Nat. Volatiles & Essent. Oils, 2021; 8(4): 9849-9860

NVEO
Natural Volatiles &
Essential Oils

Fixed Point Theorem With Integral-Type Inequality On Multiplicative Metric Spaces

Dr. A. Mary Priya Dharsini[1] , Dr. A. Leema Maria Prakasam[2] , Dr. A. Jennie Sebasty Pritha[3]

1, 2, 3 – Assistant Professor, PG and Research Department of Mathematics, Holy Cross College(Autonomous), Tiruchirappalli-620002. Mail Id: priyairudayam@gmail.com

Abstract

We prove some fixed point theorem on integral-type inequality in the setting of multiplicative metric in order to find the existence and uniqueness of the some fixed point.

Keywords: Integral-type inequality, Multiplicative metric space

Mathematics subject classification: Primary 47H10; Secondary 54H25

1. Introduction

In the domain of mathematics, science and engineering, fixed point theorem plays a significant role. The scientific basics of the fixed point theory was established in the 20^{th} century. In 1922, Stephan Banach established the theorem called "Banach fixed point theorem"based on finding the existence of solution for integral and non-linear equation. The fixed point theory for multivalued operators in metric space has been used in many works published in specially literature. The fixed point of certain important single-valued mapping also plays an important role, as their result can be applied in engineering, physics, economics and in telecommunication.

The first constructive method of calculating the fixed point of a continuous function was presented by H scarf in the year 1973.

Multiplicative metric space was first introduced by Bashirov at all in 2008 among various thing, started a new kind of spaces. The main purpose to replace usual triangular inequality by a " Multiplicative triangle inequality". By using the concept of multiplicative absolute value and multiplicative distance ozavsar and cevikel introduced multiplicative metric space observed its topological properties and defined some fixed point theorem for multiplicative contraction mapping using multiplicative space.

In 1986, Jungck introduced a lot referred to as compatible mapping that are more general of commuting and weakly commuting maps. Common fixed point theorems for two pairs of weakly compatible mapping distribution metric space have recently introduced by K. Jhaet.al and K. Jha and D. Panthi.

Recently Branciari finds a fixed point result for a single mapping fulfiling an correlation principle for an integral type inequality. In this consideration, we initiate some fixed points theorem in multiplicative metric space using integral type implied relation with integral-type inequality.

2. Preliminaries

2. Multiplicative Metric spaces

Definition 2.1:

Let Y be a nonempty set. A multiplicative metric is mapping $\nabla: \mathrm{Y} \times \mathrm{Y} \rightarrow \mathrm{R}$ satisfying the following conditions:

1) $\nabla(\mathrm{w}, \mathrm{v}) \geq 1$ for all $\mathrm{w}, \mathrm{v} \in \mathrm{Y}$ and $\nabla(\mathrm{w}, \mathrm{v})=1$ if and only if $\mathrm{w}=\mathrm{v}$;
2) $\nabla(\mathrm{w}, \mathrm{v})=\nabla(\mathrm{v}, \mathrm{w})$ for all $\mathrm{w}, \mathrm{v} \in \mathrm{Y}$
3) $\nabla(\mathrm{w}, \mathrm{v}) \leq \nabla(\mathrm{w}, \mathrm{t}) . \nabla(\mathrm{t}, \mathrm{v})$ for all $\mathrm{w}, \mathrm{v}, \mathrm{t} \in \mathrm{Y}$ (multiplicativetriangleinequality)

Theorem 2.2:

Let $\Re_1, \Re_2, \Re_3, \Re_4$ be Self-quadruple mapping of an multiplicative metric space (Y,∇), satisfying the following conditions:

a) $(\Re_1, \Re_3)$and $(\Re_2, \Re_4)$ satisfy $(\mathrm{CLR}_{\Re_3 \Re_4})$ property
b) $(\Re_1, \Re_3)$ and $(\Re_2, \Re_4)$ are weakly compatible.
c) $\Re_1, \Re_2, \Re_3, \Re_4$ satisfy the inequality

$$\eta\left(\int_0^{\frac{1}{\mu} \nabla^\mu(\Re_1 \lambda^*, \Re_2 \mathrm{u})} \gamma(\mathrm{t}) \mathrm{dt}\right) \leq \mathcal{L} \Psi_1\left(\int_0^{\varphi_1(\mathrm{M}(\lambda^*, \mathrm{u}))} \gamma(\mathrm{t}) \mathrm{dt}\right) \Psi_1\left(\int_0^{\varphi_1(\mathrm{N}(\lambda^*, \mathrm{u}))} \gamma(\mathrm{t}) \mathrm{dt}\right)$$

