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Single Server Finite Capacity Morkovian Queuing Model with Encouragement Arrival

MUDDASIR AHMAD [1] and G.JAYALALITHA [2]

[1] *Research Scholar of Mathematics, Vels Institution of Science, Technology and Advanced Studies, Pallavaram Chennai-117, Tamil Nadu*

[2] *Professor, Department Of Mathematics, and Vels Institution of Science, Technology and Advanced Studies, Pallavaram Chennai-117, Tamil Nadu*

Abstract:

this paper develops the concept of finite capacity Markovin queuing model with encouraging arrival. The steady state is analysis. The benefits to the customers by an organization with encouraging them and to become new customers joining to the firm. The Economic analysis of model is present and to develop the cost model, and also the numerical representation discussed. Encouraged arrival gives new addition to the customer for bringing new customers behavior in queuing system.

Keywords*: Queuing Model, Encouragement Arrival, Stochastic Queuing Model*

1. Introduction.

(A k Eralang 1909) introduced the concept of queuing theory as an area of research; in his work they create a model to describe the Copenhagen telephone exchange. While developing the solution to the problem it begin to realize the applicability of queuing system in many other fields and begin to develop queuing theory further. Now to become a father of queuing model. In queuing situation it is important to check the behavior of customer. In queuing situation the behavior is checked with help of Balking, Reneging, jocking and Collusion, in this situation the behavior is checked by Haight (1957, 1959) and then Ancker and Gafaria (1962). To steady the detailed literature on customer's behavior from the business point it is dynamic nature of competitive business environment and uncertain customer behavior. This implies the firm makes such an organization in making polices so to engage more and more customers to make a benefit for them. There should be low margin so the organization make comprehensive with the strategy may not loss result to the business.

In order to engage more customers the organization make some benefits to the customer they may release the discount offers so that the new customers join it. Thus customers looking and attracting with such organization so they get and visit such a particular organization. The attracted customers are termed as encouraged arrival. In this paper the term encouraged arrivals contribute to the fundamental queuing literature. Notion of customer mobilization was introduced by (jain el at 2014). They called it reverse balking. In [2] by Haight customer are attracted towards the organization by looking a large base. Whereas reverse balking deal with probability of joining and not joining the system where as the engaged customers join because they new the benefits and discounts are remaining in the organization. The analysis by 'Haid' [1] where the steady state solution is performed in a single queuing model. [3],[5],[6] by 'Haight and Gafarain' analysis the Balking and Reneging rate in 1957 and 1963. In discouraged arrival that means the retention of Balking [7],[4] by Nativg B. that the queuing model where potential of customers are discouraged. 'Kumar and Sharma' [9], [10] that the customers who are impatient with the behavior by organization retained by retention of reneging and reneging of customers. In [8] 'V. K. Gupta' give the feedback of customer regarding the behavior. [10] The probability of balking and reneging of customer will be operated and considered to retain the old customers and joining the new customers by encouraging them and will facing the deal and discount offers.

2. Mathematical Model Formulation.

The arrival occurs one by one according to the passion process with parameter $\lambda(1+\eta)$. Where η represent the percentage change in number of customer calculated from past or observed date. The organization offered discounts and the percentage change in number of customer was observed η=50% and η=120%. There is one server and the service time independent, identical and exponentially distributed with parameter μ. The customers are serviced in order to their arrival that is the queue discipline is First come First Serve. There is a single server theory which the service is provided. The capacity of the system is finite say N

3. The Differential Difference Equation Of The Model Is Given By:

$$\frac{d}{dt}p_0(t) = -\lambda(1+\eta)p_0(t) + \mu p_1(t) \tag{3.1}$$

$$\frac{d}{dt}p_n(t) = \lambda(1+\eta)p_{n-1}(t) - \{\lambda(1+\eta)+\mu\}p_n(t) + \mu\, p_{n+1}(t) \quad 1 \le n \le N-1 \tag{3.2}$$

$$\frac{d}{dt}p_N(t) = \lambda(1+\eta)p_{N-1}(t) - \mu p_N(t) \qquad n=N \tag{3.3}$$

4. Steady- State Solution Of The Model

In steady state at $t \to \infty$ **$p_n(t) = p_n$** therefore $\frac{d}{dt}p_n(t) = 0$ as $t \to \infty$

