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Development of Empirical Correlations by Regression Analysis and Curve Fitting method for Uncleaned Sugar Cane Juice with Equispaced Tape Inserts

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Abstract

The presented work focuses on the development of empirical correlations to represent the change in Reynolds number and twist ratio on friction factor and also to represent the effects of Reynolds number, Prandtl number and twist ratio on Nusselt number. The proposed experimental investigations carried out are based on the implementation of full length equispaced tape inserts by using uncleaned sugar cane juice as a test fluid. The results are proposed by using food grade Stainless Steel, Copper and Aluminium material twisted tapes with twist ratios 3.01, 5.06, 6.78 and 8.39. Empirical correlations are developed by regression analysis for friction factor and Nusselt number. The curve fitting of friction factor, heat transfer coefficient and Nusselt number are presented. The correlations are developed to understand the trend of the flow. The experimental data is compared with the results obtained by correlations. The average mean error is found for all types of tape materials. The average mean error of friction factor food grade Stainless Steel is found 7.18% with the experimental results. For Copper and Aluminum twisted tape material the average mean error of friction factor is found as 14.99% and 9.71% respectively. The average mean error of Nusselt number is found as 2.47% for food grade Stainless Steel material with the experimental data. The average mean error of Nusselt number for Copper and Aluminum are found as 3.94% and 1.75% respectively.

Keywords: friction factor, heat transfer coefficient, twist ratio, twisted tape, uncleaned sugar cane juice

Introduction

There are different equipment's used in industry to transfer the heat energy from one fluid to other fluid. These devices are known as heat exchangers. Condenser and evaporator are treated as special type of heat exchangers. The working of both heat exchangers is exactly opposite to each other. In condenser, the objective is to condense the working fluid (giving out heat to the cooling fluid) while in evaporator the main objective is to evaporate (absorb heat from the enclosed space or hot fluid) the working fluid. The main interest of design engineers is to increase the efficiency of such heat exchangers, effectively to increase the performance of the system. In 19th century, A. E. Bergles and Harmon L. Morton [1] identifies sixteen different enhancement techniques to augment convective heat transfer. The enhancement techniques are broadly classified as active, passive and compound techniques. This work deal with the pressure drop, frictional factor, enhancement of heat transfer studies in a Uncleaned Sugar Cane Juice (UNSCJ) at UWT conditions in a circular pipe for laminar flow, using full length equispaced twisted tapes of different twist ratio as 8.39, 6.78, 5.06 and 3.01. The three materials viz: food grade Stain less Steel (SS), Copper (Cu) and Aluminium (Al) are selected for full length twisted tape inserts.

Literature Review

Some researchers have proposed important correlations for laminar, transition and turbulent flow in a plain tube. E. N. Sieder and C. E. Tate [2] studied heat transfer and pressure drop of liquid in tubes. Hausen [3] reported heat and pressure drop studies in circular tube for transition flow using water as working fluid. Dittus and Boelter [4] proposed an empirical correlation for heat transfer in fully developed turbulent flow in smooth tubes. All the quantities that are measured to estimate Nusselt number, Reynolds number and friction factor are subject to certain uncertainties because of errors associated in the measurement. These individual uncertainties as well as the combined effect of these are presented here. The analysis is carried out on the basis of the suggestion made by S. J. Kline and F. A. McClintock [5] and W. G. Steele and W. G. Coleman [6].

Hong and Bergles [7] studied heat transfer enhancement in in a tube with laminar viscous liquid, with constant heat flux boundary conditions. The limitation of the correlation developed was valid for only high Prandtl number (approx. 730). They reported the increase in heat transfer rate with the insertion of twisted tape inserts.

W. J. Marner and A. E. Bergles [8] studied heat exchangers used in chemical engineering with uniform wall temperature. The test fluid was ethylene glycol. They reported uniform wall temperature heating and cooling of test fluid (Pr = 24-85, Rea=380-3470) with twisted tape of twist ratio (TR) 5.4 in a tube and tubes with internal fins and observed that, heat transfer and friction factor both increased with increase in Reynolds number.

R.K. Manglik and A. E. Bergles [9] studied the heat transfer and pressure drop correlation for twisted tape inserts for UWT condition. The test fluids were water and ethylene glycol. The enhancement mechanism was explained for laminar flow condition. The major parameters for enhancement in heat transfer were partitioning in tubes, flow disturbance, the large flow path and the secondary fluid circulation. The authors developed laminar flow correlations for the friction factor with Nusselt number.