For $\lambda^*, \mathrm{u} \in \mathrm{Y}, \mu \in (0,1)$, where

$$\varphi_1\big(\mathcal{M}(\lambda^*,u)\big)$$

$$= \frac{1}{\mu}\max\left\{\begin{matrix} \nabla(\Re_1\lambda^*,\Re_4u),\nabla(\Re_2u,\Re_4u),\nabla(\Re_4u,\Re_3\lambda^*), \\ \frac{\nabla(\Re_1\lambda^*,\Re_4u)\nabla(\Re_1\lambda^*,\Re_4\lambda^*)}{2},\frac{\nabla(\Re_2u,\Re_3\lambda^*)\nabla(\Re_1\lambda^*,\Re_2u)}{1+\nabla(\Re_2u,\Re_1\lambda^*)}, \\ \frac{\nabla(\Re_1\lambda^*,\Re_4u)\nabla(\Re_3\lambda^*,\Re_4u)}{1+\nabla(\Re_2\lambda^*,\Re_3\lambda^*)} \end{matrix}\right\}^{\mu}$$

$$\varphi_2\big(N(\lambda^*,u)\big)$$

$$= \varphi_2(\nabla(\Re_1\lambda^*,\Re_4u),\nabla(\Re_2u,\Re_4u),\nabla(\Re_4u\Re_3\lambda^*),\frac{\nabla(\Re_1\lambda^*,\Re_4u)\nabla(\Re_3\lambda^*,\Re_4u)}{1+\nabla(\Re_2\lambda^*,\Re_3\lambda^*)}),$$

Then, the self-quadruple mappings $\Re_1,\Re_2,\Re_3,\Re_4$ have a unique common fixed point $u\in$ Y.

Corollary 2.3:

Let $\Re_1$,$\Re_2$,$\Re_3$,$\Re_4$ be self-quadruple mappings of an multiplicative metric space (Y,∇), Satisfying the following condition:

(a) ($\Re_1$,$\Re_3$) and ($\Re_2$,$\Re_4$) satisfy ($CLR_{\Re_3\Re_4}$) property.

(b) ($\Re_1$,$\Re_3$) and ($\Re_2$,$\Re_4$) are weakly compatible.

(c)$\Re_1$,$\Re_2$,$\Re_3$,and $\Re_4$ Satisfy the inequality

$$\eta\int_0^{\frac{1}{\mu}\nabla^{\mu}(\Re_1\lambda^*,\Re_2y)}\gamma(t)dt \preceq \psi_1\Big(\int_0^{\varphi_1(p(\lambda^*,y))}\gamma(t)dt\Big)\,\psi_2\Big(\int_0^{\varphi_2(q(\lambda^*,y))}\gamma(t)dt\Big)$$

For λ^*, y $\in$ Y, $\mu\in(0,1)$, where

$$\varphi_1\big(P(\lambda^*,y)\big) = \frac{1}{\mu}\max\left\{\begin{matrix}\nabla(\Re_1\lambda^*,\Re_4y),\nabla(\Re_2y,\Re_4y),\nabla(\Re_4y,\Re_3\lambda^*),\frac{\nabla(\Re_1\lambda^*,\Re_4y)\nabla(\Re_1\lambda^*,\Re_3\lambda^*)}{2}, \\ \frac{\nabla(\Re_2y,\Re_3\lambda^*)\nabla(\Re_1\lambda^*,\Re_2y)}{1+(\Re_2y,\Re_1\lambda^*)},\frac{\nabla(\Re_1\lambda^*,\Re_4y)\nabla(\Re_3\lambda^*,\Re_4y)}{1+(\Re_2\lambda^*,\Re_3\lambda^*)},\end{matrix}\right\}^{\mu},$$

$$\varphi_2\big(Q(\lambda^*,y)\big) = \varphi_2(\nabla(\Re_1\lambda^*,\Re_4y),\nabla(\Re_2y,\Re_4y),\nabla(\Re_4y,\Re_3\lambda^*),\frac{\nabla(\Re_1\lambda^*,\Re_4y)\nabla(\Re_3\lambda^*,\Re_4y)}{1+(\Re_2y,\Re_3\lambda^*)})$$

Then, the self-quadruple mappings $\Re_1$,$\Re_2$,$\Re_3$, $\Re_4$ have a unique common fixed point $y \in Y$.

