

Bacterial Foraging Optimized Block Diagonalization Algorithm in MU-MIMO System using Different Fading Channel Models

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Abstract

MIMO applications' diversity tends to improve system architecture in order to compensate for increased hardware and software requirements. A single user can be served every transmission interval in single-user MIMO systems. This maximises the throughput of a single user, but it has the disadvantage of ignoring the benefits of multi-user variation. Multi-user MIMO systems have emerged as the primary method for satisfying the demands. Eliminating co-channel interference is one of the most difficult challenges in MU-MIMO. Block diagonalization (BD) is a linear precoding approach that totally removes multi-user interference (MUI) in MU-MIMO broadcast channels, while it is wasteful in terms of processing. To address this, we have created a unique algorithm which optimizes (Bacterial Foraging Optimization) and evaluates it on a range of fading channel models, including as Rician, Rayleigh, Nakagami, and Alpha-Mu, as well as tests it for changes in the number of users and order of modulation. The simulation results indicate that both the Block diagonalization and Bacterial Foraging Optimized block diagonalization techniques show inverse relation between BER and SNR for all the four fading channels. By employing the Bacterial Foraging optimization along with the traditional Block Diagonalization technique in the pre-coders, we can greatly reduce the BER for a given SNR value. This in-turn saves a lot of power in transmission of the signal. The Nakagami channel out-performs all other channels and can be considered as a practical viable channel for present day communications.

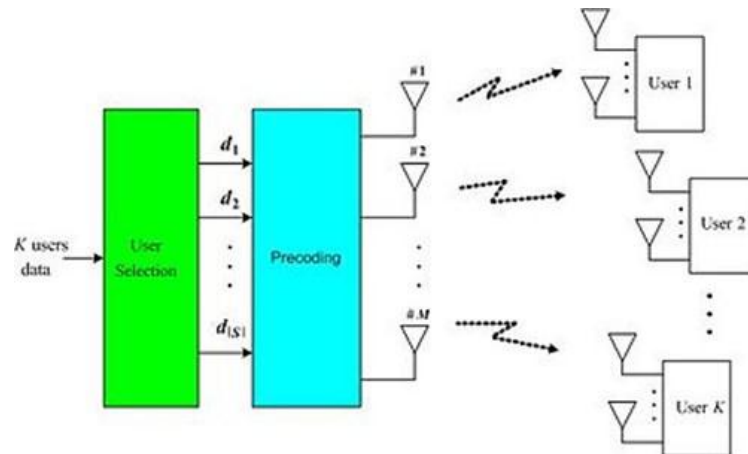
Keywords: Bacterial foraging optimization, Block diagonalization, Fading channels, Multi-user MIMO

Introduction :

MIMO technology employs multiple antennas in both the transmitter and receiver [3]. By utilising the spatial domain, MIMO improves the spectral efficiency of a wireless communication system [4]. Co-channel interference removal is one of the most difficult challenges in MU-MIMO [5], because multiple users on the same frequency, results in increase in user interference [6]. We will be able to boost the system's capacity if we can minimize inter-user interference within the cell, which is the main goal of our strategy. The Channel Status Information (CSI) is crucial because it is required for the design of our algorithm [7].

Co-channel interference generated by multiple users is currently countered using current techniques, almost completely removing it, but at the expense of signal processing on both the transmitter and receiver.[8][9][10]. The ability to eliminate inter-user interference is highly dependent on channel information, which changes over time and is frequency selective, hence in real-world situations, eliminating interference is only partial. A return channel is required to relay the channel state to the transmitter, allowing it to construct the correct pre-coding matrices and prevent interference between channels. It greatly influences the performance of the system. If we do not have the return channel well dimensioned, it will not achieve the desired end. It is a very easy solution to implement in a cellular system, since only matrices are handled. However, it does involve additional signal processing at base stations.

Fig 1. Representation of a multi-user MIMO system's downlink channel [15]



Problem statement and Approach :

Rayleigh, Rician, Nakagami, and Alpha-Mu channels will be considered for our research. We also state there will be Non-Line of Sight (NLOS) between the sender and receiver following the most common Rayleigh channel. As a result of being flat, all frequency bands fade at the same rate. Users will be deemed to be far away in terms of distance, so each user's channel will be distinct from the others. Because we recognize its usefulness at the transmitter, we will treat Channel Status Information (CSI) at the transmitter as the best feedback channel between the base station and the user. Finally, multiply the number of receiving antennas by the number of users to get the number of broadcasting antennas.