C. Shivkumar and M. Raja Rao [10] studied spirally corrugated tubes with twisted tape inserts. The compound laminar flow heat transfer augmentation to power law fluid, but the authors did not find any better results compared with twisted tape insert in a plain circular tube.

S. K. Saha, U. N. Gaitonde and A. W. Date [11] studied pressure drop and heat transfer characteristics under constant wall heat flux boundary conditions. The experiments were conducted on circular tube fitted with regularly spaced twisted tape inserts. They studied the effect of the pinching of place and the use of connecting elements by rods for a better thermohydraulic performance. As the tape width reduced, it causes poor results.

V. V. Pinjala and M. Rajarao [12] studied laminar flow pseudo-plastic type power law fluid, in a circular pipe, subjected to uniform wall temperature condition and gave a predictive correlation to fit the analytical data.

S. K. Agarwal and M. Rajarao [13] studied the experimental findings for servo therm oil (Pr = 195-375). The experiment was performed for UWT heating and cooling of oil in a circular tube (Rea = 70-4000) with twisted tape inserts (TR = 2.41-4.84). The Nusselt number was found to be 2.28 -5.35 and 1.21-3.7 times more than the plain tube. They developed a correlation representing effect of heat transfer on friction factor, for practical application.

M. Loknath [14] studied horizontal tube and tube fitted with half-length and full-length twist inserts to analyze the pressure drop and heat transfer of water with laminar flow, maintained at constant heat flux conditions. The author founds the half-length twisted tapes performs better than that of full-length twisted tapes.

S. Al-Fahed, L. M. Chamra and W. Chakroun [15] studied the pressure drop and heat transfer coefficient in tubes of shell and tube heat exchanger with a plain, micro fin, and twisted-tape inserted. Oil was considered as a working fluid under UWT. The set of twisted-tapes includes three different twist ratios each with two different widths. The results showed with the decrease in twist ratio, the twisted-tape gave more heat transfer enhancement. The loose-fit (W =10:8 mm) is recommended to be used in applications where low twist ratios (TR = 5.4 and 3.6) and high pressure drop situations are expected.

S. K. Saha, A. Dutta and S. K. Dhal [16] studied a horizontal tube and tube fitted with regularly spaced twisted tapes for the analysis of a pressure drop and heat transfer, on laminar flow of viscous fluid, subjected to constant wall heat flux conditions. It was observed that pinching (a twisted tape placed exactly at the centre of the tube) of twisted tape insert in a tube performed better than a twisted tape insert. Experimental analysis showed that a non-zero phase angle, in between the segmented twisted tape gave poor results.

A.G. Patil [17] presented pseudo plastic type power law fluid in a circular tube and the friction factor and heat transfer characteristics of laminar swirl flow, using varying width full length twisted tapes under a UWT. The reduced width twisted tapes were more effective for enhancing laminar swirl flow heat transfer. The reduced width twisted tapes effect in better heat transfer, savings in input pumping power and in cost of tape material. The 17-60% reduction in friction factor and 5-24 % reduction in Nusselt number was observed for 15-50% reduction in tape-width.

S. K. Saha and P. Langille [18] reported heat transfer and pressure drop characteristics in a circular tube. A test tube was fitted with full length, short length and regularly spaced inserts connected by thin circular rod under constant heat flux conditions. The test fluid was water and viscous fluid. The different strips had geometry as rectangular, square and crossed in cross-section with change in aspect ratio. The experimental findings showed that the tape length, Twist ratio, space ratio, Reynolds number, Prandtl number and the rod diameter govern the characteristics.

P. K. Sarma, T. Subramanyam, P. S. Kishore, V. Dharma Rao and Kakac Sadik [19] studied the methods the prediction of heat transfer coefficients with twisted tape inserts in a tube. The wall shear and the temperature gradient are modified with friction coefficient correlation which leads to heat transfer augmentation from the tube wall. The predictions from this theory were compared with reported correlations of a tube with twisted tape inserts. The presented theoretical results were converted in the form of a correlation.

Regression Analysis And Curve Fitting

Empirical correlations are developed to represent the change in Reynolds number and twist ratio on friction factor and also to represent the effects of Reynolds number, Prandtl number and twist ratio on Nusselt number.