By considering $\Re_1 = \Re_2$ and $\Re_3(t) = \Re_4(t) = t$ in theorem, we have the following result.

Corollary 2.4:

Let $\Re_1$ be a self-mapping of an multiplicative metric space (Y,∇), satisfying the following integral condition

$$\eta\Big(\int_0^{\frac{1}{\mu}\nabla^{\mu}(\Re_1\lambda^*,\Re_2 y)} \gamma(t)dt \leqq \mathcal{L}\psi_1\Big(\int_0^{\varphi_1(p(\lambda^*,y))} \gamma(t)dt\Big)\, \psi_2\Big(\int_0^{\varphi_2(q(\lambda^*,y))} \gamma(t)dt\Big)$$

For $\lambda^*, y \in Y$, $\mu \in (0,1)$, where

$$\varphi_1(P(\lambda^*,y)) = \frac{1}{\mu}\max\begin{Bmatrix} \nabla(\Re_1\lambda^*,y), \nabla(\Re_1 y,y), \nabla(y,\lambda^*), \frac{\nabla(\Re_1\lambda^*,y)\nabla(\Re_1\lambda^*,\lambda^*)}{2}, \\ \frac{\nabla(\Re_1 y,\lambda^*)\nabla(\Re_1\lambda^*,\Re_1 y)}{1+(\Re_1 y,\Re_1\lambda^*)}, \frac{\nabla(\Re_1\lambda^*,y)\nabla(\lambda^*,y)}{1+(\Re_1\lambda^*,\lambda^*)} \end{Bmatrix}^{\mu},$$

$$\varphi_2(Q(\lambda^*,y)) = \varphi_2(\nabla(\Re_1\lambda^*,y), \nabla(\Re_1 y,y), \nabla(y,\lambda^*)\frac{\nabla(\Re_1\lambda^*,y)\nabla(\lambda^*,y)}{1+(\Re_1 y,\lambda^*)})$$

Then, $\Re_1$ has a unique common fixed point $y \in Y$.

3. SOME FIXED POINT THEOREM WITH INTEGRAL-TYPE INEQUALITY ON MULTIPLICATIVE METRIC SPACE

Theorem 3.1:

Let $\Re_1$,$\Re_2$,$\Re_3$,$\Re_4$ be self-quadruple mappings of an multiplicative metric space (Y,∇), Satisfying the following condition:

(a) $(\Re_1,\Re_3)$ and $(\Re_2,\Re_4)$ satisfy $(CLR_{\Re_3\Re_4})$ property.

(b) $(\Re_1,\Re_3)$ and $(\Re_2,\Re_4)$ are weakly compatible.

(c) $\Re_1,\Re_2,\Re_3$, and $\Re_4$ Satisfy the inequality

$$\eta\left(\int_0^{\frac{1}{\mu}\nabla^{\mu}(\Re_1\lambda^*,\Re_2 y)} \gamma(t)dt\right) \leqq \mathcal{L}\psi_1$$

$$\left(\int_0^{\varphi_1(p(\lambda^*,y))} \gamma(t)dt\right)\psi_2\left(\int_0^{\varphi_2(q(\lambda^*,y))} \gamma(t)dt\right)$$

(2.1)

For $\lambda^*, y \in Y$, $\mu \in (0,1)$, where

$$\varphi_1(P(\lambda^*,y)) = \frac{1}{\mu}\max\left\{\begin{matrix}\nabla(\Re_1\lambda^*,\Re_4 y),\nabla(\Re_2 y,\Re_4 y),\nabla(\Re_4 y,\Re_3\lambda^*),\dfrac{\nabla(\Re_1\lambda^*,\Re_4 y)\nabla(\Re_1\lambda^*,\Re_3\lambda^*)}{2}, \\ \dfrac{\nabla(\Re_2 y,\Re_3\lambda^*)\nabla(\Re_1\lambda^*,\Re_2 y)}{1+(\Re_2 y,\Re_1\lambda^*)}\end{matrix}\right\}^{\mu}$$

$$\varphi_2(Q(\lambda^*,y)) = \varphi_2(\nabla(\Re_1\lambda^*,\Re_4 y),\nabla(\Re_2 y,\Re_4 y),\nabla(\Re_4 y,\Re_3\lambda^*))$$

Then, the self-quadruple mappings $\Re_1,\Re_2,\Re_3,\Re_4$ have a unique common fixed point $y \in Y$.

