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Analysis Of Multi Server Queues With Pentagonal Fuzzy Number By Flexible Alpha Cut Method

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ABSTRACT

Fuzzy queuing structuresilluminateaccessibility and heterogeneity in all domains. Queues and congestion are common prevalence we face in all corners.This paper analyses the procedure for various fuzzy performance measures of the multi-server queuing model based on the alpha-cut with its arithmetic operations to handle uncertain parameters. The fuzzy estimators $\tilde{\lambda}$ and$\tilde{\mu}$are based on Poisson and exponential distribution. A progressive and flexible technique called flexible alpha-cuts method is applied to the proposed method with numerical example.

KEY WORDS: Fuzzy set, queuing model, pentagonal fuzzy number, flexible alpha cut.

I. INTRODUCTION

Waiting line techniques form an integral part in fields of operations research, transmission and interconnection frameworks, which was rooted in the 20th century spread rapidly in all areas which later on flourished with constraints to apply randomness with uncertainty in all possibilities to produce innovation with optimum utility.Multiserver systems, a generalization of M/M/1 queue,are depicted with similar servers or with different types of servers to impartassistance to the consumers arriving into the queue. Many realistic predicaments can be modelled with such systems, to enrich present circumstances.

Queuing models marked renowned applicability in real time systems. Since the last three decades these techniques were massively researched by numerous investigators and has extended its utility on uncertain grounds. Fuzzy waiting line models have been discussed by various investigators like Zadeh , L.A., Li ,R.J., and Lee ,E.S., Negi ,D.S., and Lee ,E.S.,, Kao ,C., Li ,C., and Botzoris ,G.N., Papadopoulos ,B.K., and Sfiris ,D.S., applied fuzzy estimators to the performance of M/M/s queuing systems.J.P. Mukeba. K[9] has analyzed that fuzzy queue characteristics can be constructed by a new process called "flexible α-cuts method". We extend this idea by taking fuzzy multi-server queues and applying flexible α-cuts method with parametric non-linear programming approach to explore its perceptivity.The α-cuts arithmetic is applied for defuzzification, interval arithmetic for performing classical arithmetic and characteristic function of α-cuts is needed for fuzzification.

Pentagonal Fuzzy Number: A pentagonal fuzzy number given by A= (a, b, c, d, e) has a membership function

$$\mu_A(x) = \begin{cases} L_1(x) = \frac{x-a}{b-a} & \text{when } a \le x \le b \\ L_2(x) = \frac{x-a}{c-b} & \text{when } b \le x \le c \\ 1 & \text{when } x = c \\ R_1(x) = \frac{d-x}{d-c} & \text{when } c \le x \le d \\ R_2(x) = \frac{e-x}{e-d} & \text{when } d \le x \le e \\ 0 & \text{otherwise} \end{cases}$$

ESTIMATION PROCEDURE FOR FLEXIBLE α-CUT:

The objective of Flexible alpha cut is accomplished only when classical queue formulation and fuzzy waiting line input specifications are perceptible. Introduce fuzzy queue characteristic with input parameters and estimate fuzzy characteristic and constraints compatibleto the parameters of classical model. Implementing basic arithmetic operations in R and Zadeh's extension principle the crisp characteristic is illuminated to fuzzy characteristic in F(R).

STEPS:

1. Generate the alpha-cuts for all input variables.
2. Apply alpha-cuts fuzzy arithmetic to fuzzy queue characteristic.
3. Use interval arithmetic to obtain lower and upper bounds whose reciprocals signify the membership function as

$$\eta_{\tilde{\phi}}(x) = \begin{cases} (\tilde{\phi}^L)^{-1}(x), & \tilde{\phi}^L(0) \le x \le \tilde{\phi}^L(1) \\ (\tilde{\phi}^U)^{-1}(x), & \tilde{\phi}^L(1) < x \le \tilde{\phi}^U(0) \\ 0, \text{otherwise.} \end{cases}$$

4. Put α = 0 in step (3) to get the support bounds, and its most possible value.

II.MODEL DESCRIPTION

The frequently used queuing model is M\M\S_F or Erlang delay model. It takes a single queue with S_F identical servers. Customers arrived based on Poisson process and service time follows exponential distribution. This model stipulates three specifications such as average arrival rate ' λ_F ' average service rate $\frac{1}{\mu_F}$ and the number of servers'S_F' to obtain the performance measures with sufficient data. The performance measures are:

