Nat. Volatiles & Essent. Oils, 2021; 8(4): 10041-10048

An Algorithm For Solving Quadratic Hesitant Fuzzy Transportation Problem

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Abstract:

This paper presents a quadratic transportation problem under hesitant fuzzy environment. Mathematical model of quadratic transportation problem is formulated with hesitant supply and demand. The cost coefficients are considered as quadratic functions. The proposed hesitant method would provide better optimal pair than the other existing methods. The numerical problem is solved to show the efficiency of the proposed method.

Key Words: Transportation Problem , Quadratic Transportation Problem, Hesitant Fuzzy set

1. Introduction:

The transportation problem plans a network of distribution of goods from different locations to different destinations with minimal costs. Time minimizing transportation problem are important when it is required to transport perishable goods. Sometimes there may exists emergency situations such as those requiring police services, fire services, ambulance services , etc. when the time of transportation is ofgreaterimportancethan cost of transportation. Somemethodsforminimizingthetimeoftransportationhavebeenestablished.Insuchsituations rather than minimizing the cost ,the objective is to minimize the maximum time totransport all supply to destinations satisfying certain conditions in respect of availabilities atsourcesandrequirements at thedestinations.

CTP, being a well structured problem, has been studied extensively in the literature. It was Hitchcock (1941) who firstly developed the basic transportation problem and suggested constructive method of solution. Later on, Dantzig (1963) formulated the CTP as an LPP and provided the solution. A number of researchers have worked to find suitable methods of solving CTP using different assumptions including Charnes and Cooper (1954), Shih (1987), Arsham and Khan (1989), Adlakha and Kowalski (1999), Ji and Chu (2002), Korukoglu and Balli (2011), Sudhakaret al (2012), are some among others. Recently Sharma et al (2013) have introduced an easy and interesting method named, "A modified zero suffix method" for finding optimal solution for CTP.

The linear functions are the most useful and widely used in operational research. Also quadratic functions and quadratic problems are the least difficult ones to handle out of all nonlinear programming problems. A fair number of functional relationships occurring in the real world are truly quadratic. For example kinetic energy carried by a rocket or an atomic particle is proportional to the square of its velocity, in statistics, the variance of a given sample of observations is a quadratic function of the values that constitute the sample. So there are countless other non-linear relationships occurring in nature, capable of being approximated by quadratic functions.

QTP and its indirect solution techniques have been discussed by researchers including Hochbaumet al (1992), Megiddo and Tamir (1993), Eduardo et al (2003), Cosares and Hochbaum (1994) are some among others. The techniques developed by these researchers are very much complicated and time-consuming. Recently, Adlakha and Kowalski (2013) have introduced an analytical algorithm for QTP which is based on finding absolute point (AP) in QTP by tracing graph for each quadratic cost function which is time-consuming and tedious and needs computer application for tracing graphs of each quadratic cost function.

A bibliography of Quadratic programming problems can be found in Nicholas(2001). Using fuzzy triangular technique, Ramesh(2013) proposed a fuzzy method to solve interval transportation problems. In Arora(2004) &Khurana(2009), the authors studied fixed charge indefinite quadratic transportation problems and fixed charge bicriterion quadratic transportation problems. In Das(1999) using fuzzy technique, a new method is proposed for interval transportation problems by considering the right bound and midpoint of interval.

Due to the characteristics of hesitant fuzzy sets (HFSs), one hesitant fuzzy element (HFE), which is the basic component of HFSs, can express the evaluation values of multiple decision makers on the same alternative under a certain attribute. Thus, the HFS has its unique advantages in group decision making (GDM), based on which, many scholars have conducted in-depth research on the applications of HFSs in GDM.

This paper presents a quadratic transportation problem under hesitant fuzzy environment. Mathematical model of quadratic transportation problem is formulated with hesitant supply and demand. The cost coefficients are considered as quadratic functions. The proposed hesitant method would provide better optimal pair than the other existing methods. The numerical problem is solved to show the efficiency of the proposed method.

2. Basic Definitions:

2.1 Hesitant Fuzzy Set :Torra (2009)

A hesitant fuzzy set HF on Y is defined in terms of a function h(y) from Y to the subset of values in the interval [0, 1].

If $\rho([0,1])$ is the power set of [0,1] then h is the function from Y to $\rho([0,1])$.

$$h: Y \rightarrow \rho([0,1])$$

Mathematically it can be stated that $HF = \{ (y_i , h(y_i)) : y_i \in Y \}$ where $h(y_i)$ is a set of several values in [0,1].In general each member of $h(y_i)$ is called a hesitant fuzzy element denoted by h_i

2.2 Score Function of Hesitant:

Let $h = \{ (y_i , h(y_i) ; y_i \in Y\}$ be a hesitant fuzzy element. Then the score value of h is defined as s(h) = $y_i - \frac{1}{k}\sum_{i=1}^{k} h(y_i)$

3. Mathematical Model of Quadratic Hesitant Fuzzy Transportation Problem:

m : Number of origins

n : Number of sources

$f(x_{ij})$: Quadratic Cost function

a_i^H ; Hesitant Availability of i-th source

b_j^H: Hesitant Demand of j-th destination

$$\text{Min } z = \sum_{i=1}^{n} \sum_{j=1}^{m} f(x_{ij})$$

Subject to

$$\sum_{j=1}^{n} x_{ij} \leq a_i^H \quad \forall\, i = 1,2, \dots m$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j^H \quad \forall\, j = 1,2, \dots n$$

$x_{ij} \geq 0 ; i = 1,2, \dots m , j = 1,2, \dots n$

4. Algorithm for finding solution of quadratic hesitant transportation problem:

Step 1: Determine the score value of each source and demand using the definition of score function.