Proof:

By the help of our supposition of the $(CLR_{\Re_3\Re_4})$ property of the pairs $(\Re_1,\Re_2)$, and $(\Re_3,\Re_4)$, we assume two sequence $\{\lambda_n^*\}$ and $\{y_n\}$ in multiplicative metric space (Y,∇) such that

$$\lim_{n\to\infty}\Re_1(\lambda_n^*) = \lim_{n\to\infty}\Re_3(\lambda_n^*) = \lim_{n\to\infty}\Re_2\, y_n = \lim_{n\to\infty}\Re_4\, y_n = w, \qquad (2.2)$$

for $w \in \Re_3(Y) \cap \Re_4(Y)$

$$\lim_{n\to\infty}\Re_1(\lambda_n^*) = \lim_{n\to\infty}\Re_3(\lambda_n^*) = \lim_{n\to\infty}\Re_2\, y_n = \lim_{n\to\infty}\Re_4\, y_n = w = \Re_3 v \qquad (2.3)$$

We show that $\Re_1 v = \Re_3 v$.

For this, we follow by the contradiction, that is, $\Re_1 v \neq \Re_3 v$ and define the following sequence:

$y_{2n} = \Re_1(\lambda_{2n}^*) = \Re_4\lambda_{2n+1}^*$ and

$$y_{2n+1} = \Re_2(\lambda^*_{2n+1}) = \Re_3(\lambda^*_{2n+2})$$

by the help of (2.1), we have

$$\eta\left(\int_0^{\frac{1}{\mu}\nabla^{\mu}(\Re_1 v, \Re_2 y_n)} \gamma(t)dt\right) \leqq \mathcal{L}\psi_1$$

$$\left(\int_0^{\varphi_1(p(v,y_n))} \gamma(t)dt\right)\psi_2\left(\int_0^{\varphi_2(q(v,y_n))} \gamma(t)dt\right)(2.4)$$

For $\lambda^* = v$ and $y = y_n$ in the equality (2.4), where

$$\varphi_1(P(v,y_n)) = \frac{1}{\mu}\max\left\{\begin{matrix}\nabla(\Re_1 v, \Re_4 y_n), \nabla(\Re_2 y_n, \Re_4 y_n), \nabla(\Re_4 y_n, \Re_3 v), \frac{\nabla(\Re_1 v, \Re_4 y_n)\nabla(\Re_1 v, \Re_3 v)}{2}, \\ \frac{\nabla(\Re_2 y_n, \Re_3 v)\nabla(\Re_1 v, \Re_2 y_n)}{1+((\Re_2 y_n, \Re_1 v))}\end{matrix}\right\}^{\mu}$$

(2.5)

$$(\varphi_2(Q(v,y_n)) = \varphi_2(\nabla(\Re_1 v, \Re_4 y_n), \nabla(\Re_2 y_n, \Re_4 y_n), \nabla(\Re_4 y_n, \Re_3 v))$$

(2.6)

Taking the $\lim_{n\to\infty}$ in (2.5), (2.6) and (2.4), respectively, we get

$$\lim_{n\to\infty}\varphi_1(P(v,y_n)) = \frac{1}{\mu}\lim_{n\to\infty}\max$$

$$\left\{\begin{matrix}\nabla(\Re_1 v, \Re_4 y_n), \nabla(\Re_2 y_n, \Re_4 y_n), \nabla(\Re_4 y_n, \Re_3 v), \frac{\nabla(\Re_1 v, \Re_4 y_n)\nabla(\Re_1 v, \Re_3 v)}{2}, \\ \frac{\nabla(\Re_2 y_n, \Re_3 v)\nabla(\Re_1 v, \Re_2 y_n)}{1+((\Re_2 y_n, \Re_1 v))}\end{matrix}\right\}^{\mu}$$

$$= \frac{1}{\mu}\max\left\{\begin{matrix}\nabla(\Re_1 v, w), \nabla(w,w), \nabla(w,w), \frac{\nabla(\Re_1 v, w)\nabla(\Re_1 v, y)}{2}, \\ \frac{\nabla(w,w)\nabla(\Re_1 v, w)}{1+((\Re_2 y_n, \Re_1 v))}\end{matrix}\right\}^{\mu} \quad (2.7)$$