A. Friction factor

For food grade SS material,

$$f = (\mathrm{y})^{-0.267} (Re)^{-0.159} \tag{1}$$

For Cu material,

$$f = (\mathrm{y})^{-0.384} (Re)^{-0.195} \tag{2}$$

For Al material,

$$f = (\mathrm{y})^{-0.589} (Re)^{-0.125} \tag{3}$$

B. Nusselt number

For food grade SS material, $Nu = (y)^{-0.239}(Re)^{1.145}(Pr)^{-2.602}$ (4)

For Cu material,

$$Nu = (y)^{-0.279}(Re)^{0.986}(Pr)^{-1.810} \tag{5}$$

For Al material, $Nu = (y)^{-0.218}(Re)^{0.907}(Pr)^{-1.582}$ (6)

Table I values of average mean error, correlation coefficient and R^2 for friction factor

Material	Average mean error	correlation coefficient	R^2
Food grade SS	7.18%	0.9984	0.9969
Cu	14.99%	0.9951	0.9902
Al	9.71%	0.9982	0.9965

Table II values of average mean error, correlation coefficient and R^2 for Nusselt number.

Material	Average mean error	correlation coefficient	R^2
Food grade SS	2.47%	0.9999	0.9999
Cu	3.94%	0.9999	0.9998
Al	1.75%	0.9999	0.9999

C. Curve fitting

Curve fitting is proposed to know the trend of friction factor with change in Reynolds number. Fig. 1 shows the polynomial curve fitting for food grade SS material with twist ratios 3.01, 5.06, 6.78 and 8.39 respectively. The trend of friction factor with open pipe without twisted tape is also represented. The trends for different cases are formed by using Origin 8.0 software.

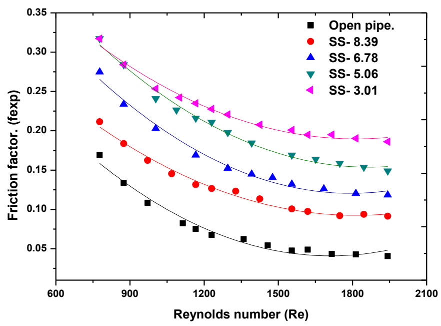

Fig. 1 Curve fitting of *f* and *Re* for SS material

For open pipe,

$f = 1E\text{-}07Re^2 - 0.0005Re + 0.4358$ (7)

SS TR 3.01,

f = 1E-07Re^2 - 0.0004Re + 0.5526 (8)

SS TR 5.06,

f = 1E-07Re^2 - 0.0005Re + 0.6109 (9)

SS TR 6.78,

f = 1E-07Re^2 - 0.0005Re + 0.5716 (10)

SS TR 8.39,

f = 1E-07Re^2 - 0.0004Re + 0.4365 (11)

The correlation obtained from polynomial curve fitting for Cu and Al with twist ratios 3.01, 5.06, 6.78 and 8.39 are given by equations 12 to 19 respectively.

For Cu material

Cu TR 3.01,

f = 1E-07Re^2 - 0.0004Re + 0.4846 (12)

Cu TR 5.06,

f = 1E-07Re^2 - 0.0004Re + 0.473 (13)

Cu TR 6.78,

f = 2E-07Re^2 - 0.0005Re + 0.5146 (14)

Cu TR 8.39,

f = 1E-07Re^2 - 0.0004Re + 0.4235 (15)

For Al material,

Al TR 3.01,

f = 9E-08Re^2 - 0.0003Re + 0.4542 (16)

Al TR 5.06,

f = 9E-08Re^2 - 0.0004Re + 0.4445 (17)

Al TR 6.78,

f = 1E-07Re^2 - 0.0004Re + 0.4989 (18)

Al TR 8.39,

f = 1E-07Re^2 - 0.0004Re + 0.4258 (19)

The correlations between heat transfer coefficient and Reynolds number for the materials food grade SS with TR 3.01,5.06,6.78 and 8.39 are shown in Fig. 2. Equations 21 yo 24 shows the correlation between heat transfer coefficient and Reynolds number for food grade SS with TR 3.01, 5.06, 6.78 and 8.39 respectively.