1.Positive delay: $P_D = 1 - \sum_{n=0}^{S_F=1} PF_n$

2. Average time spent in line:$WF_q = {P_D}/{[(1-\rho)S_F\mu_F]}$

3. Utilization factor: $\rho = {\lambda_F}/{S_F\mu_F}$ where $\rho < 1$

4. Probability that n customers are in the system at a given time

$$PF_n = \begin{cases} \frac{\lambda_F^n}{n!\mu_F^n} PF_0 & (1 \le n \le s_F) \\ \frac{\lambda_F^n}{S_F^{n-s_F} S_F!\mu_F^n} PF_0 & (n \ge s_F) \end{cases} \text{ where } PF_0 = \left[\sum_{n=0}^{s_F-1} \frac{(\rho s_F)^n}{n!} + \frac{\rho^{s_F} s_F{}^{s_F+1}}{s_F!(s_F - \rho s_F)}\right]^{-1} \text{ where } \rho < 1$$

III. NUMERICAL EXAMPLE

Let the arrival and service stream be$\tilde{\lambda}_{PF} = [1,2,3,4,5]$,$\tilde{\mu}_{PF} = [6,7,8,9,10]$; $S_{PF} = 3$ servers

The interval of confidence for different levels of α are:

$\tilde{\lambda}_{PF\alpha} = [2\alpha + 1, 5 - 2\alpha]$ and$\tilde{\mu}_{PF\alpha} = [6 + 2\alpha, 10 - 2\alpha]$, $0 \le \alpha \le 1$

$$(i)\tilde{\rho}_{PF\alpha} = \frac{\tilde{\lambda}_{PF\alpha}}{S_{PF}\tilde{\mu}_{PF\alpha}} = \frac{[2\alpha+1, 5-2\alpha]}{[18+12\alpha, 30-12\alpha]} = \{\min F_1(\alpha), \max F_1(\alpha)\}$$

$$= \begin{cases} f_{11}(\alpha) = \frac{2\alpha+1}{18+6\alpha} \\ f_{12}(\alpha) = \frac{2\alpha+1}{30-6\alpha} \\ f_{13}(\alpha) = \frac{5-2\alpha}{18+6\alpha} \\ f_{14}(\alpha) = \frac{5-2\alpha}{30-6\alpha} \end{cases} \text{ where } \min F_1(\alpha) = \min\{f_{11}(\alpha), f_{12}(\alpha), f_{13}(\alpha), f_{14}(\alpha)\}$$

$$= \begin{cases} f_{11}(\alpha) = \frac{2\alpha+1}{18+6\alpha} \\ f_{12}(\alpha) = \frac{2\alpha+1}{30-6\alpha} \\ f_{13}(\alpha) = \frac{5-2\alpha}{18+6\alpha} \\ f_{14}(\alpha) = \frac{5-2\alpha}{30-6\alpha} \end{cases} \text{ where } \max F_1(\alpha) = \max\{f_{11}(\alpha), f_{12}(\alpha), f_{13}(\alpha), f_{14}(\alpha)\}$$

whose solutions are$\min F_1(\alpha) = \frac{2\alpha+1}{18+6\alpha}$; $\max F_1(\alpha) = \frac{5-2\alpha}{30-6\alpha}$. Hence $\tilde{\rho}_{PF\alpha} = \left[\frac{2\alpha+1}{18+6\alpha}, \frac{5-2\alpha}{30-6\alpha}\right]$

(ii) Fuzzy probability when system is empty is $P_{PF0} = \frac{1}{\sum_{n=0}^{s_{PF}-1}\frac{(s_{PF}\rho_{PF})^n}{n!}+\frac{(s_{PF}\rho_{PF})^{s_{PF}}}{s_{PF}!(1-\rho_{PF})}}$

$=\frac{1}{1+\left[\frac{10\alpha+5}{18+6\alpha},\frac{25-10\alpha}{30-6\alpha}\right]+\frac{\{\min F_2(\alpha),\max F_2(\alpha)\}}{2}+\frac{\{\min F_3(\alpha),\max F_3(\alpha)\}}{\left[\frac{30+24\alpha}{30-6\alpha},\frac{78-24\alpha}{18+6\alpha}\right]}}$ where $\min F_2(\alpha)=\frac{(10\alpha+5)^2}{(18+6\alpha)^2}$; $\max F_2(\alpha)=\frac{(25-10\alpha)^2}{(30-6\alpha)^2}$