$$= \frac{1}{\mu}\max\left\{\nabla(\Re_1 v, w), 1, 1, \nabla(\Re_1 v, w), 1\right\}^{\mu}$$

$$= \frac{1}{\mu}\nabla^{\mu}(\Re_1 v, w)$$

$$\lim_{n\to\infty}\varphi_2(Q(v,y_n)) = \lim_{n\to\infty}\varphi_2(\nabla(\Re_1 v, \Re_4 y_n), \nabla(\Re_2 y_n, \Re_4 y_n), \nabla(\Re_4 y_n, \Re_3 v))$$

$$= \varphi_2(\nabla(\Re_1 v, w), \nabla(w, w), \nabla(w,w)) \quad (2.8)$$

$$= \varphi_2(\nabla(\Re_1 v, w), 1, 1)$$

= 1, and

$$\lim_{n\to\infty}\eta\left(\int_0^{\frac{1}{\mu}\nabla^{\mu}(\Re_1 v,\Re_2 y_n)}\right) \leqq \mathcal{L}\lim_{n\to\infty}\psi_1$$

$$\left(\int_0^{\varphi_1(p(v,y_n))}\gamma(t)dt\right)\lim_{n\to\infty}\psi_2\left(\int_0^{\varphi_2(q((v,y_n)))}\gamma(t)dt\right)(2.9)$$

$$= \mathcal{L}\lim_{n\to\infty}\psi_1\left(\int_0^{\frac{1}{\mu}\nabla^{\mu}(\Re_1 v,w)}\gamma(t)dt \quad \lim_{n\to\infty}\psi_2\left(\int_0^1\gamma(t)dt\right)\right.$$

by the use of (2.3), (2.7), and (2.8) in (2.9), we have

$$\eta\left(\int_0^{\frac{1}{\mu}\nabla^{\mu}(\Re_1 v,w)}\gamma(t)dt\right) \leqq \psi_1\left(\int_0^{\frac{1}{\mu}\nabla^{\mu}(\Re_1 v,w)}\gamma(t)dt\right)$$

Which is a contradiction of $\eta(t) > \psi_1(t)$. This contradiction is due to our supposition $\Re_1 v \neq \Re_3 v$, and hence $\Re_1 v = \Re_3 v$. Also from (2.2), we have w ϵ $\Re_4(Y)$. This implies w = $\Re_4(u)$, for some u ϵ Y.

Now, we show that $\Re_4(u) = \Re_2(u)$, for this we assume the contrary path, that is $\Re_4(u) \neq \Re_2(u)$. By putting $\lambda^* = \lambda_n^*$ and w=u in (2.4), and following the same lines as above for proof of $\Re_3 v = \Re_1 v = w$, we can get $\Re_4(u) = \Re_2(u) = y$.

Consequently, we have

$$\Re_4(u) = \Re_2(u) = \Re_3 v = \Re_1 v = w$$

Since, $(\Re_1, \Re_2)$ and $(\Re_3, \Re_4)$ are weakly compatible. Therefore, $\Re_3 v = \Re_1 v$ implies $\Re_1\Re_3 v = \Re_3\Re_1 v$ which implies $\Re_1 w = \Re_3 w$.

Similarly, we have $\Re_4 w = \Re_2 w$. Next, we show that w is a common fixed point of $\Re_1, \Re_2, \Re_3, \Re_4$. For this, let assume that $\Re_1 w \neq w$, and putting $\lambda^* = w$ and $y = u$ in (2.4) we have

$$\eta\left(\int_0^{\frac{1}{\mu}\nabla^{\mu(}\Re_1 w,\Re_2(u))}\gamma(t)dt\right) \leqq \mathcal{L}\psi_1\left(\int_0^{\varphi_1(p(w,u))}\gamma(t)dt\right)\psi_2\left(\int_0^{\varphi_2\left(q((w,u))\right)}\gamma(t)dt\right)$$

where,

$$\varphi_1\big(P(w,u)\big) = \frac{1}{\mu}\max\begin{Bmatrix}\nabla\big(\Re_1 w,\Re_4(u)\big),\nabla(\Re_2(u),\Re_4(u)),\nabla(\Re_4(u),\Re_3 w),\\ \frac{\nabla(\Re_1 w,\Re_4(u))\nabla(\Re_1 w,\Re_3 w)}{2},\\ \frac{\nabla(\Re_2(u),\Re_3 w)\nabla(\Re_1 w,\Re_2(u))}{1+\big((\Re_2(u),\Re_1 w)\big)}\end{Bmatrix}^{\mu}$$

