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Interval-Valued Intuitionistic Fuzzy Cognitive Map Based On Llkelihood Concept For Multi-Attribute Decision Making

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Abstract

In this paper, Interval Valued Intuitionistic Fuzzy Cognitive Map (IVIFCM) and likelihood concept are combined to produce an integrated algorithm for Multi-attribute decision making (MADM). To know the efficacy of the proposed new formulated algorithm, food farming investment in organic or inorganic agriculture is taken as a multi-attribute problem. The Interval Valued Intuitionistic Fuzzy decision matrix andFuzzy Cognitive Map (FCM)matrix are collected from the expert in the form of linguistic variable. Finally the result of the adopted problem concludes the effectiveness of the proposed integrated algorithm.

Keyword: Interval Valued Intuitionistic Fuzzy Cognitive Map, Multi-attribute decision making, best investment in farming, linear programming model

1. Introduction:

In today's world, decision making shows the importance in entire life style. The set of alternatives are ranked or selected the feasible one regarding the information which is gained from the subject expert is the main process of Decision making. In Decision making process, Multiple attribute decision making (MADM) plays a vital role. In MADM problem, alternativesare examined with respect to the attribute and gives the finest solution to make decision for the concern problem. [Feng, F., Xu, Z., Fujita, H., & Liang, M. (2020).].

Due to some vagueness in fuzzy information Atanassov introduced the Intuitionistic fuzzy set(IFS)because fuzzy set have membership function only. To overcome the difficulty in fuzzy set, Atanassov introducing the non-membership function along with the membership function. IFS is the generalization of the Fuzzy set. As an extension of IFS, Interval valued intuitionistic fuzzy set is proposed by Atanassov and Gargov. In that extension, membership and non-membership are assigned in the form interval numbers. [Liu, P. (2017).]

In 1976, Axelrod introduced the tool cognitive map. Cognitive map is developed for studying the behavior of the decision making. Fuzzy cognitive map was introduced by Bart kosko in 1986 as an generalization of cognitive map. Fuzzy Cognitive map is a combination of artificial neural network and fuzzy logic.To Study and describing the perception of the expert for the given problem are most significance during this time. Based on the subject expert, fuzzy cognitive map is worked. [Tsadiras, A., & Zitopoulos, G. (2017).].

The focus of this paper work is to combine Interval Valued Intuitionistic Fuzzy Cognitive Map and likelihood concept to propose a new integrated algorithm. Interval valued Intuitionistic fuzzy cognitive map is utilized by its decision matrix and FCM matrixis obtained to get the final decision matrix. The part of likelihood concept is to identify the auxiliary linear programming model from the final decision matrix and solved it to attain the relative closeness coefficient interval, then the final results are found and arranged in the decreasing order regarding the optimal degree membership which is obtained from the likelihood matrix. To scrutinize the proposed algorithm, a multi-attribute problem food farming investment in organic or inorganic agriculture is chosen.

2. Preliminaries

Intuitionistic fuzzy sets [Atanassov, K. T. (1986) &Atanassov, K. T. (1999)**]**

Intuitionistic fuzzy set A in X is defined as follows:

$A= \{<x,\mu_A(x), \gamma_A(x)>\}: x\in E\}$

where μ_A:E→[0,1] and γ_A: E→[0,1] and every $x\in E : 0 \le \mu_A + \gamma_A \le 1$. Here μ_A called as membership degree and γ_A called as non-membership degree.

Interval-valued intuitionistic fuzzy set [Atanassov, K., & Gargov, G. (1989)]**]**

Consider A on X be an Interval valued intuitionistic fuzzy set which is defined asA ={, $\{< x, [\mu_A^L(x), \mu_A^U(x)], [\gamma_A^L(x), \gamma_A^U(x)] >| x \in X\}$ in which $[\mu_A^L(x), \mu_A^U(x)]$ represent the interval membership function and $[\gamma_A^L(x), \gamma_A^U(x)]$ represent the interval non-membership function.