For open pipe,

h=3.70E-04Re^2 - 0.15241Re + 572.90 (20)

For SS material,

SS TR 3.01,

h=1.91E-04Re^2 + 0.07376Re + 194.95 (21)

SS TR 5.06,

h=2.33E-04Re^2 + 0.08879Re + 296.79 (22)

SS TR 6.78,

h=2.39E-04Re^2 + 0.14766Re + 323.86 (23)

SS TR 8.39,

h=4.49E-04Re^2 - 0.19944Re + 622.97 (24)

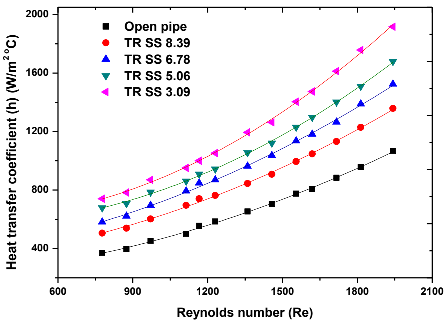

Fig. 2 Curve fitting of *h* and *Re* for SS material

For Cu material,

Cu TR 3.01,

h =2.14E-04Re^2 + 0.1077Re + 244.51 (25)

Cu TR 5.06,

h =2.14E-04 Re^2 + 0.1335Re + 312.07 (26)

Cu TR 6.78,

h =3.21E-04 Re^2 - 0.1206Re + 525.30 (27)

Cu TR 8.39,

h =3.8E-04 Re^2 - 0.1497Re + 564.97 (28)

For Al material,

Al TR 3.01,

h =2.14E-04Re^2 + 0.1077Re + 244.51 (29)

Al TR 5.06,

h =2.14E-04Re^2 + 0.1335Re + 312.07 (30)

Al TR 6.78,

h =3.21E-04Re^2 - 0.1206Re + 525.30 (31)

Al TR 8.39,

h =3.8E-04Re^2 - 0.1497Re + 564.97 (32)

The curve fitting between Nusselt number and Reynolds number for food grade SS with TR 3.01, 5.06, 6.78 and 8.39 is shown in Fig.3. The polynomial fitting for open pipe is proposed in equation 33. The polynomial fitting for food grade SS is proposed in equations 34 to 37.

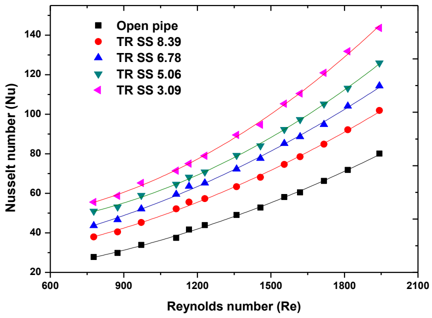

Fig. 3 Curve fitting of *Nu* and *Re* for Food grade SS material

For open pipe,

$$Nu = 1.43E\text{-}05 Re^2 + 0.00553 Re + 14.62 \quad (33)$$

For food grade SS,

SS TR 3.01,

$$Nu = 1.75E\text{-}05 Re^2 + 0.00663 Re + 22.25 \quad (34)$$

SS TR 5.06,

$$Nu = 1.79E\text{-}05\ Re^2 + 0.01107 Re + 24.29 \quad (35)$$

SS TR 6.78,

$$Nu = 2.77E\text{-}05\ Re^2 - 0.01143 Re + 42.96 \quad (36)$$

SS TR 8.39,

$$Nu = 3.36E\text{-}05\ Re^2 - 0.01496 Re + 46.72 \quad (37)$$

Following polynomial curve fitting equations from 38 to 41 and 42 to 45 are proposed for Cu and Al material with TR 3.01,5.06,6.78 and 8.39 respectively.

For Cu material,

Cu TR 3.01,

$$Nu = 1.60E\text{-}05\ Re^2 + 0.00808 Re + 18.33 \quad (38)$$

Cu TR 5.06,

$$Nu = 1.54E\text{-}05\ Re^2 + 0.01002 Re + 23.40 \quad (39)$$

Cu TR 6.78,

$$Nu = 2.4E\text{-}05\ Re^2 - 0.00904 Re + 39.39 \quad (40)$$

Cu TR 8.39,

$$Nu = 2.85E\text{-}05\ Re^2 - 0.01123 Re + 42.32 \quad (41)$$

For Al material,

Al TR 3.01,

Nu =1.32E-05 Re^2 + 0.0085Re + 18.14 (42)

Al TR 5.06,

Nu =1.41E-05 Re^2 + 0.00597Re + 22.41 (43)

Al TR 6.78,

Nu =1.90E-05 Re^2 - 0.00619Re + 33.97 (44)

Al TR 8.39,

Nu =2.17E-05 Re^2 - 0.00709Re + 35.94 (45)

