Nat. Volatiles & Essent. Oils, 2021; 8(6): 4818-4827

Unconditional Decomposition Of Wiza Property For Operators In Banach Space

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Abstract

In this paper we study a linear transform between bounded linear operators for every continuous surjective algebra homomorphism from $\mathrm{X} \rightarrow \mathrm{Y}$ on Banach space has the Wiza property. Through this study we get Banach space X which satisfies the Wiza property if and only if it satisfies the rule deduced from the main result. This rule is satisfactory for a Banach space with is omorphicbases for finite co type and Neumann norm (p-spaces).

Key words (Haar Basis, Schauder Decomposition, Neumann Space)

1. Introduction

We will start our study in this paper with the problem mentioned by Horváth [1], does space $\mathrm{L}^{\mathrm{p}} = \mathrm{L}^{\mathrm{p}}(0,1)$ have Wiza property? And based on our main result, Corollary (1.14) , the space L^{p}have aWiza property , but we do not know whetherL^{1} it has the property, and also in [2, theorem 4.3.10] the space L^{∞} have the Wiza property because $\mathrm{L}^{\infty}$is isomorphic as a Banach space tol^{∞} . Also previously identified the spaces L^{p} for $1 \leq \mathrm{p} \leq \infty$,have the Wiza property [1]. If X, Y banach spaces we say that X have Wiza property if the linear transform from the space $\mathrm{L}(\mathrm{X})$ on X onto $\mathrm{L}(\mathrm{Y})$is injective and Banach space X called isomorphic as Banach space to Y in [3] .

Before stating our theorems we need the following definitions:

(1.1)Definition (unconditional schauder decomposition)(USD)

Let $(E_\alpha)_{\alpha\in A}$ a family of closed subspaces of X, we called $(E_\alpha)_{\alpha\in A}$ an USD, for X if $\forall x \in X$ there is a unique representation $x = \sum_{\alpha\in A} x_\alpha$ so that the convergence unconditional and $\forall\alpha \in A$, the vector $x_\alpha \in E_\alpha$. We conclude that $E_\alpha \cap E_\beta = \{0\}$, When $\alpha \neq \beta$, and there are idempotents P_α on X such that $P_\alpha X = E_\alpha$, $P_\alpha P_\beta = 0$, And therefore P_α are in $L(X)$.

(1.2)Definition

Let B subset of A then the family $\{\sum_{\alpha\in F} P_\alpha : F \subset B \text{ finite}\}$ is bounded in $L(X)$ and convergence to P_β has a range $\overline{\text{span}}_{\alpha\in\beta} E_\alpha$.

(1.3)Definition(suppression constant)

We define the suppression constant of the decomposition by $\{\|\sum_{\alpha\in F} P_\alpha\| : F \subset A \text{ finite}\}$. that is mean $\|P_\beta\|$ on definition(1.2)is bounded for all subsets B of A by this suppression constant.

(1.4)Remark

The family $(e_\alpha)_{\alpha\in A}$ forms USD basis for X where $E_\alpha = K_{e_\alpha}$ (K is the scalar field), in the following we use schauder decomposition of E_α is a finite dimension, this decomposition is called finite dimensional decomposition (FDD) and discussed in [4,section 1.9].

(1.5)Definition

Let $(\mathbf{E}_\alpha)_{\alpha\in A}$ a family of conditional schauder decomposition (CSD) for X is boundedly complete if $\{\|\sum_{\alpha\in F} x_\alpha\|_X : F \subset A \text{ finite}, x_\alpha \in E_\alpha\}$ is bounded, then the sum $\sum_{\alpha\in F} x_\alpha$ convergence in X

(1.6)Definition

Let $(E_\alpha)_{\alpha\in A}$ a family of (CSD) for X we say that decomposition is discrete lower, upper p estimate respectively if there exist a constant $C < \infty$ so that $x_1, \cdots\cdots\cdots, x_n$ are finitely many vectors in X , such that $\forall\alpha \in A$, there is at most one i for which $\{x = \sum_{i=1}^n x_i, P_\alpha x_i \neq 0, 1 \leq i \leq n\}$ the inequality

$$\left\|\sum_{i=1}^n x_i\right\| \geq \frac{1}{C}\left(\sum_{i=1}^n \|x_i\|^p\right)^{\frac{1}{p}}, \text{respectively}, \left\|\sum_{i=1}^n x_i\right\| \geq C\left(\sum_{i=1}^n \|x_i\|^p\right)^{\frac{1}{p}}$$

If $F_1, \cdots\cdots\cdots, F_n$ are a discrete finite subset of $A, \forall x \in X$, we say that decomposition has a discrete lower p estimate with constant C , then

$$\|x\| \geq \frac{1}{C}\left(\sum_{j=1}^n \left\|\sum_{\alpha\in F_j} P_\alpha x\right\|^p\right)^{\frac{1}{p}}$$

Where P_α is idempotent [5].

(1.7)Definition (Type and Co type)

If p, q (type ,co type) respectively then every USD decomposition for X has a discrete (upper p , lower q) estimate where the constant depend only on the definition (1.3) of decomposition and p, qconstant of X such that if $1 < p \leq 2$ then every USD for subspace of a quotient of L^p has a discrete (upper p , lower 2) estimate , while $2 \leq p < \infty$ then every USD decomposition for X has a discrete (upper 2 , lower p) estimate [2, Theorem 6.2.14].we will used this definition in the following theorem when there is a surjective homomorphism from $L(Y)$onto $L(X)$ for transferring information from Y to X.