$$=\begin{cases} f_{31}(\alpha)=\frac{(10\alpha+5)^2}{(18+6\alpha)^2}\cdot\frac{10\alpha+5}{18+6\alpha} \\ f_{32}(\alpha)=\frac{(10\alpha+5)^2}{(18+6\alpha)^2}\cdot\frac{25-10\alpha}{30-6\alpha} \\ f_{33}(\alpha)=\frac{(25-10\alpha)^2}{(30-6\alpha)^2}\cdot\frac{10\alpha+5}{18+6\alpha} \\ f_{34}(\alpha)=\frac{(25-10\alpha)^2}{(30-6\alpha)^2}\cdot\frac{25-10\alpha}{30-6\alpha} \end{cases} \text{where } \min F_3(\alpha)=\min\{f_{31}(\alpha),f_{32}(\alpha),f_{33}(\alpha),f_{34}(\alpha)\}$$

$$=\begin{cases} f_{31}(\alpha)=\frac{(10\alpha+5)^2}{(18+6\alpha)^2}\cdot\frac{10\alpha+5}{18+6\alpha} \\ f_{32}(\alpha)=\frac{(10\alpha+5)^2}{(18+6\alpha)^2}\cdot\frac{25-10\alpha}{30-6\alpha} \\ f_{33}(\alpha)=\frac{(25-10\alpha)^2}{(30-6\alpha)^2}\cdot\frac{10\alpha+5}{18+6\alpha} \\ f_{34}(\alpha)=\frac{(25-10\alpha)^2}{(30-6\alpha)^2}\cdot\frac{25-10\alpha}{30-6\alpha} \end{cases} \max F_3(\alpha)=\max\{f_{31}(\alpha),f_{32}(\alpha),f_{33}(\alpha),f_{34}(\alpha)\}$$

$\min F_3(\alpha)=\frac{(10\alpha+5)^3}{(18+6\alpha)^3}$ and $\max F_3(\alpha)=\frac{(25-10\alpha)^3}{(30-6\alpha)^3}$

$P_{PF0}=\frac{1}{1+\left[\frac{10\alpha+5}{18+6\alpha},\frac{25-10\alpha}{30-6\alpha}\right]+\{\min F_4(\alpha),\max F_4(\alpha)\}+\{\min F_5(\alpha),\max F_5(\alpha)\}}$

where $\min F_4(\alpha)=\frac{(10\alpha+5)^2}{2(18+6\alpha)^2}$ and $\max F_4(\alpha)=\frac{(25-10\alpha)^2}{2(30-6\alpha)^2}$

$$=\begin{cases} f_{51}(\alpha)=\frac{(10\alpha+5)^3}{(18+6\alpha)^3}\cdot\frac{30+24\alpha}{30-6\alpha} \\ f_{52}(\alpha)=\frac{(10\alpha+5)^3}{(18+6\alpha)^3}\cdot\frac{78-24\alpha}{18+6\alpha} \\ f_{53}(\alpha)=\frac{(25-10\alpha)^3}{(30-6\alpha)^3}\cdot\frac{30+24\alpha}{30-6\alpha} \\ f_{54}(\alpha)=\frac{(25-10\alpha)^3}{(30-6\alpha)^3}\cdot\frac{78-24\alpha}{18+6\alpha} \end{cases} \text{where } \min F_5(\alpha)=\min\{f_{51}(\alpha),f_{52}(\alpha),f_{53}(\alpha),f_{54}(\alpha)\}$$

$$=\begin{cases} f_{51}(\alpha)=\frac{(10\alpha+5)^3}{(18+6\alpha)^3}\cdot\frac{30+24\alpha}{30-6\alpha} \\ f_{52}(\alpha)=\frac{(10\alpha+5)^3}{(18+6\alpha)^3}\cdot\frac{78-24\alpha}{18+6\alpha} \\ f_{53}(\alpha)=\frac{(25-10\alpha)^3}{(30-6\alpha)^3}\cdot\frac{30+24\alpha}{30-6\alpha} \\ f_{54}(\alpha)=\frac{(25-10\alpha)^3}{(30-6\alpha)^3}\cdot\frac{78-24\alpha}{18+6\alpha} \end{cases} \text{where } \max F_5(\alpha)=\max\{f_{51}(\alpha),f_{52}(\alpha),f_{53}(\alpha),f_{54}(\alpha)\}$$

$\min F_5(\alpha)=\frac{(10\alpha+5)^3}{(18+6\alpha)^3}\cdot\frac{30+24\alpha}{30-6\alpha}$ and $\max F_5(\alpha)=\frac{(25-10\alpha)^3}{(30-6\alpha)^3}\cdot\frac{78-24\alpha}{18+6\alpha}$