Addition and Multiplication operators of interval valued intuitionistic fuzzy set:

$$A \oplus B = \{< x, [\mu_A^L(x) + \mu_B^L(x) - \mu_A^L(x) . \mu_B^L(x), \mu_A^U(x) + \mu_B^U(x) - \mu_A^U(x). \mu_B^U(x)], [\gamma_A^L(x). \gamma_B^L(x), \gamma_A^U(x). \gamma_B^U(x)] > / \, x \in X >\}$$

$$A \otimes B = \{< x, [\mu_A^L(x) + \mu_B^L(x) - \mu_A^U(x) . \mu_B^U(x), \gamma_A^L(x) + \mu_B^L(x) - \gamma_A^L(x). \gamma_B^u(x), [\gamma_A^U(x). \gamma_B^U(x), \gamma_A^U(x)\gamma_B^U(x)] >/ x \in X\}$$

Interval valued Intuitionistic fuzzy cognitive map [Hajek, P., & Prochazka, O. (2017, June)].**:**

Kosko and Dickerson defined <C, E> as a pair for FCM. Here the set of concepts is represented by C and adjacency matrix is represented by E. $e_{ji} \in [-1, 1]$ is the crisp weight contained in adjacency matrix. $c_j \in C$ is the concept represented in the form of fuzzy set. The connection between the concepts are represented depend either in the form of positive or negative. To figure dynamic procedure FCM is utilized in diverse iteration step t, where t= 1, 2, 3,...T . Here T is the length of the adopted sequence.

The position of the concept $c_i(t) \in [0, 1]$ explains the position of FCM in the iteration t .The position for successive iteration t+1 is evaluated as:

$$c_i(t+1) = f(c_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^{N} c_i(t) \times e_{ji})$$

According to the IVIFS, every concept $c_j \in C$ is replaced. After that, in the iteration t the position of i^{th} concept is calculated as

$$c_i(t) = \{\left[\mu_{c_i}^L(x), \mu_{c_i}^U(x)\right], \left[\gamma_{c_i}^L(x), \gamma_{c_i}^U(x)\right]\}(t).$$

Consider the weight $e_{ji} \in E$ is also denoted by IVIFS.

$$e_{ji} = \{\left[\mu_{e_{ji}}^L(x), \mu_{e_{ji}}^U(x)\right], \left[\gamma_{e_{ji}}^L(x), \gamma_{e_{ji}}^U(x)\right]\}$$

Applying the addition and multiplication operators, the succeeding concepts position are determined by $c_i(t+1)$.

$c_i(t+1) =$

$$f(\{[\mu_{c_i}^L(x),\mu_{c_i}^U(x)],[\gamma_{c_i}^L(x),\gamma_{c_i}^U(x)]\}(t)\oplus_{\substack{j=1\\ j\neq i}}^{n}(\{[\mu_{c_i}^L(x),\mu_{c_i}^U(x)],[\gamma_{c_i}^L(x),\gamma_{c_i}^U(x)]\}(t)\otimes\{[\mu_{e_{ji}}^L(x),\mu_{e_{ji}}^U(x)],[\gamma_{e_{ji}}^L(x),\gamma_{e_{ji}}^U(x)]\}$$

))

Likelihood Concept [Li, D. F. (2010)]**:**

The likelihood a>b is defined as

$$P(a>b) = \begin{cases} 1, a < b \\ 0, a \geq b \end{cases}$$

Where a and b be any two real numbers.

The likelihood of $a \geq b$ (a and b be any two interval numbers)is defined as

$$p(a \geq b) = \max\left\{1 - \max\left\{\frac{b^+ - a^-}{L(a) + L(b)}, 0\right\}, 0\right\}$$

Where $a = [a^-, a^+]$, $b = [b^-, b^+]$, $L(a) = [a^+ - a^-]$ and $L(b) = [b^+ - b^-]$.

3. Integrating algorithm for Interval valued Intuitionistic fuzzy multi attribute decision making.

The following procedure is formulated for MADM by integrating IVIFCM[Li, D. F. (2010)] and likelihood concept [Hajek, P., & Prochazka, O. (2017, June)].

Step 1: Obtain the IVIFdecision matrix from the expert through linguistic variable.

Step 2: Obtain the FCMmatrix and its value via linguistic variable from the Expert.