Step 2: Check the given quadratic fuzzy transportation is balanced otherwise convert the unbalanced into balanced by including row/column.

Step 3: Find the estimated transportation cost matrix using

$$a_{ij} \min\{a_i, b_j\} + b_{ij} \min\{a_i, b_j\} + cij$$

Step 4: Subtract the least element of each row from all the elements of the corresponding row of the estimated cost matrix

Step 5: Subtract the least element of each column from all the elements of the corresponding column obtained in step 4 to get the transformed estimated cost matrix

Step 6: Find the suffix value of each zero cell using the following formula.

$$(p, q) = \max\{\min_{j \neq q} d'_{pj}, \quad \min_{i \neq p} d'_{ip}$$

Step 7 :Assign the cell having the greatest suffix value and delete the exhausted row/column to get the reduced estimated cost matrix

Step 8: In the reduced table update for those cells whose is reduced by k as

$$d'_{ij} - (a_{ij} * k + b_{ij})$$

Step 9: Repeat steps 2 to 6 till all the demands and supplies are exhausted.

5. Numerical Example:

Consider the following quadratic hesitant transportation problem with three sources and three destinations. The cost coefficients are expressed as quadratic functions. Supplies and demands are hesitant fuzzy elements. Our aim is to obtain the optimal solution for quadratic fuzzy transportation problem.

Table: 1

	D1	**D2**	**D3**	**Supply**

S1	$2x^2 + x$	$3x^2 + 2x$	$x^2 + 4x$	(3 ; 0.4,0.5,0.6)
S2	$x^2 + 3x$	$2x^2 + 2x$	$3x^2 + x$	(3; 0.6,0.7)
S3	$3x^2 + 3x$	$2x^2 + 3x$	$4x^2 + x$	(3; 0.5,0.6,0.7)
Demand	(2; 0.3,0.4)	(5; 0.5, 0.6, 0.7)	(2; 0.3, 0.4)	

Using the ranking function Table 1 can be written as follows

Table: 2

	D1	**D2**	**D3**	**Supply**
S1	$2x^2 + x$	$3x^2 + 2x$	$x^2 + 4x$	2.5
S2	$x^2 + 3x$	$2x^2 + 2x$	$3x^2 + x$	2.35
S3	$3x^2 + 3x$	$2x^2 + 3x$	$4x^2 + x$	2.4
Demand	1.65	4.4	1.65	

Formulate the estimated cost matrix

Table: 3

	D1	**D2**	**D3**	**Supply**
S1	$2(1.65)^2 + 1.65 = 7.1$	$3(2.5)^2 + 2(2.5) = 23.75$	$(1.65)^2 + 4(1.65) = 9.32$	2.5
S2	$(1.65)^2 + 3(1.65) = 7.67$	$2(2.35)^2 + 2(2.35) = 15.75$	$3(1.65)^2 + (1.65) = 9.82$	2.35
S3	$3(1.65)^2 + 3(1.65) = 9.82$	$2(2.4)^2 + 3(2.4) = 18.72$	$4(1.65)^2 + 1.65 = 12.54$	2.4
Demand	1.65	4.4	1.65	

Now total supply is not equal to total demand , then Table: 3 becomes

Table: 4

	D1	**D2**	**D3**	**Supply**
S1	7.1	23.75	9.32	2.5
S2	7.67	15.75	9.82	2.35
S3	9.82	18.72	12.54	2.4
S4	0	0	0	0.45
Demand	1.65	4.4	1.65	

Using Step 4 & Step 5 , reduced cost matrix is shown below:

Table: 5

	D1	D2	D3	Supply
S1	0	16.66	2.23	2.5
S2	0	8.08	2.15	2.35
S3	0	8.9	2.72	2.4
S4	0	0	0	0.45
Demand	1.65	4.4	1.65	

Allocate the value to the zeros using Step:6

Table:6

	D1	D2	D3	Supply
S1	$0_{2.23}$	16.66	2.23	2.5
S2	$0_{2.15}$	8.08	2.15	2.35
S3	$0_{2.72}$	8.9	2.72	2.4
S4	0_{0}	$0_{8.08}$	$0_{2.15}$	0.45
Demand	1.65	4.4	1.65	

The cell (4,2) has the highest zero suffix value , so allocate the maximum number of units to the cell (4,2).Following the procedure , optimum solution of the above problem is

Table : 7

	D1	D2	D3	Supply
S1	7.1	23.75 **0.85**	9.32 **1.65**	2.5
S2	7.67 **1.65**	15.75 **2.35**	9.82	2.35
S3	9.82	18.72 **0.75**	12.54	2.4
S4	0	0 **0.45**	0	0.45
Demand	1.65	4.4	1.65	

Total transportation cost = 23.75(0.85)+9.32(1.65) + 7.67(1.65) + 15.75(2.35) + 18.72(0.75)

= Rs.99.2735

6. Conclusion:

In this paper mathematical model of quadratic transportation problem is formulated with hesitant supply and demand. The cost coefficients are considered as quadratic functions. Finally, the proposed model is highly applicable for solving the real- life transportation problem and by following this model, the decision maker will be more benefited to take right decision for giving more information. In future the Proposed methodology of this paper can be extended to solve multi-objective quadratic hesitant fuzzy transportation problem. Also this methodology may be useful to extract better solution of decision making problems in supply chain management.

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