$P_{PF0}=\left[\frac{10368\alpha^4+106272\alpha^3+396576\alpha^2+629856\alpha+349920}{19680\alpha^4+282624\alpha^3+773064\alpha^2+962976\alpha+468120},\frac{10368\alpha^4-189216\alpha^3+1283040\alpha^2-3823200\alpha+4212000}{30048\alpha^4-598176\alpha^3+3812040\alpha^2-10111200\alpha+4722030}\right]$

(iii) Fuzzy expected number of items in the queue: $L_{PFq} = \frac{\left(\frac{\tilde{\lambda}_{PF\alpha}}{\tilde{\mu}_{PF\alpha}}\right)^{s_{PF}} \cdot \tilde{\rho}_{PF\alpha}}{s_{PF}!\left(1-\tilde{\rho}_{PF\alpha}\right)^2} P_{PF0}$

$= \frac{\{\min F_6(\alpha), \max F_6(\alpha)\}\left[\frac{2\alpha+1}{6+2\alpha}, \frac{5-2\alpha}{10-2\alpha}\right] \cdot \left[\frac{2\alpha+1}{18+6\alpha}, \frac{5-2\alpha}{30-6\alpha}\right]}{6\left(\left[\frac{25-4\alpha}{30-6\alpha}, \frac{17+4\alpha}{18+6\alpha}\right]\right)^2} P_{PF0}$

$$= \begin{cases} f_{61}(\alpha) = \frac{2\alpha+1}{6+2\alpha} \cdot \frac{2\alpha+1}{6+2\alpha} \\ f_{62}(\alpha) = \frac{2\alpha+1}{6+2\alpha} \cdot \frac{5-2\alpha}{10-2\alpha} \\ f_{63}(\alpha) = \frac{5-2\alpha}{10-2\alpha} \cdot \frac{2\alpha+1}{6+2\alpha} \\ f_{64}(\alpha) = \frac{5-2\alpha}{10-2\alpha} \cdot \frac{5-2\alpha}{10-2\alpha} \end{cases} \text{where } \min F_6(\alpha) = \min\{f_{61}(\alpha), f_{62}(\alpha), f_{63}(\alpha), f_{64}(\alpha)\}$$

$$= \begin{cases} f_{61}(\alpha) = \frac{2\alpha+1}{6+2\alpha} \cdot \frac{2\alpha+1}{6+2\alpha} \\ f_{62}(\alpha) = \frac{2\alpha+1}{6+2\alpha} \cdot \frac{5-2\alpha}{10-2\alpha} \\ f_{63}(\alpha) = \frac{5-2\alpha}{10-2\alpha} \cdot \frac{2\alpha+1}{6+2\alpha} \\ f_{64}(\alpha) = \frac{5-2\alpha}{10-2\alpha} \cdot \frac{5-2\alpha}{10-2\alpha} \end{cases} \text{where } \max F_6(\alpha) = \max\{f_{61}(\alpha), f_{62}(\alpha), f_{63}(\alpha), f_{64}(\alpha)\}$$

$\min F_6(\alpha) = \frac{(2\alpha+1)^2}{(6+2\alpha)^2}$ and $\max F_6(\alpha) = \frac{(5-2\alpha)^2}{(10-2\alpha)^2}$

$L_{PFq} = \frac{\{\min F_7(\alpha), \max F_7(\alpha)\} \cdot \left[\frac{2\alpha+1}{18+6\alpha}, \frac{5-2\alpha}{30-6\alpha}\right]}{6\left(\left[\frac{25-4\alpha}{30-6\alpha}, \frac{17+4\alpha}{18+6\alpha}\right]\right)^2} P_{PF0}$ with $\min F_7(\alpha) = \frac{(2\alpha+1)^3}{(6+2\alpha)^3}$ and $\max F_7(\alpha) = \frac{(5-2\alpha)^3}{(10-2\alpha)^3}$

$$L_{PFq} = \frac{\left[\frac{(2\alpha+1)^3}{(6+2\alpha)^3}, \frac{(5-2\alpha)^3}{(10-2\alpha)^3}\right] \cdot \left[\frac{2\alpha+1}{18+6\alpha}, \frac{5-2\alpha}{30-6\alpha}\right]}{6\left(\left[\frac{25-4\alpha}{30-6\alpha}, \frac{17+4\alpha}{18+6\alpha}\right]\right)^2} P_{PF0} = \frac{\{\min F_8(\alpha), \max F_8(\alpha)\}}{6\{\min F_9(\alpha), \max F_9(\alpha)\}} P_{PF0}$$