Step 3: Calculate the Final Decision matrix from the FCM graph and IVIFCM matrix

Step4:Determineand solve the Auxiliary linear programming model to obtain the relative closeness coefficient intervals for the alternatives.

$$c_i^u = \max\left\{\sum_{j=1}^{n}[t_j\,\mu_{ij}^u + y_j\left(1-\gamma_{ij}^l\right)\right\}$$

$$\text{s.t}\quad \begin{cases} z\omega_j^l \leq t_j \leq z\omega_j^u \quad (j = 1,2 \ldots . n) \\ z\rho_j^l \leq y_j \leq z\rho_j^u \quad (j = 1,2 \ldots . n) \\ \sum_{j=1}^{n}(t_j+y_j) = 1 \\ z \geq 0 \end{cases}$$

-------------------------------- (1)

and

$$c_i^l = \min\left\{ \sum_{j=1}^{n} [t_j \, \mu_{ij}^l + y_j \left(1 - \gamma_{ij}^u\right) \right\}$$

$$\text{s.t} \quad \begin{cases} z\omega_j^l \le t_j \le z\omega_j^u & (j = 1,2 \ldots . n) \\ z\rho_j^l \le y_j \le z\rho_j^u & (j = 1,2 \ldots . n) \\ \sum_{j=1}^{n}(t_j + y_j) = 1 \\ z \ge 0 \end{cases}$$

-------------------------------- (2)

STEP 5: Determine the likelihood of $x_i \ge x_j$ of alternative x_i and x_j by using

$$p\left(x_i \ge x_j\right) = p\left(c_i \ge c_j\right) = \max\left\{1 - \max\left(\frac{C_k^u - C_i^l}{L(C_i) + L(C_k)}, 0\right), 0\right\}$$

-------------------------------- (3)

where $C_i = [C_i^l, C_i^u]$, $C_k = [C_k^l, C_k^u]$, $L(C_i) = C_i^u - C_i^l$ and $L(C_k) = C_k^u - C_k^l$

andObtain the likelihood matrix (i.e) pairwise comparison of the alternative from **Step** 5.

$$\text{(i.e.) } P = (p_{ik})_{mxn} = \begin{array}{c} \\ X_1 \\ X_2 \\ \vdots \\ X_m \end{array} \begin{array}{c} \begin{array}{cccc} X_1 & X_2 & \cdots & X_m \end{array} \\ \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mm} \end{pmatrix} \end{array}$$

STEP 6: Find out the optimal degree $\theta_i$by utilizing equation (4) for the alternatives

$\theta_i = \frac{1}{m(m-1)} \left(\sum_{k=1}^{m} P_{ik} + \frac{m}{2} - 1\right)$--------------------- (4)

STEP 7: To identify the best alternative arrange the optimal degree $\theta_i$in the non-increasing order.

4. Problem description:

Suppose a farmer wants to do investment in farming. There are two type of farming are there. One is organic farming and another one is inorganic farming. So the farmer want to decide whether he put his investment in organic or inorganic for that four alternatives are taken. They are A1- Organic field farming, A2- Organic farming in greenhouses with soil, A3- Inorganic farming in greenhouses with soil and A4- Inorganic field farming. The alternatives are want to analyze to know the best outcome so that some attributes are considered to evaluate the alternatives. The attributes are X1-Previously applied production system and technologies, X2- Annual average net income, X3- Increase in labor requirement, X4- Need for alternation and X5- Soil Characteristic. In decision making to evaluate the alternative, the importance of attributes is needed. For that, the importance of the attributes are gathered from the experts are given below: X1 - [0.7, 0.8][0.1, 0.2], X2 - [0.6, 0.75][0.3, 0.4], X3 - [0.45, 0.55][0.15, 0.35], X4 - [0.25, 0.35][0.5, 0.6] and X5 - [0.15, 0.25][0.8, 0.9]

Step1: obtained IVIF decision matrix from the experts through linguistic variable.

Table 1: IVIF decision matrix

	A1	A2	A3	A4
X1	[0.4, 0.5] [0.3, 0.4]	[0.6, 0.7] [0.1, 0.2]	[0.2, 0.3] [0.5, 0.6]	[0.6, 0.7] [0.1, 0.2]
X2	[0.8, 0.9] [0, 0.1]	[0.4, 0.5] [0.3, 0.4]	[0.6, 0.7] [0.1, 0.2]	[0.6, 0.7] [0.1, 0.2]
X3	[0.6, 0.7] [0.1, 0.2]	[0.2, 0.3] [0.5, 0.6]	[0.4, 0.5] [0.3, 0.4]	[0.8, 0.9] [0, 0.1]
X4	[0.8, 0.9] [0, 0.1]	[0.8, 0.9] [0, 0.1]	[0, 0.1] [0.7, 0.8]	[0, 0.1] [0.7, 0.8]
X5	[0.8, 0.9] [0, 0.1]	[0.8, 0.9] [0, 0.1]	[0.6, 0.7] [0.1, 0.2]	[0, 0.1] [0.7, 0.8]