Assume you have a multi-user communication system where a single base station feeds a large number of mobile stations. Let the number of antennas at the base station be N_p and those at the mobile station be N_q respectively. As R independent user, with $R.N_q$, the antennas communicate with the BS via radio waves N_p antennas, assuming end-to-end communication for downlink $(R.N_q) \times N_p$ MIMO system. We shall treat Channel Status Information (CSI) in the transmitter as the best feedback channel between the base station and the user since we recognize its usefulness. Multiply the number of receiving antennas by the number of users to get the number of transmitting antennas. In multiuser communication system, multiple antennas allows the base station to transmit the multiple user data stream to be decoded by each user in downlink. By considering R independent user, where $X \in \mathbb{C}^{N_p \times 1}$ is the transmit signal from the BS and $R_a \in \mathbb{C}^{N_q \times 1}$ with received signal at the a^{th} user, where, $a = 1, 2, 3, \dots, R$.

Let $Ch_a \in \mathbb{C}^{N_q \times N_p}$ represent the channel gain between BS and the a^{th} user. The received signal at the a^{th} user is expressed as:

$$R_a = Ch_a X + Z_a \tag{1}$$

$$a = 1, 2, 3, \dots, R$$

Where $Z_a \in \mathbb{C}^{N_q \times 1}$ is the additive zero mean circular complex Gaussian random vector [.] for all users. Where X is the set of transmitted signal (Ch_1, \dots, Ch_R) . The fundamental challenge with data

transmission through the Broadcast Channel (BC) is that coordinated signal recognition on the receiver side is difficult, necessitating downlink interference cancellation.

Block Diagonalization for Broadcast Channel Transmission :

We'll look at a technique known as Block Diagonalization as one of the many solutions available to ease the problem we're dealing with, namely co-channel interference created by multiple users engaging in the same frequency band inside a cell. It uses a linear pre-coding matrix to supply each user with numerous frames of data while reducing inter-user interference. Inter-user interference is swiftly neutralized when the pre-coding matrix is multiplied by the channels of the interfering users, and only the signal is transferred to the intended user, as we'll see when we solve the mathematical component in the next section. Inter-symbolic interference (ISI) will still be present in the receiver even if the interference from the other user is cancelled.

Let $N_{Q,a}$ denotes the number of antennas for the a^{th} user. Where $a = 1,2,3, \dots R$.

For the a^{th} signal $\tilde{x}_a \in C^{N_{Q,a} \times 1}$, the received signal, $R_a \in C^{N_{Q,a} \times 1}$ can be expressed as:

$$R_a = Ch_a \sum_{R=1}^K p_R \tilde{x}_k + Z_a$$

$$= Ch_a p_a \tilde{x}_a + \sum_{R=1, R \neq a}^K Ch_R p_R \tilde{x}_R + Z_a \quad (2)$$

Where $Ch_a \in C^{N_{Q,a} \times N_P}$ is channel matrix between BS and a^{th} user.

$w_a \in C^{N_a \times N_{Q,a}}$ is the precoded matrix for the a^{th} user and Z_a denotes the noise vector.

From equation (2), $\{Ch_u p_R\}_{a \neq R}$ increases interference to a^{th} user unless,

$$Ch_a p_R = O_{N_Q \times N_{Q,a}}, \forall a \neq R \quad (3)$$

Where $O_{N_Q \times N_{Q,a}}$ is a zero matrix.

The precoder was designed to fulfil the total power constraints, $p \in C^{N_P \times N_{Q,a}}$ must be unitary, $a = 1,2,3, \dots R$.

From equation (3), the interference free received signal is,

$$R_a = Ch_a p_a \tilde{x}_a + Z_a \quad (4)$$

$a = 1,2,3, \dots R$.

For obtaining the value of \tilde{x}_a , various signal detection algorithms now can be employed for estimation.

To obtain $[P_R]_{R=1}^K$, let us take channel matrix of all users except a^{th} user.

$$\tilde{Ch}_a = [(Ch_1)^{Ch} \dots (Ch_{a-1})^{Ch} (Ch_{a+1})^{Ch} \dots Ch_R]^{Ch} \quad (5)$$

Where $N_{Q,total} = \sum_{a=1}^R N_{Q,a} = N_P$

$$\tilde{Ch}_a p_a = O_{(N_{Q,total} - N_{Q,a}) \times N_{Q,a}} \quad (6)$$

$a = 1,2,3, \dots R$.