$$= \begin{cases} f_{81}(\alpha) = \frac{(2\alpha+1)^3}{(6+2\alpha)^3} \cdot \frac{2\alpha+1}{18+6\alpha} \\ f_{82}(\alpha) = \frac{(2\alpha+1)^3}{(6+2\alpha)^3} \cdot \frac{5-2\alpha}{30-6\alpha} \\ f_{83}(\alpha) = \frac{(5-2\alpha)^3}{(10-2\alpha)^3} \cdot \frac{2\alpha+1}{18+6\alpha} \\ f_{84}(\alpha) = \frac{(5-2\alpha)^3}{(10-2\alpha)^3} \cdot \frac{5-2\alpha}{30-6\alpha} \end{cases} \text{where } \min F_8(\alpha) = \min\{f_{81}(\alpha), f_{82}(\alpha), f_{83}(\alpha), f_{84}(\alpha)\}$$

$$= \begin{cases} f_{81}(\alpha) = \frac{(2\alpha+1)^3}{(6+2\alpha)^3} \cdot \frac{2\alpha+1}{18+6\alpha} \\ f_{82}(\alpha) = \frac{(2\alpha+1)^3}{(6+2\alpha)^3} \cdot \frac{5-2\alpha}{30-6\alpha} \\ f_{83}(\alpha) = \frac{(5-2\alpha)^3}{(10-2\alpha)^3} \cdot \frac{2\alpha+1}{6+2\alpha} \\ f_{84}(\alpha) = \frac{(5-2\alpha)^3}{(10-2\alpha)^3} \cdot \frac{5-2\alpha}{30-6\alpha} \end{cases} \max F_8(\alpha) = \max\{f_{81}(\alpha), f_{82}(\alpha), f_{83}(\alpha), f_{84}(\alpha)\}$$

$$\min F_8(\alpha) = \frac{(2\alpha+1)^3}{(6+2\alpha)^3} \cdot \frac{2\alpha+1}{18+6\alpha}; \max F_8(\alpha) = \frac{(5-2\alpha)^3}{(10-2\alpha)^3} \cdot \frac{5-2\alpha}{30-6\alpha}$$

On simplification we get,

$\mathrm{minF}_9(\alpha) = \frac{(25-4\alpha)^2}{(30-6\alpha)^2}$;$\mathrm{maxF}_9(\alpha) = \frac{(17+4\alpha)^2}{(18+6\alpha)^2}$; $\mathrm{minF}_{10}(\alpha) = \frac{6(25-4\alpha)^2}{(30-6\alpha)^2}$;$\mathrm{maxF}_{10}(\alpha) = \frac{6(17+4\alpha)^2}{(18+6\alpha)^2}$

$\mathrm{minF}_{11}(\alpha) = \frac{(2\alpha+1)^3}{(6+2\alpha)^3}.\frac{2\alpha+1}{18+6\alpha}.\frac{(30-6\alpha)^2}{6(25-4\alpha)^2}$ and $\mathrm{maxF}_{11}(\alpha) = \frac{(5-2\alpha)^3}{(10-2\alpha)^3}.\frac{5-2\alpha}{30-6\alpha}.\frac{(18+6\alpha)^2}{6(17+4\alpha)^2}$

$\mathrm{minF}_{12}(\alpha) = \frac{(2\alpha+1)^3}{(6+2\alpha)^3}.\frac{2\alpha+1}{18+6\alpha}.\frac{(30-6\alpha)^2}{6(25-4\alpha)^2}.\mathrm{P}_{\mathrm{PFL0}}$; $\mathrm{maxF}_{12}(\alpha) = \frac{(5-2\alpha)^3}{(10-2\alpha)^3}.\frac{5-2\alpha}{30-6\alpha}.\frac{(18+6\alpha)^2}{6(17+4\alpha)^2}.\mathrm{P}_{\mathrm{PFR0}}$

