1)(\beta_i - 1)^{r_i}} - \sum_{i=1}^M \frac{(1 - z_i)^{r_i} (1 + r_i z_i) (c_i - e_i s_{c_i}) Ad_i (Ch_i + b_i)}{(r_i+1)} \\ + \sum_{i=1}^M \frac{(c_i - e_i s_{c_i}) Ad_i}{(r_i+1)} \left( \frac{Ch_i}{(\beta_i)^{r_i}} + b_i \right) + \frac{\sum_{i=1}^M S_{p_i}}{P^2} \end{aligned} \quad \dots(22)$$

Let us equate the equations (21) and (22) to zero , we get

$$(1 - z_i)^{r_i} - \frac{z_i^{r_i}}{(\beta_i - 1)^{r_i}} - \frac{b_i}{(Ch_i + b_i)} = 0 \quad \dots(23)$$

$$\begin{aligned} \sum_{i=1}^M \frac{r_i z_i^{r_i+1} (c_i - e_i s_{c_i}) Ad_i (Ch_i + b_i)}{(r_i+1)(\beta_i - 1)^{r_i}} - \sum_{i=1}^M \frac{(1 - z_i)^{r_i} (1 + r_i z_i) (c_i - e_i s_{c_i}) Ad_i (Ch_i + b_i)}{(r_i+1)} \\ + \sum_{i=1}^M \frac{(c_i - e_i s_{c_i}) Ad_i}{(r_i+1)} \left( \frac{Ch_i}{(\beta_i)^{r_i}} + b_i \right) + \frac{\sum_{i=1}^M S_{p_i}}{P^2} = 0 \end{aligned} \quad \dots(24)$$

Therefore the following function has the unique solution  $z_i^*$  is

$$(1 - z_i)^{r_i} - \frac{z_i^{r_i}}{(\beta_i - 1)^{r_i}} - \frac{b_i}{(Ch_i + b_i)} = 0 \text{ on interval } \left( 0, \frac{\beta_i - 1}{\beta_i} \right)$$



From equation (23) we have

$$\frac{z_i^{r_i}}{(\beta_i - 1)^{r_i}} = (1 - z_i)^{r_i} - \frac{b_i}{(Ch_i + b_i)} = 0 \quad \dots(25)$$

By substituting equation (25) in equation (24) we get

$$\sum_{i=1}^M \frac{(c_i - e_i s_{c_i}) Ad_i (Ch_i + b_i) (1 - z_i)^{r_i}}{(r_i + 1)} - \sum_{i=1}^M \frac{(r_i) (c_i - e_i s_{c_i}) Ad_i (b_i - z_i)}{(r_i + 1)} + \sum_{i=1}^M \frac{(c_i - e_i s_{c_i}) Ad_i}{(r_i + 1)} \left( \frac{Ch_i}{(\beta_i)^{r_i}} + b_i \right) + \frac{\sum_{i=1}^M S_{p_i}}{p^2} = 0 \quad \dots(26)$$

$$P = \sqrt{\frac{\sum_{i=1}^M S_{p_i}}{\sum_{i=1}^M \left[ \left( \frac{(c_i - e_i s_{c_i}) Ad_i}{(r_i + 1)} \right) \left( (Ch_i + b_i) (1 - z_i)^{r_i} + r_i b_i z_i - \left( \frac{Ch_i}{(\beta_i)^{r_i}} + b_i \right) \right) \right]}}$$

The best cycle length  $P^*$  for given  $\vec{p}$  is

$$P^* = \sqrt{\frac{\sum_{i=1}^M S_{p_i}}{\sum_{i=1}^M \left[ \left( \frac{(c_i - e_i s_{c_i}) Ad_i}{(r_i + 1)} \right) \left( (Ch_i + b_i) (1 - z_i)^{r_i} + r_i b_i z_i - \left( \frac{Ch_i}{(\beta_i)^{r_i}} + b_i \right) \right) \right]}}$$

#### IV. NUMERICAL EXAMPLE:

Consider a production system with one item and also the following values for the input parameters.  $S_{p_i} = 100$ ,  $Ch_i = 4$ ,  $b_i = 5$ ,  $Ad_i = 1200$ ,  $c = 100$ ,  $e = 2$ ,  $\beta = 1.5$  and also the index of the power demand pattern  $r = 3$ ,  $z_i^* = 0.161603$

Optimum values for different cases are given below:

Scenario	Production cost ( $p_{c_i}$ )	$P^*$
Case 1	10	0.0519
Case 2	15	0.0555
Case 3	20	0.0600
Case 4	25	0.0657
Case 5	30	0.0734

From the above table, it is clear that in increase in production cost increases the total cost of the system.

#### V. CONCLUSION

In this paper, a specific demand rate known as power demand has been bestowed on an economic

production model. It is assumed that consumer demand is directly related to price, and that production rate fluctuates proportionally to demand rate. Multiple products are considered to be in the inventory system, and shortages are permitted and guaranteed. Mathematical modelling and improvement tactics are commonly used as the best solutions for a variety of product issues. Because attaining optimal inventory policies for the second state of affairs is difficult, a simple heuristic approach for multi-items is planned. Many examples are provided, such as the model's applications victimizing various parameter values. The results show that when the assembly rate parameter is fixed, if the unit cost increases, the overall profit and therefore the economic ton size fall, although the shortest cycle time and therefore the optimum increment rise. Because the production rate increases, the entire function, the simplest programming amount, and thus the best ton amount, decreases little. When the index of power demand is fixed, if the unit cost rises, the overall profit performance and thus the economic ton amount fall, however the simplest worth and thus the best cycle duration rise.

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