$$=\frac{1}{\mu}\max\begin{Bmatrix}\nabla(\Re_1 w,w),\nabla(w,w),\nabla(w,\Re_3 w),\\ \frac{\nabla(\Re_1 w,w)\nabla(\Re_1 w,\Re_3 w)}{2},\\ \frac{\nabla(w,\Re_3 w)\nabla(\Re_1 w,w))}{1+\big((\Re_2(u),\Re_1 w)\big)}\end{Bmatrix}^{\mu}$$

$$=\frac{1}{\mu}\nabla^{\mu(}\Re_1 w,w),$$

and

$$\varphi_2\big(Q(w,u)\big) = \varphi_2(\nabla(\Re_1 w,\Re_4(u)),\nabla(\Re_2(u),\Re_4(u)),\nabla(\Re_4(u),\Re_3 w))$$

$$= \varphi_2(\nabla(\Re_1 w,w)),\nabla(w,w),\nabla(w,\Re_1 w))$$

$$= \varphi_2\big(\nabla(\Re_1 w,w),1,\nabla(w,\Re_1 w)\big)$$

$$= 1$$

by the use of (2.3), (2.7) and (2.8) in (2.9), we have

$$\eta\left(\int_0^{\frac{1}{\mu}\nabla^{\mu(}\Re_1 w,w)}\gamma(t)dt\right) \leqq \psi_1\left(\int_0^{\frac{1}{\mu}\nabla^{\mu(}\Re_1 w,w)}\gamma(t)dt)\right)$$

For μ ϵ (0,1) which is a contradiction of $\eta(t) > \psi_1(t)$. This contradiction is due to our supposition that $\Re_1 w \neq w$, and hence $\Re_1 w = \Re_3 w = w$. Similarly, we can show $\Re_2 w = \Re_4 w = w$.

Ultimately, we have $\Re_1 w = \Re_3 w = \Re_2 w = \Re_4 w = w$.

Finally, we have that the common fixed point of the self-quadruple mapping $\mathfrak{R}_1, \mathfrak{R}_2, \mathfrak{R}_3, \mathfrak{R}_4$ is unique. For this, we again presume a contrary path, that is, let there exist two different fixed points such that

$$\mathfrak{R}_1 w_1 = \mathfrak{R}_3 w_1 = w_1, \mathfrak{R}_2 w_2 = \mathfrak{R}_2 w_2 = w_2$$

For some $w_1, w_2 \in Y$ such that $w_1 \neq w_2$. By putting $\lambda^* = w_1$and $y = w_2$ in (2.4), we have

$$\eta\left(\int_0^{\frac{1}{\mu}\nabla^{\mu}(\mathfrak{R}_1 w_1, \mathfrak{R}_2 w_2)} \gamma(t)dt\right) = \eta\left(\int_0^{\frac{1}{\mu}\nabla(w_1, w_2)} \gamma(t)dt\right) \quad (2.10)$$

$$\leq \eta\psi_1\left(\int_0^{\varphi_1(p(w_1,w_2))} \gamma(t)dt\right)$$

$$\psi_2\left(\int_0^{\varphi_2(q(w_1,w_2))} \gamma(t)dt\right),$$

Where

$$\varphi_1(P(w_1, w_2)) = \frac{1}{\mu}\max\begin{Bmatrix} \nabla(\mathfrak{R}_1 w_1, \mathfrak{R}_4 w_1), \nabla(\mathfrak{R}_2 w_2, \mathfrak{R}_4 w_2), \nabla(\mathfrak{R}_4 w_2, \mathfrak{R}_3 w_1), \\ \frac{\nabla(\mathfrak{R}_1 w_1, \mathfrak{R}_4 w_2)\nabla(\mathfrak{R}_1 w_1, \mathfrak{R}_3 w_1)}{2}, \\ \frac{\nabla(\mathfrak{R}_2 w_2, \mathfrak{R}_3 w_1)\nabla(\mathfrak{R}_1 w_1, \mathfrak{R}_2 w_2)}{1+((\mathfrak{R}_2 w_2, \mathfrak{R}_1 w_1))} \end{Bmatrix}^{\mu} \quad (2.11)$$