Step2`: Obtained FCMmatrix and its value via linguistic variable from the experts

	X1	X2	X3	X4	X5
X1	-	[0.1, 0.2] [0.6, 0.7]	[0.3, 0.4] [0.4, 0.6]	[0.1, 0.2] [0.6, 0.7]	[0.3, 0.4] [0.4, 0.6]
X2	[0.5, 0.6] [0.2, 0.3]	-	[0.5, 0.6] [0.2, 0.3]	[0.7, 0.8] [0, 0.1]	[0.7, 0.8] [0, 0.1]
X3	[0.1, 0.2] [0.6, 0.7]	[0.1, 0.2] [0.6, 0.7]	-	[0.7, 0.8] [0, 0.1]	0

X4	[0.4, 0.5] [0.3, 0.4]	[0.7, 0.8] [0, 0.1]	[0.3, 0.4] [0.4, 0.6]	-	[0.7, 0.8] [0, 0.1]
X5	[0.4, 0.5] [0.3, 0.4]	[0.3, 0.4] [0.4, 0.6]	[0.1, 0.2] [0.6, 0.7]	[0.7, 0.8] [0, 0.1]	-

Step 3: Final decision matrix is found from Step1 and Step 2

Table 2: Final Decision matrix

	A1	A2	A3	A4
X1	[0.83, 0.93] [0.003, 0.02]	[0.84, 0.93] [0.003, 0.02]	[0.58, 0.71] [0.02, 0.09]	[0.74, 0.88] [0.009, 0.048]
X2	[0.93, 0.98] [0, 0.16]	[0.80, 0.92] [0, 0.028]	[0.68, 0.82] [0.018, 0.07]	[0.71, 0.85] [0.14, 0.07]
X3	[0.84, 0.94] [0.002, 0.24]	[0.62, 0.78] [0.01, 0.102]	[0.62, 0.78] [0.027, 0.09]	[0.76, 0.96] [0, 0.024]
X4	[0.97, 0.99] [0, 0.0007]	[0.94, 0.98] [0, 0.004]	[0.61, 0.80] [0.004, 0.04]	[0.75, 0.91] [0, 0.02]
X5	[0.96, 0.99] [0, 0.002]	[0.94, 0.98] [0, 0.005]	[0.88,0.95] [0.004, 0.03]	[0.52, 0.63] [0.02, 0.11]

Step4:the following relative closeness coefficient intervals are obtained by solving the auxiliary linear programming model:

$$C_1^L = 0.9041222 \; C_1^U = 0.9801458$$

$$C_2^L = 0.8752414 \; C_2^U = 0.9558125$$

$$C_3^L = 0.7692135 \; C_3^U = 0.8937979$$

$$C_4^L = 0.8038681 \; C_4^U = 0.9270$$

Step 5: Determined likelihood matrix

$$P = \begin{pmatrix} 0.5 & 1 & 1 & 0.88 \\ 0.33 & 0.5 & 0.9 & 0.74 \\ 0 & 0.09 & 0.5 & 0.36 \\ 0.11 & 0.25 & 0.63 & 0.5 \end{pmatrix}_{4x4}$$

Step 6: optimal degree membership for the alternatives are found from Step 5

θ_1=0.365 θ_2= 0.289 θ_3= 0.162 θ_4 = 0.124

Step7: the best alternative for the concern problem is found by arranging the optimal degree membership in the non-increasing order.

$\theta_1 > \theta_2 > \theta_3 > \theta_4$

Conclusion

This paper is presented to find out the food farming investment in organic or inorganic agriculture to evaluate the integrated algorithm effectiveness. In today's world, either Food consumption or farming plays a vital role, and a lot of information available regarding farming gives vagueness for the decision-maker. For that, the values for IVIF and FCM matrix are collected from the subject expert in the linguistic variable form. The integrated procedure is done by evaluating the multi-attribute problem and it shows that organic agriculture is the best one of all. Finally, the problem enhances that the present integrated algorithm expresses its reliability and effectiveness.

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