On further substitution and simplification, the solution is obtained as,

$$\mathrm{L}_{\mathrm{PFq}} = \left[\frac{\begin{array}{c} 5971968\alpha^{10} + 13436928\alpha^9 - 222455808\alpha^8 - 854737920\alpha^7 + 1151470080\alpha^6 \\ +10919090304\alpha^5 + 22363807104\alpha^4 + 21776967936\alpha^3 + 11228232960\alpha^2 \\ +2960323200\alpha + 314928000 \end{array}}{\begin{array}{c} 90685440\alpha^{10} + 1256988672\alpha^9 - 2252289024\alpha^8 - 80405084160\alpha^7 \\ -254633137920\alpha^6 + 781480396800\alpha^5 + 6897390955776\alpha^4 \\ +19059354390528\alpha^3 + 27311436092160\alpha^2 + 20956382208000\alpha \\ +6825189600000 \end{array}}, \right.$$

$$\left. \frac{\begin{array}{c} 5971968\alpha^{10} - 13876288\alpha^9 + 1094363136\alpha^8 - 3253976064\alpha^7 - 6634483200\alpha^6 \\ +71474659200\alpha^5 - 153393264000\alpha^4 - 108358560000\alpha^3 + 986191200000\alpha^2 \\ -1570266000000\alpha + 852930000000 \end{array}}{\begin{array}{c} 138461184\alpha^{10} - 4348698624\alpha^9 + 48996154368\alpha^8 - 185971203072\alpha^7 \\ -720543962880\alpha^6 + 8850564403200\alpha^5 - 23634948960000\alpha^4 \\ -35195817600000\alpha^3 + 330443301600000\alpha^2 - 693009216000000\alpha \\ +507038940000000 \end{array}} \right]$$

(iv) Fuzzy expected number of items in the system:

$\mathrm{L}_{\mathrm{PFs}} = \mathrm{L}_{\mathrm{PFq}} + \frac{\tilde{\lambda}_{\mathrm{PF}\alpha}}{\tilde{\mu}_{\mathrm{PF}\alpha}} = \mathrm{L}_{\mathrm{PFq}} + \{\mathrm{minF}_{13}(\alpha), \mathrm{maxF}_{13}(\alpha)\}$

$$= \begin{cases} f_{131}(\alpha) = \frac{2\alpha+1}{6+2\alpha} \\ f_{132}(\alpha) = \frac{2\alpha+1}{10-2\alpha} \\ f_{133}(\alpha) = \frac{5-2\alpha}{6+2\alpha} \\ f_{134}(\alpha) = \frac{5-2\alpha}{10-2\alpha} \end{cases} \text{where } \mathrm{minF}_{13}(\alpha) = \min\{f_{131}(\alpha), f_{132}(\alpha), f_{133}(\alpha), f_{134}(\alpha)\}$$

$$= \begin{cases} f_{131}(\alpha) = \frac{2\alpha+1}{6+2\alpha} \\ f_{132}(\alpha) = \frac{2\alpha+1}{10-2\alpha} \\ f_{133}(\alpha) = \frac{5-2\alpha}{6+2\alpha} \\ f_{134}(\alpha) = \frac{5-2\alpha}{10-2\alpha} \end{cases} \text{where } \mathrm{maxF}_{13}(\alpha) = \max\{f_{131}(\alpha), f_{132}(\alpha), f_{133}(\alpha), f_{134}(\alpha)\}$$

whose solutions are $\mathrm{minF}_{13}(\alpha) = \frac{2\alpha+1}{6+2\alpha}$ and $\mathrm{maxF}_{13}(\alpha) = \frac{5-2\alpha}{10-2\alpha}$

On substitution and simplification,

$$=\left[\frac{\begin{array}{c}193314816\alpha^{11}+266736844\alpha^{10}-3611879424\alpha^{9}-166106668032\alpha^{8}-592496847360\alpha^{7}+1337074656768\alpha^{6}\\+14686504464384\alpha^{5}+45193836515328\alpha^{4}+73835344848384\alpha^{3}+69297490552320\alpha^{2}+34625153203200\alpha\\+6827079168000\end{array}}{\begin{array}{c}181370880\alpha^{11}+3058089984\alpha^{10}+3037353984\alpha^{9}-174323902464\alpha^{8}-991696780800\alpha^{7}+35161966080\alpha^{6}\\+18483664292352\alpha^{5}+79503054515712\alpha^{4}+168978998527488\alpha^{3}+205781380968960\alpha^{2}+139388672448000\alpha\\+40951137600000\end{array}}\right.$$

$$,\frac{\begin{array}{c}288866304\alpha^{11}-975175424\alpha^{10}+123253291008\alpha^{9}-634374761472\alpha^{8}-491961116160\alpha^{7}+2151314277\\-92544253056000\alpha^{5}+491003251240\alpha^{4}+839921659200000\alpha^{3}-3051237384000000\alpha^{2}+44965324800\\-2543724000000000\end{array}}{\begin{array}{c}276922368\alpha^{11}-10082009088\alpha^{10}+141479294976\alpha^{9}-861903949824\alpha^{8}+418624104960\alpha^{7}+249065684\\-135775541952000\alpha^{5}+165957854400000\alpha^{4}+1012844779200000\alpha^{3}-4690451448000000\alpha^{2}+79441700\\-5070389400000000\end{array}}$$