$$= \frac{1}{\mu}\max\begin{Bmatrix} \nabla(w_1, w_1), \nabla(w_2, w_2), \nabla(w_2, w_1), \\ \frac{\nabla(w_1, w_2)\nabla(w_1, w_1)}{2}, \\ \frac{\nabla(w_2, w_1)\nabla(w_1, w_2)}{1+((w_2, w_1))} \end{Bmatrix}^{\mu}$$

$$= \frac{1}{\mu}\nabla^{\mu}(w_1, w_2),$$

and

$$\varphi_2(Q(w_1, w_2)) = \varphi_2(\nabla(\mathfrak{R}_1 w_1, \mathfrak{R}_4 w_2), \nabla(\mathfrak{R}_2 w_2, \mathfrak{R}_4 w_2), \nabla(\mathfrak{R}_4 w_2, \mathfrak{R}_3 w_1))$$

$$= \varphi_2(\nabla(w_1, w_2), \nabla(w_2, w_2), \nabla(w_2, w_1)) \quad (2.12)$$

$$= \varphi_2(\nabla(w_1, w_2), 1, \nabla(w_2, w_1))$$

$$= 1$$

By the use of (2.11), (2.12) in (2.10), we have

$$\eta\left(\int_0^{\frac{1}{\mu}\nabla^{\mu}(w_1,w_2)}\gamma(t)dt\right) \leqq \psi_1\left(\int_0^{\frac{1}{\mu}\nabla^{\mu}(w_1,w_2)}\gamma(t)dt\right),$$

Which is a contradiction of the fact $\eta(t) > \psi_1$. Thus $w_1 = w_2$ and therefore the common fixed point of the self-quadruple mappings. $\mathfrak{R}_1, \mathfrak{R}_2, \mathfrak{R}_3, \mathfrak{R}_4$ is unique.

If we assume that $\varphi_2(t) = \frac{t}{\mathcal{L}}$, we have the following corollary.

Corollary 3.2:

Let $\mathfrak{R}_1$,$\mathfrak{R}_2$,$\mathfrak{R}_3$,$\mathfrak{R}_4$ be self-quadruple mappings of an multiplicative metric space (Y,∇), Satisfying the following condition:

(a) ($\mathfrak{R}_1$,$\mathfrak{R}_3$) and ($\mathfrak{R}_2$,$\mathfrak{R}_4$) satisfy ($CLR_{\mathfrak{R}_3\mathfrak{R}_4}$) property.

(b) ($\mathfrak{R}_1$,$\mathfrak{R}_3$) and ($\mathfrak{R}_2$,$\mathfrak{R}_4$) are weakly compatible.

(c)$\mathfrak{R}_1$,$\mathfrak{R}_2$,$\mathfrak{R}_3$,and $\mathfrak{R}_4$ Satisfy the inequality

$$\eta\left(\int_0^{\frac{1}{\mu}\nabla^{\mu}(\mathfrak{R}_1\lambda^*,\mathfrak{R}_2 y)}\gamma(t)dt\right) \leqq \psi_1\left(\int_0^{\varphi_1(p(\lambda^*,y))}\gamma(t)dt\right)\psi_2\left(\int_0^{\varphi_2(q(\lambda^*,y))}\gamma(t)dt\right)$$

for λ^*, y ϵ Y, μ ϵ (0,1), where

$$\varphi_1(P(\lambda^*,y)) = \frac{1}{\mu}\max\begin{Bmatrix}\nabla(\mathfrak{R}_1\lambda^*,\mathfrak{R}_4 y), \nabla(\mathfrak{R}_2 y,\mathfrak{R}_4 y), \nabla(\mathfrak{R}_4 y,\mathfrak{R}_3\lambda^*), \frac{\nabla(\mathfrak{R}_1\lambda^*,\mathfrak{R}_4 y)\nabla(\mathfrak{R}_1\lambda^*,\mathfrak{R}_3\lambda^*)}{2}, \\ \frac{\nabla(\mathfrak{R}_2 y,\mathfrak{R}_3\lambda^*)\nabla(\mathfrak{R}_1\lambda^*,\mathfrak{R}_2 y)}{1+(\mathfrak{R}_2 y,\mathfrak{R}_1\lambda^*)},\end{Bmatrix}^{\mu},$$

$$\varphi_2(Q(\lambda^*,y)) = \varphi_2(\nabla(\mathfrak{R}_1\lambda^*,\mathfrak{R}_4 y), \nabla(\mathfrak{R}_2 y,\mathfrak{R}_4 y), \nabla(\mathfrak{R}_4 y,\mathfrak{R}_3\lambda^*))$$

Then, the self-quadruple mappings $\mathfrak{R}_1$,$\mathfrak{R}_2$,$\mathfrak{R}_3$, $\mathfrak{R}_4$ have a unique common fixed point y ϵ Y

By considering$\mathfrak{R}_1 = \mathfrak{R}_2$ and $\mathfrak{R}_3(t) = \mathfrak{R}_4(t) = t$ in theorem, we have the following result.