Hence, precoding matrix $P_a \in C^{N_P \times N_{Q,a}}$ should exist in null space of \tilde{Ch}_a and precoders should satisfy the equation (6). Singular value decomposition (SVD) for this would be \tilde{V}_a^{zero} of \tilde{Ch}_a . This is expressed using non-zero singular values and zero singular values.

$$\tilde{Ch}_a = \tilde{U}_a \tilde{\Lambda}_a [\tilde{V}_a^{non zero} \tilde{V}_a^{zero}]^{Ch} \quad (7)$$

Where $\tilde{V}_a^{non zero} \in C^{(N_{Q,total} - N_{Q,a}) \times N_P}$ and $\tilde{V}_a^{zero} \in C^{N_{Q,a} \times N_P}$ are composed of non-zero singular vectors values as well as zero singular values, respectively, and are made up of right singular vectors.

From equation (7) multiplying $\widetilde{C}h_a$ with \widetilde{V}_a^{zero} , we get following term,

$$\widetilde{C}h_a \widetilde{V}_a^{zero} = 0 \tag{8}$$

When both the channel gain and the SVD variables are multiplied together, the result is zero. Signal interference has been minimised if there are no received signals at the destination end. Thus, $P_a = \widetilde{V}_a$ can be employed to pre-code the signal of a^{th} user.

From equation (9), the pre-coding matrix may be seen to be made up of zeros and non-zero singular values $P_a = \widetilde{V}_a$ for the a^{th} user. Where \widetilde{V}_a is composed of zeros and non-zero singular values. Size of \widetilde{V}_a depends on size of $\widetilde{C}h_a$. If \widetilde{V}_a is when a matrix is huge, it has a greater number of non-zero singular values, causing equation (3) to be true:

$$\widetilde{C}h_a P_R > 0 \quad \forall a \neq R \tag{9}$$

If \widetilde{V}_u is a smaller matrix since it has fewer non-zero singular values, resulting in equation (3):

$$\widetilde{C}h_a P_R < 0 \quad \forall a \neq R \tag{10}$$

Because the channel matrix is not adequately block diagonalized, user a suffers from significant co-channel interference in both scenarios. Thus size of \widetilde{V}_a for optimal performance, it should be at its best. Since the size of \widetilde{V}_a depends on size of $\widetilde{C}h_a$, we can manipulate the size of $\widetilde{C}h_a$ by allocating the appropriate number of receiving antennas to each user N_{RX} . To compute an optimal value of N_{RX} the objective function employed is:

$$\min f(\widetilde{C}h_a) = |\widetilde{C}h_a P_R| \tag{11}$$

Where,

$$\widetilde{C}h_a = f(N_{RX}) \tag{12}$$

The value of N_{RX} can be found as,

$$N_{RX} = G(N_{RX}) \tag{13}$$

Where G is a Bacterial Foraging Optimization-optimized operator.

Bacterial Foraging Optimization (BFO) :

Natural selection supports the transmission of genes from animals with superior food-finding, handling, and getting skills, while weeding out those with worse skills. Animals aim to maximize energy use per unit of time based on their physical and environmental attributes. Four stages are used to explain bacteria's food search strategy (particularly, Escherichia Coli, which is found in human intestines): Chemotaxis, Bacteria Group Intelligence (Swarming), Reproduction, and Dispersion are all examples of bacterial behaviour.

Bacteria can move around thanks to their stiff flagella. Chemotaxis is a type of movement utilised in the quest for food that can be divided into two sorts or modes: swimming and falling together. Chemo-taxis is the motion of a group of bacteria in multiple directions, analogous to swimming or running. Both forms of locomotion alternate during the bacteria's life. Chemotaxis is caused by clockwise rotation of the flagella, while Swimming is caused by counter-clockwise rotation. The major aims and motivations of Bacteria's social intelligence for the quest for food are the possibility of finding concentrated nutrients, the potential to group assault a very large prey to kill and digest it, and defence from Bacteria group predators. Grouping, mechanisms for communication, and collective intelligence impact the success of each Bacteria group member's quest for food.

It is required to communicate information on the concentration of nutrients (ideal point) with other bacteria in order to lure the rest of the bacteria to the algorithm's optimal direction of convergence.