(v)Fuzzy expected time an item spends waiting in the queue:

$$W_{PFq}=\frac{L_{PF\alpha}}{\tilde{\lambda}_{PF\alpha}}=\frac{[L_{PFL\alpha},L_{PFR\alpha}]}{[2\alpha+1,5-2\alpha]}=\{\min F_{14}(\alpha),\max F_{14}(\alpha)\}$$

$$=\begin{cases}f_{141}(\alpha)=\frac{L_{PFL\alpha}}{2\alpha+1}\\ f_{142}(\alpha)=\frac{L_{PFL\alpha}}{5-2\alpha}\\ f_{143}(\alpha)=\frac{L_{PFR\alpha}}{2\alpha+1}\\ f_{144}(\alpha)=\frac{L_{PFR\alpha}}{5-2\alpha}\end{cases}\text{where } \min F_{14}(\alpha)=\min\{f_{141}(\alpha),f_{142}(\alpha),f_{143}(\alpha),f_{144}(\alpha)\}$$

$$=\begin{cases}f_{141}(\alpha)=\frac{L_{PFL\alpha}}{2\alpha+1}\\ f_{142}(\alpha)=\frac{L_{PFL\alpha}}{5-2\alpha}\\ f_{143}(\alpha)=\frac{L_{PFR\alpha}}{2\alpha+1}\\ f_{144}(\alpha)=\frac{L_{PFR\alpha}}{5-2\alpha}\end{cases}\text{where } \max F_{14}(\alpha)=\max\{f_{141}(\alpha),f_{142}(\alpha),f_{143}(\alpha),f_{144}(\alpha)\}$$

$\min F_{14}(\alpha)=\frac{L_{PFL\alpha}}{2\alpha+1}$ and $\max F_{14}(\alpha)=\frac{L_{PFR\alpha}}{5-2\alpha}$

Finally, on simplification we get,

$$W_{PFq}$$

$$=\left[\frac{\begin{array}{c}5971968\alpha^{10}+13436928\alpha^{9}-222455808\alpha^{8}-854737920\alpha^{7}+1151470080\alpha^{6}\\+10919090304\alpha^{5}+22363807104\alpha^{4}+21776967936\alpha^{3}+11228232960\alpha^{2}\\+2960323200\alpha+314928000\end{array}}{\begin{array}{c}193314816\alpha^{11}+2667368448\alpha^{10}-3611879424\alpha^{9}-166106668032\alpha^{8}-592496847360\alpha^{7}\\+1337074656768\alpha^{6}+14686504464384\alpha^{5}+45193836515328\alpha^{4}\\+73835344848384\alpha^{3}+69297490552320\alpha^{2}+34625153203200\alpha\\+6827079168000\end{array}}\right.$$

$$\left. ,\frac{\begin{gathered}5971968\alpha^{10}-13876288\alpha^{9}+1094363136\alpha^{8}-3253976064\alpha^{7}-6634483200\alpha^{6}\\+71474659200\alpha^{5}-153393264000\alpha^{4}-108358560000\alpha^{3}+986191200000\alpha^{2}\\-1570266000000\alpha+852930000000\end{gathered}}{\begin{gathered}-288866304\alpha^{11}+9715175424\alpha^{10}-123253291008\alpha^{9}+636474761472\alpha^{8}+491961116160\alpha^{7}\\-21513142771200\alpha^{6}+92544253056000\alpha^{5}-49100325120000\alpha^{4}\\-839921659200000\alpha^{3}+3051237384000000\alpha^{2}-4496532480000000\alpha\\+2543724000000000\end{gathered}}\right]$$

(vi)Fuzzy expected time an item spends waiting in the queue:

$$W_{PFs} = W_{PFq} + \frac{1}{\mu_{PF\alpha}} = W_{PFq} + \{\min F_{15}(\alpha), \max F_{15}(\alpha)\}$$

whose solutions are $\min F_{15}(\alpha) = \frac{1}{6+2\alpha}$ and $\max F_{15}(\alpha) = \frac{1}{10-2\alpha}$

$$W_{PFs} = [\ W_{PFLq},\ W_{PFRq}] + \left[\frac{1}{6+2\alpha}, \frac{1}{10-2\alpha}\right]$$