Results and Discussions

A. Empirical correlations for friction factor

For food grade SS material, $f = (\mathrm{y})^{-0.267} (Re)^{-0.159}$

For Cu material, $f = (\mathrm{y})^{-0.384} (Re)^{-0.195}$

For Al material, $f = (\mathrm{y})^{-0.589} (Re)^{-0.125}$

The mean error and average mean error are found and listed in Table III

Table III Average mean error of friction factor for food grade SS, Cu and Al

Material	Average mean error
Food grade SS	7.18%
Cu	14.99%
Al	9.71%

The average mean error is found by comparing the experimental and predicted values of friction factor with respect to the Reynolds number. It is found that the correlation developed for food grade SS with minimum error of 7.18% as compared with the correlations developed for Cu and Al material.

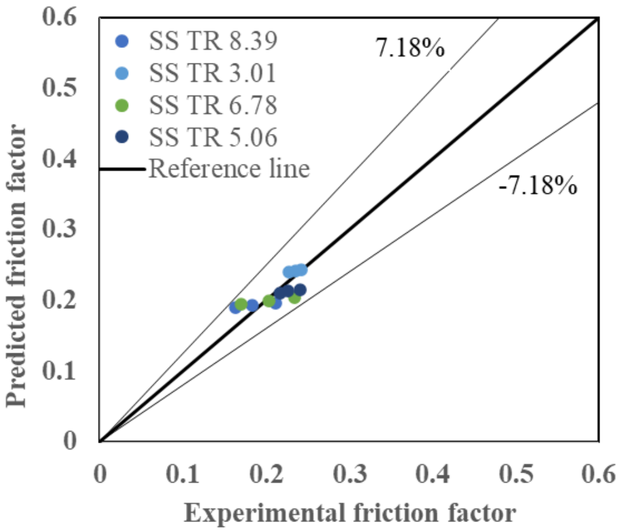

Fig. 4 Experimental friction factor with predicted value for food grade SS material

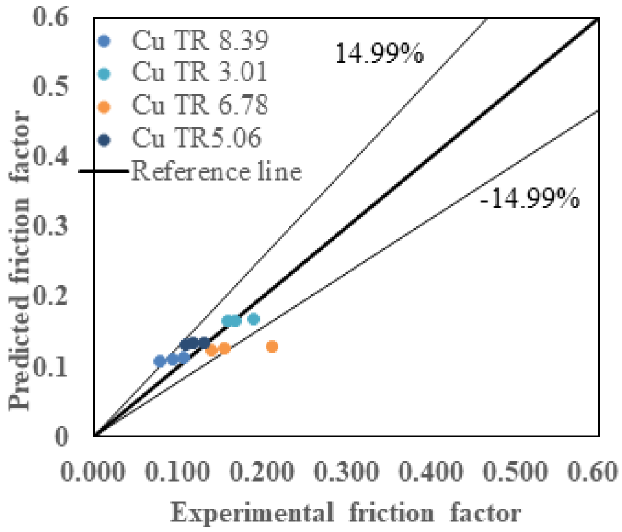

Fig. 5 Experimental friction factor with predicted value for Cu material

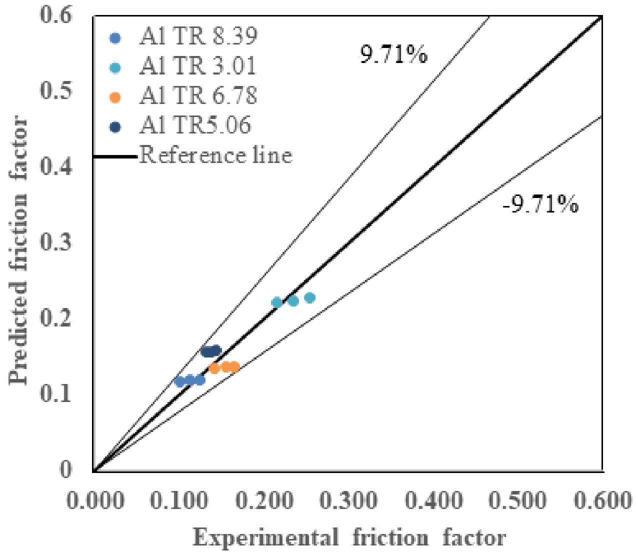

Fig. 6 Experimental friction factor with predicted value for Al material

B. Empirical correlations for Nusselt number:

For food grade SS material, $Nu = (y)^{-0.239}(Re)^{1.145}(Pr)^{-2.602}$

For Cu material, $Nu = (y)^{-0.279}(Re)^{0.986}(Pr)^{-1.810}$

For Al material $Nu = (y)^{-0.218}(Re)^{0.907}(Pr)^{-1.582}$

The mean error and average mean error for empirical correlations were tabulated in Table 4.