This is referred to as swarming. A penalty function is introduced to the function to be optimized in order to do this. This penalty function is based on each Bacterium's relative distance from the Bacterium with the best ability (during the search process). When all Bacteria converge to the desired solution, the penalty function becomes zero. After being involved in multiple steps of Chemotaxis, the original set of bacteria is fit for reproduction. The Bacterium is divided into two identical copies as part of the reproductive process. This is represented in the optimization technique, which involves replacing half of the population with lower fitness capabilities with Bacteria with better fitness capabilities. The total population of bacteria remains unaltered during this time. When a collection of Bacteria disperses to a new site, the usual evolutionary process is disturbed. To avoid stagnation at local minimums, this approach is utilised to replace a new set of bacteria near the meal location.

Variables (var_{max} and var_{min}), their aptitudes are calculated and the bacteria with the best aptitude as $var_{global}^{(0)}$.

The different constants used by the algorithm are initialized:

1. Maximum cycles of Chemotaxis (Max_{chemo}).
2. Maximum swim length (Max_{nado}).
3. Maximum reproduction cycles (Max_{reprod}).
4. Maximum Dispersion cycles ($Max_{dispers}$).
5. Maximum number of total iterations $Max_{cycle} = Max_{reprod} \cdot Max_{chemo}$
6. Reproduction ratio of bacteria (S_r).
7. The depth of the attractant released by the $d_{attract}$ cell.
8. The width of the attracting signal $w_{attract}$.
9. The height of the repellent effect $d_{repellant}$.
10. The width of the repellent $w_{repellant}$.
11. The probability of elimination P_{el} .
12. Other positive constants: c_{max} ; c_{min} ; d_1 ; d_2 .

Scatter-kill entails walking through the population and picking the germs to apply killing in a hap-hazard manner using a P_{el} probability. This is accomplished by eradicating these bacteria and replacing them with new ones that are disseminated in a random area, preserving the population number.

The breeding process kills half of the population of bacteria with the worst fitness and doubles the population of bacteria with the best fitness in order to maintain population expansion. The following equation is used to apply swarming to the bacteria's abilities:

$$J_i^{(j+1,R,l)} = J_i^{(j,R,l)} - \sum_{numBact} d_{attract} x \left(e^{-w_{attract} x \sigma_i^{(j+1,R,l)}} \right) + \sum_{numBact} d_{repellant} x \left(e^{-w_{repellant} x \sigma_i^{(j+1,R,l)}} \right) \quad (14)$$

Where,

$$\sigma_i^{(j+1,R,l)} = \sigma_i^{(j,R,l)} + \sum_{numBact} \left(var_{global}^{(j,R,l)} - var_i^{(j,R,l)} \right)^2 \quad (15)$$

The variables for the next iteration are determined using the maximum and minimum values set by the previous iteration (var_{max} and var_{min}) as:

$$var_i^{(j+1,R,l)} = var_i^{(j,R,l)} + (c^{var_i} - d_2) \times \frac{\Delta(var_i)}{\Delta} \quad (16)$$

Where,

$$\Delta^{(var_i)} = (2 \cdot \text{round}(\text{rand}(\cdot)) - 1) \cdot \text{rand}(\cdot); \Delta\Delta = \sqrt{\Delta^{(var_i)} \times \Delta'^{(var_i)}} \quad (17)$$

And

$$c^{var_i} = c_{max} - (c_{max} - c_{min}) \times \frac{j \times R}{max_{cycle}} \quad (18)$$

Swimming is applied obtaining $\Delta^{(var)}$ and $\Delta\Delta$ from Eq. (17) and the new variables are calculated with their restrictions:

$$var_i^{(j+1,R,l)} = var_i^{(j,R,l)} + (c^{var_i} + d_1) \times \frac{\Delta^{(var_i)}}{\Delta\Delta} \quad (19)$$

Communication Channels :

Rayleigh Channel :

A Rayleigh distribution occurs when the line of sight (LOS) between the sender and receiver is interrupted by obstacles or its level is below the noise level. In these circumstances, the sum of the multipath contributions allows the link to exist. Most connections with mobile telephony are made through Rayleigh distribution links, which allows coverage inside buildings without a direct line to the antenna. This typically occurs in mobile communications and AM and FM broadcasting.