As equation (3.1) and (3.3)

$$0 = -\lambda(1+\eta)p_0 + \mu p_1 \tag{4.1}$$

$$0 = \lambda(1+\eta)p_{n-1} - \{\lambda(1+\eta)+\mu\}p_n + \mu\, p_{n+1} \quad 1 \le n \le N-1 \tag{4.2}$$

$$0 = \lambda(1+\eta)p_{N-1} - \mu p_N \tag{4.3}$$

Now from equation (4.1) then

$$-\lambda(1+\eta)p_0 + \mu p_1 = 0 \tag{4.4}$$

$$\lambda(1+\eta)p_0 = \mu p_1$$

$$p_1 = \frac{\lambda(1+\eta)}{\mu} \qquad (4.5)$$

Substitute n=1 in equation (4.2) we get

$$\mu p_2 = \lambda(1+\eta)p_0 - \{\lambda(1+\eta)+\mu\}p_1$$

$- \mu\, p_2 = \{- \lambda(1+\eta)p_0 + \mu p_1\} - \lambda(1+\eta)\, p_1$ using (4.4) we get

$$p_2 = \frac{\lambda(1+\eta)}{\mu}\, p_1 \qquad (4.6)$$

Solving recursively equation (3.2) and (3.3)

$$p_n = \text{probability \{n customers in the system\}} \left[\frac{\lambda(1+\eta)}{\mu}\right]^n p_0 \quad 1 \leq n \leq n-1 \qquad (4.7)$$

And the probability that system is full given by

$$p_N = pr.\,system\ is\ full\ \left[\frac{\lambda(1+\eta)}{\mu}\right]^N p_0 \qquad (4.8)$$

Thus similarly $p_n = \left[\frac{\lambda(1+\eta)}{\mu}\right]^n p_0$ (4.9)

Using the normality condition $\sum_{n=0}^{N} p_n = 1$

Then $p_0 = \frac{1}{1+\sum_{n=1}^{N}\left[\frac{\lambda(1+\eta)}{\mu}\right]^n}$

5. Measures Of Performances :

Expected system size $Ls = \sum_{n=1}^{N} n\, p_n$

$$Ls = \sum_{n=1}^{N} n \left[\frac{\lambda(1+\eta)}{\mu}\right]^n p_0$$

Expected queue length $Lq = \sum_{n=1}^{N}(n-1)\, p_n$

$$Lq == \sum_{n=1}^{N}(n-1) \left[\frac{\lambda(1+\eta)}{\mu}\right]^n p_0$$

6. **Numerical Illustration of the System** :

In Table 1 :

It provides numerical results for different performances measures to steady the variation in performance of Ls and Lq with respect to the λ.

N=10 μ=3 and η=0.5

Average arrival	Expected system	Expected queuing

rate (λ)	size (Ls)	length (Lq)
2	5	4.090909
2.1	5.482345	4.556565
2.2	5.955435	4.985432
2.3	6.345474	5.386754
2.4	6.717843	5.746574
2.5	7.013245	6.056543
2.6	7.315432	6.338756
2.7	7.566452	6.577657
2.8	7.777564	6.786574
2.9	7.964323	6.975355
3	8.123241	7.135667
3.1	8.276751	7.276578
3.2	8.396778	7.396435
3.3	8.503425	7.507676
3.4	8.607868	7.607678
3.5	8.698635	7.677564
3.6	8.763423	7.768243

In this table it indicates the increasing probability of arrival rate. This implies that increase in Expected system size as well as Expected length of queue.

In Table 2:

Process it provides numerical results for different performances measures to steady the variation in performance of Ls and Lq with respect to the μ.

N=10 λ=3 and η=0.5

Expected service rate μ	Expected system size Ls	Expected length of queue Lq
3	8.124535	7.134535
3.1	7.977685	6.978765
3.2	7.806758	6.816476
3.3	7.626547	6.638745
3.4	7.437889	6.453546
3.5	7.236871	6.257658
3.6	7.038732	6.056754
3.7	6.829821	5.848765
3.8	6.597896	5.634536
3.9	6.375895	5.414536
4	6.148769	5.195668

4.1	5.918769	4.964651
4.2	5.697695	7.749832
4.3	5.458748	4.538743
4.4	5.227367	4.307685
4.6	5.000000	4.097896
4.7	4.788574	3.888796

In this table it indicates the increasing the probability of service rate. This implies that decrease in Expected system size as well as Expected length of queue.

7. Economic Analysis:

Economic analysis of the model is discussed by developing Total Expected cost (TEC), Total Expected Profit (TEP) and Total Expected Revenue (TER).

To develop cost-profit analysis model by using the following symbols.

λ= means inter arrival rate.

μ= means service rate

c_h =holding cost per unite per unite time.

C_s=cost per service per unite time.

C_L =cost associated to each lost unite per unit of time.

R= Revenue earned per unite time

TEP= total expected profit.

TEC= total expected cost

TES=total expected service.