Table IV Average mean error of Nusselt number for food grade SS, Cu and Al

Material	Average mean error
Food grade SS	2.47%

Cu	3.94%
Al	1.75%

The average mean error for Al material correlation is found with minimum error of 1.75% as compared to food grade SS and Cu material.

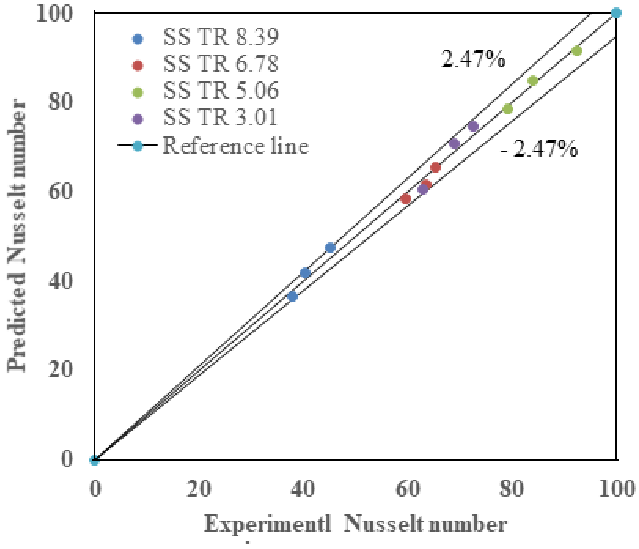

Fig. 7 Experimental Nusselt number with predicted value for SS material

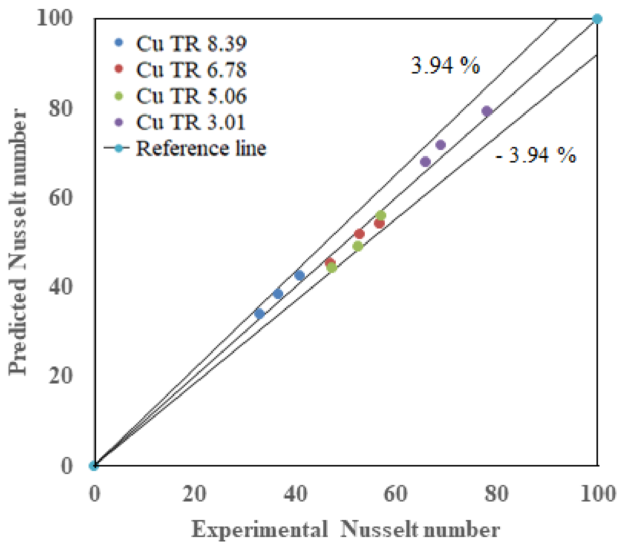

Fig. 8 Experimental Nusselt number with predicted value for Cu material

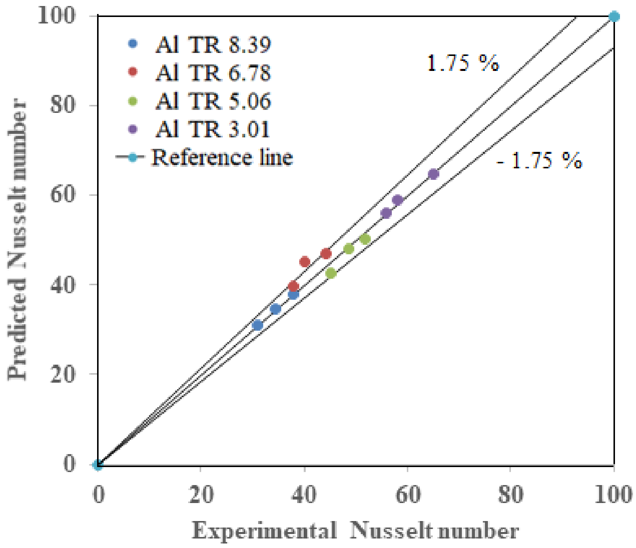

Fig. 9 Experimental Nusselt number with predicted value for Al material

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