α	L_{PFq}	L_{PFs}	W_{PFq}	W_{PFs}

Finally, on simplification we get,

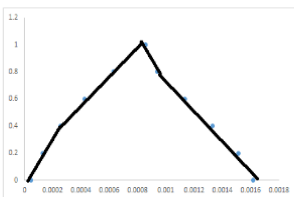
$$W_{PFs} =$$

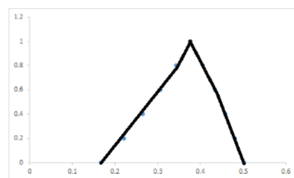
$$\left[\frac{\begin{gathered}205258752\alpha^{11}+2730074112\alpha^{10}-3976169472\alpha^{9}-169150878720\alpha^{8}-595322334720\alpha^{7}+1365821657856\\+14796748820416\alpha^{5}+45371573293824\alpha^{4}+73988463121920\alpha^{3}+69370780596480\alpha^{2}\\+34643544998400\alpha+6828968736000\end{gathered}}{\begin{gathered}386629632\alpha^{12}+6494625792\alpha^{11}+8780451840\alpha^{10}-353884612608\alpha^{9}-2181633702912\alpha^{8}-880831770624\\+37395456869376\alpha^{6}+178506699816960\alpha^{5}+418833708788736\alpha^{4}\\+581607050194944\alpha^{3}+485035249720320\alpha^{2}+221405077555200\alpha\\+40962475008000\end{gathered}}\right.$$

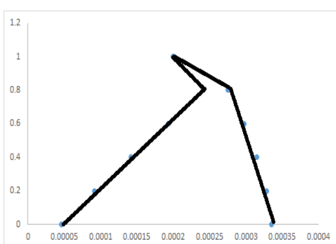
$$,\frac{\begin{gathered}-300810240\alpha^{11}+10040647680\alpha^{10}-126770780160\alpha^{9}+651826344960\alpha^{8}+592111019520\alpha^{7}-217224369\\+93565786176000\alpha^{5}-50417540440000\alpha^{4}-842977627200000\alpha^{3}+3064239828000000\alpha^{2}\\-4513941000000000\alpha+2552253300000000\end{gathered}}{\begin{gathered}577732608\alpha^{12}-22319013888\alpha^{11}+343658336256\alpha^{10}-2501282433024\alpha^{9}+5359825382400\alpha^{8}+47945896\\-400219933824000\alpha^{6}+1023643180800000\alpha^{5}-1188840067200000\alpha^{4}\\-1450169136000000\alpha^{3}+3950543880000000\alpha^{2}-5005277280000000\alpha\\+2543724000000000\end{gathered}}$$

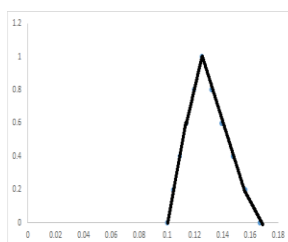
0	[0.0000460.001682]	[0.166712,0.501682]	[0.000046,0.000335]	[0.166712,0.100335]
0.2	[0.000127,0.001513]	[0.220013,0.480676]	[0.000091,0.000328]	[0.156341,0.104494]
0.4	[0.000256,0,001330]	[0.264962,0.457852]	[0.000142,0.000315]	[0.147201,0.109011]
0.6	[0.000426,0.001133]	[0.305981,0.432955]	[0.000193,0.000297]	[0.139082,0.113935]
0.8	[0.000628,0.000939]	[0.342734,0.405701]	[0.000241,0.000275]	[0.131820,0.119329]
1	[0.000854,0.000743]	[0.374854,0.375743]	[0.000284,0.000247]	[0.125284,0.125300]

If α runs from 0 to 1, the bounds of real intervals in the above calculations, describe the membership functions of fuzzy queue characteristics presented in table and graphs as follows:









IV. CONCLUSION

Modeling Queuing Systems throw light in diminishing congestion problem occurring in all aspects of life. Fuzzy queuing approaches marks its richness in rectifying complex data linked with ambiguity. Multiple server queues with flexible alpha-cut method is discussed for pentagonal fuzzy numbers which throws light on wider exploration of the support bounds, extremities, range of the system performance characteristics and its most possible value.

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