Thus the total expected cost of the system (TEC) is given by

$$TEC= C_s\mu + C_h\sum_{n=1}^{N} n\left[\frac{\lambda(1+\eta)}{\mu}\right]^n p_0 + C_L\lambda\left[\frac{\lambda(1+\eta)}{\mu}\right]^N p_0$$

Where $p_0 = \frac{1}{1+\sum_{n=1}^{N}\left[\frac{\lambda(1+\eta)}{\mu}\right]^n}$

Total Expected Revenue of the system is given by

$$TER= R \times \mu \times (1- P_0)$$

Total Expected Profit is given by.

TEP= TER-TEC

In Table 3 :

The variation of Total Expected cost, Total Expected Profit and Total Expected Revenue is calculated with respect λ

$N = 10$, $\mu = 3$, $\eta = 0.5$ $C_s = 10$, $C_L = 15$, $C_h = 2$, $R = 200$

Average rate of arrival (λ)	Total Expected cost (TEC)	Total Expected Revenue (TER)	Total Expect Profit (TEP)
2	42.773456	545.458975	502.724567
2.1	44.587643	557.763675	513.185643
2.2	46.493489	567.626343	521.135986
2.3	48.425678	575.353753	526.935673
2.4	50.355634	581.333256	530.983668
2.5	52.279675	585.904246	533.936975
2.6	54.166854	589.364678	535.194576
2.7	56.024656	591.964566	535.935896
2.8	57.865987	593.296432	536.066564
2.9	59.666754	595.394523	535.726549
3	61.347231	596.493456	535.056755
3.1	63.173456	597.138746	534.145348
3.2	64.899873	597.945793	533.046342
3.3	66.59467	598.717675	531.826574
3.4	68.264578	598.779675	530.505239
3.5	69.928453	599.046986	529.117623
3.6	71.574567	599.256755	527.686432

The table indicates that with increasing in arrival rate the total expected profit increases rapidly and reaches at maximum value at certain level then starting falling down. This is because the service rate is being fixed, after certain level with increasing load on service, cost increases rapidly than revenue.

Table 4 :

The variation of Total Expected cost, Total Expected Profit and Total Expected Revenue is calculated with respect μ

$N = 10$, $\lambda = 3$, $\eta = 0.5$ $C_s = 13$, $C_L = 15$, $C_h = 2$, $R = 200$

Average rate of service (μ)	Total Expected cost (TEC)	Total Expected Revenue (TER)	Total Expected Profit (TEP)
3	70.436537	596.499453	526.059854

3.1	70.475342	615.279832	544.808676
3.2	70.514365	633.732376	563.218764
3.3	70.554565	651.814895	581.250989
3.4	70.606785	669.439830	598.838732
3.5	70.655673	686.555362	615.895398
3.6	70.714566	703.085623	632.379341
3.7	70.794585	718.974567	648.196734
3.8	70.893484	734.187443	663.298764
3.9	71.013684	748.633567	677.626589
4	71.173468	762.313575	691.137564
4.1	71.372466	775.165686	703.794987
4.2	71.602468	787.354987	715.578795
4.3	71.885778	798.355689	726.468765
4.4	72.214576	808.687865	736.463456
4.5	72.592478	818.189543	745.597453
4.6	73.075798	826.874532	753.857654

The table indicates that with increasing service rate the total expected profit increases rapidly and reaches at maximum value. The revenue goes high and the firm keeps on increase with an improving the service rate.

8. **Special case:**

When η=0

P_n =pr{ n customers in the system} =$\left[\frac{\lambda}{\mu}\right]^n p_0 = \rho^n p_0 \quad 1 \le n \le N-1$

P_N =pr{ the system full } =$\left[\frac{\lambda}{\mu}\right]^N p_0 = \rho^N p_0$

P_0 =pr{ the system is empty } = $\frac{1}{1+\sum_{n=1}^{N}\left[\frac{\lambda}{\mu}\right]^n}$

$$P_0 = \frac{1-\left[\frac{\lambda}{\mu}\right]^n}{1-\left[\frac{\lambda}{\mu}\right]^{n+1}} = \frac{1-\rho}{1-\rho^{n+1}}$$

Where $\rho=\frac{\lambda}{\mu}<1$, is the traffic intensity

There for the system reduces the classical single server queuing model with finite capacity

9. Conclusion :

This paper studies a single server queuing model with encouraged arrival. The result of the paper indicates that the use of immense for any organization encountering the phenomenon of encouraged customers and load on service. To implement such strategy in this model that develops the effective planning, the economic analysis of the facility can be measured and the financial aspect of the business can also be observed.

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