The short-term fading has a Rician or Rayleigh distribution depending on the direct wave is of greater or less intensity. The Rayleigh PDF represents multipath fading. And it is:

$$p(r) = \begin{cases} \frac{r}{\sigma^2} \exp\left[-\frac{r^2}{2\sigma^2}\right] & 0 \leq r \leq \infty \\ 0 & r < 0 \end{cases} \quad (20)$$

Rician Channel :

In mobile telephony, it is suitable for outdoor communications and in open spaces where the transmitting antenna has a direct beam line with the receiver. A Rician distribution is characterized by the existence of direct vision (LOS) plus several reflected waves between emitter and transmitter and its power level is above the level of the multipath signals received. The Rician distribution is given by the following expression:

$$p(r) = \frac{r}{\sigma^2} e^{-\frac{(r^2+r_s^2)}{2\sigma^2}} I_0\left(\frac{rr_s}{\sigma^2}\right) \quad (21)$$

Where:

σ : standard deviation

r_s : amplitude of the direct beam.

r : amplitude of the envelope of the received signal.

I_0 : is the first-class, zero-order modified Bessel function.

The Rician distribution is usually described in terms of a parameter, K , which defines the rate between the power of the deterministic signal and the variance of the multipath, this relationship is given by equation 22:

$$K = \frac{LOS \text{ power}}{Diffracted \text{ power}} = \frac{r_s}{2\sigma^2} \quad (22)$$

The above equation can be expressed in dB as shown in equation 23:

$$K = 20 \log \frac{r_s}{2\sigma^2} \quad (23)$$

K is known as the Rician factor and completely specifies the distribution.

Nakagami Channel :

In wireless systems it is quite usual to model the amplitude of the envelope due to fast or small scale fading using a Rayleigh or Rician statistic, however, taking into account experimental data obtained in measurement campaigns of UWB channels indoors found that the Nakagami-m distribution is more versatile in the sense that it presents greater flexibility and adjusts better to experimentally obtained results to model the envelope of the received signal than the Rayleigh and Rician distributions. Nakagami-m contains the Rayleigh distribution for $m = 1$, the unilateral Gaussian distribution for $m = 1/2$, and the Rician distribution for $m > 1$, performing a one-to-one mapping between the form parameter m and the Rician K factor (quotient between the power of the deterministic or specular component and the random power).

$$p(r) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega_p}\right)^m r^{2m-1} \exp\left\{-\frac{mr^2}{\Omega_p}\right\}$$

(24)

The α - μ fading channel :

The $\alpha - \mu$ generalized distribution is used to better represent the small scale fading variations, considering transmission conditions without direct line of sight. The parameters that give name to the distribution (α and μ) are associated with the non-linearity of the environment and the number of multipath sets, respectively. The probability density function (pdf) of the distribution can be defined as:

$$f_r(R) = \frac{\alpha\mu^{\alpha\mu-1}}{\hat{r}^{\alpha\mu}\Gamma(\mu)} \exp\left(-\mu\frac{r^\alpha}{\hat{r}^\alpha}\right)$$

(25)

Where $\Gamma(\cdot)$ is the Gamma function,

$$\hat{r} = \sqrt[\alpha]{E(R^\alpha)}$$

(26)

Where $E(\cdot)$ denotes the hope operator and R the fading envelope.

The c fading model includes among its special cases the distribution of Nakagami-m, where $\alpha = 2$ and μ can assume different values depending on the conditions of the communication environment, and Weibull where $\mu = 1$ and α can assume different values depending on the conditions of the communication environment. For the special case where $\alpha = 2$ and $\mu = 1$, the Rayleigh fading model is obtained.

Results and Discussions :

The simulation has been carried out using MATLAB environment. The first set of simulation results shows the plots of BER vs SNR using Block Diagonalization technique for the four Rician, Rayleigh, Alpha-Mu and Nakagami fading channel models. Figure 2 illustrates the variation of BER and corresponding SNR values for the four fading channel models using the Block diagonalization technique. The Block Diagonalization technique is further supported by the Bacterial Foraging Optimization soft computing technique in order to improve the performance of the system in terms of SNR vs BER as exhibited in figure 3.

Further the simulation results are obtained for the Bacterial Foraging Optimized Block Diagonalization technique for the four Rician, Rayleigh, Alpha-Mu and Nakagami fading channels. The performance of BER vs SNR are plotted for the four channels are displayed in figure 3. It is evident from the figure that the BER values decrease for an SNR value of 10 dB and shows an inverse relation between the two for all the four fading channels.

Fig. 2. Comparative graph of BER for Block Diagonalization using Alpha-Mu, Rayleigh, Rician and Nakagami fading channels.