Corollary 3.3:

Let $\mathfrak{R}_1$ be a self-mapping of an multiplicative metric space (Y,∇), satisfying the following integral condition

$$\eta\left(\int_0^{\frac{1}{\mu}\nabla^{\mu}(\Re_1\lambda^*,\Re_2 y)}\gamma(t)dt\right)\leqq \mathcal{L}\psi_1\left(\int_0^{\phi_1\left(p(\lambda^*,y)\right)}\gamma(t)dt\right)\psi_2\left(\int_0^{\phi_2\left(q(\lambda^*,y)\right)}\gamma(t)dt\right)$$

Forλ^*, y ϵ Y, μ ϵ (0,1), where

$$\phi_1\left(P(\lambda^*,y)\right)=\frac{1}{\mu}\max\left\{\begin{matrix}\nabla(\Re_1\lambda^*,y),\nabla(\Re_1 y,y),\nabla(y,\lambda^*),\frac{\nabla(\Re_1\lambda^*,y)\nabla(\Re_1\lambda^*,\lambda^*)}{2}, \\ \frac{\nabla(\Re_1 y,\lambda^*)\nabla(\Re_1\lambda^*,\Re_1 y)}{1+(\Re_1 y,\Re_1\lambda^*)}\end{matrix}\right\}^{\mu},$$

$$\phi_2\left(Q(\lambda^*,y)\right)=\phi_2(\nabla(\Re_1\lambda^*,y),\nabla(\Re_1 y,y),\nabla(y,\lambda^*)$$

Then,$\Re_1$ has a unique common fixed point y ϵ Y.

References:

[1] A. A. N. Abdou, Common fixed point results for compatible-type mappings in multiplicative metric spaces, J.NonlinearSci.Appl.,9 (2016),2244–2257.

[2] R.P.Agarwal,M.A.El-Gebeily,D.O'Regan,Generalizedcontractionsinpartiallyorderedmetricspaces,Appl.Anal.,87 (2008),109–116.

[3] R. P. Agarwal, E. Karapınar, B. Samet, An essential remark on fixed point resultson multiplicativemetricspaces,Fixed Point TheoryAppl.,2016 (2016).

[4] R. P. Agarwal, M. Meehan, D. O'Regan, Fixed point theory and applications,CambridgeUniversityPress,Cambridge,(2001).

[5] M. U. Ali, T. Kamram, E. Karapınar, An approach to existence of fixed points ofgeneralizedcontractivemultivaluedmappingsofintegraltypeviaadmissiblemapping,Abstr.Appl.Anal.,2014(2014).

[6] H. Aydi, E. Karapınar, ˙I. S¸. Yuce, ¨ Quadruple fixed point theorems in partiallyorderedmetricspacesdependingonanotherfunction,Appl.Math.,2012 (2012).

[7] D. Baleanu, R. P. Agarwal, H. Khan, R. A. Khan, H. Jafari, On the existence ofsolution for fractional differential equations of order 3 < δ1 6 4, Adv. Difference Equ.,2015 (2015).

[8] D. Baleanu, H. Khan, H. Jafari, R. A. Khan, M. Alipour, On existence results forsolutionsofacoupledsystemofhybridboundaryvalueproblemswithhybridconditions,Adv.Di

fferenceEqu.,2015(2015).

[9] S. Banach, Sur les operation dans les ensembles abstraits et leur application auxequations integrals,Fund.Math.,3(1922),133–181.

[10] V.Berinde, M.Borcut, Triple fixed point theorems for contractive type mappings in partially ordered metric spaces, Non linear Anal.,74(2011),

4889–4897.

[11] M.-F.Bota, A.Petrus¸el,G.Petrus¸el ,B.Samet, Coupled fixed point theorems for single-valued operators in b-metric spaces, Fixed Point Theory Appl.,2015(2015).