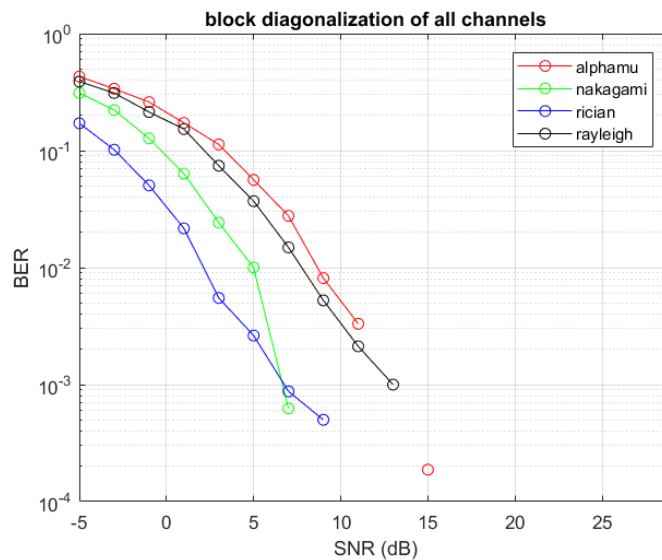
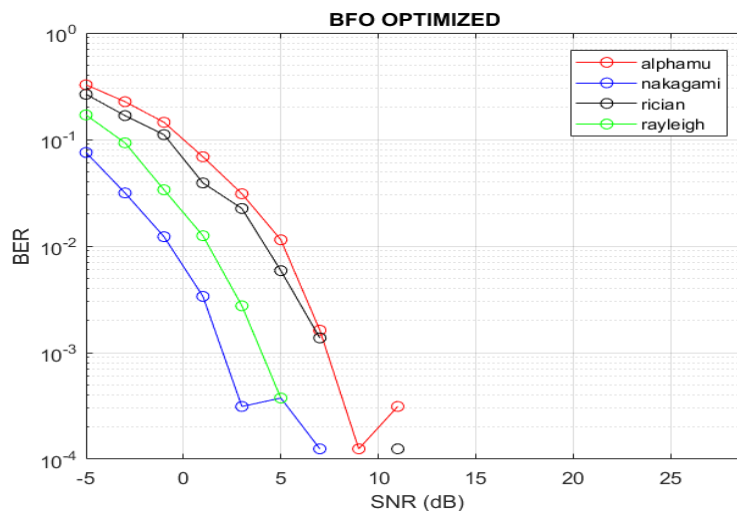


Fig. 3. Comparative graph of BER vs SNR for Bacterial Foraging Optimized Block Diagonalization using Alpha-Mu, Rayleigh, Rician and Nakagami fading channels



A detailed comparison of the SNR performance at BER 10^{-3} for the four communication channels for BD and BFO- BD techniques is made in table 1 which clearly demonstrates that that by extending BFO-BD techniques there is very significant improvement seen in the SNR for achieving a BER of 10^{-3} dB in all the four fading channels. It is also seen from the table that the Nakagami channel exhibits the lowest SNR ~ 2.3 dB and the best improvement in terms of BER performance compared to the other three fading channels .

Table 1: Comparison of the SNR values for Block diagonalization and Bacterial forging optimized Block diagonalization techniques for the four fading channels

TECHNIQUE USED	SNR REQUIRED TO ACHIEVE BER 10^{-3}			
	FADING CHANNEL MODELS			
	ALPHA-MU	RICIAN	RAYLEIGH	NAKAGAMI
Block diagonalization Technique	12.5 dB	11 dB	7.5 dB	6 dB
Proposed Bacterial Forging Optimized Block Diagonalization technique	7.5 dB	7 dB	4 dB	2.3 dB

Conclusion :

From the analysis of the simulation results carried out in the present paper the following conclusion can be drawn:

- It is found that both the Block diagonalization technique and Bacterial forging optimized Block diagonalization technique shows inverse relation between BER and SNR for all the four fading communication channels namely Rician, Alpha MU Rayleigh and Nakagami channels.
- By employing the Bacterial Foraging optimization along with the traditional Block Diagonalization technique in the pre-coders, we can greatly reduce the bit error rate . This in-turn saves a lot of power in transmission of the signal and error probability.
- The Nakagami channel out-performs all other channels and can be considered as a practical viable channel for present day communications.

CONFLICTS OF INTEREST :

The authors have no conflicts of interest to declare.

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