(1.8) Theorem

Let $(E_\alpha)_{\alpha \in A}$ a family of USD for X that has a discrete lower p estimate , $1 \leq p < \infty$and $Y \supseteq X$, if $A_1, \cdots\cdots\cdots, A_n$ disjoint of subset of A and P_{Aj} is basis defined in definition (1.2) and $K_1, \cdots\cdots\cdots, K_n$ are operators in $L(Y)$, then there exist a constant $C < \infty$ is the discrete lower p constant of $(E_\alpha)_{\alpha \in A}$such that

$$\left\| \sum_{i=1}^{n} K_i P_i \right\| \leq C \left(\sum_{i=1}^{n} \|k_i\|^q \right)^{\frac{1}{q}}, \qquad \frac{1}{p} + \frac{1}{q} = 1$$

Proof.

Let$x \in X$. then

$$\left\| \sum_{i=1}^{n} K_i P_{ix} \right\| \leq \sum_{i=1}^{n} \|K_i\| \, \|P_{ix}\| \leq \left(\sum_{i=1}^{n} \|k_i\|^q \right)^{\frac{1}{q}} \left(\sum_{i=1}^{n} \|P_{ix}\|^p \right)^{\frac{1}{p}}$$

$$\leq \mathbf{C} \left(\sum_{i=1}^{n} \|k_i\|^q \right)^{\frac{1}{q}} \|x\|$$

(1.9)Definition (Almost discrete)

Let $(E_n)_{n=1}^{\infty}$ a family of sets and $(E_1 \cap E_2), (E_2 \cap E_3), \cdots\cdots\cdots\cdots, (E_{n-1} \cap E_n)$ is finite we say that E_n is an almost discrete.

(1.10)Definition$[$ $\mathbf{property}$ $(\ast)]$

Let $\{N_\tau : \tau < C\}$ is an almost discrete continuum of natural numbers of infinite sets for each$\tau < C$, and let $(E_n)_{n=1}^{\infty}$ is unconditional FDD for X defined in (1.4), then X is symmetric to closed linear span of subspaces. Subsymmetric bases and the sum direct of two banach spaces are obvious examples of FDDs that have property ($\ast$).In corollary (1.14) and proposition (1.11). We review that the Haar basis for L^Phas property (⁎), and the consequence by using the definition (1.10).

(1.11) proposition

Let $(E_n)_{n=1}^{\infty}$ is unconditional FDD for X and (E_n) has property (⁎) see before is an almost discrete family $\{N_\tau : \tau < \mathcal{C}\}$in (1.10).LetΨ is a nonzero, non-injective continuous homomorphism from $L(X)$onto a Banach algebra $\mathcal{G}$.Then for each $\tau < \mathcal{C}, \Psi(P_{N_\tau})$is a nonzero idempotent in $\mathcal{G}$.Furthermore, if F is finite subset then there is constant $C < \infty$ such that $\left\|\sum_{\tau\in F} \Psi(P_{N_\tau})\right\|_{\mathcal{G}} \leq C$. If $\mathcal{G}$ is a sub-algebra of $L(Y)$ for some Banach space Y, then $\Psi(P_{N_\tau})$ is a family of computingAccessories to Y of projections related with USD for a subspace$Y_0$of$Y$.

Proof.

Let F is a finite subset of $\{\tau : \tau < \mathcal{C}\}$, $N_\tau \cap N_\delta \subset \mathcal{H}$ so that $\mathcal{H}$ is finite set of natural numbers for all $\{\tau, \delta\} \in F$. P_{N_τ}has a range symmetric to X and Ψ is not zero then $\Psi(P_{N_\tau})$ is nonzero idempotent in $\mathcal{G}$. Assume that $\mathcal{S}_\tau = P_{N_\tau/\mathcal{H}}$ and $\{P_{N_\tau} - \mathcal{S}_\tau, \forall\, \tau \in F\}$ we find that the basis projections from X onto $\overline{\text{span}}\{E_n : n \in N_\tau\}$ are closed spans of disjoint subsets of $(E_n)_{n=1}^{\infty}$ and Ψ is nontrivial ideal in $L(X)$contains the finite rank operators such that $\Psi(P_{N_\tau}) = \Psi(\mathcal{S}_\tau)$for each $\tau \in F$ so

$$\left\|\sum_{\tau\in F} \Psi(\mathcal{S}_\tau)\right\|_{\mathcal{G}} \leq \left\|\sum_{\tau\in F} \mathcal{S}_\tau\right\| \|\Psi\| \leq C\|\Psi\|,$$

whereC is the suppression constant (1.3). The last statement is now clear.

After this preliminary we will mention the main theorem in this article.

(1.12) Theorem

A banach space X has a Wiza property if $(E_n)_{n=1}^{\infty}$ is unconditional FDD such that $(E_n)_{n=1}^{\infty}$ has a property (⁎) and has a discrete lower p estimate $[\text{Therem } (1.8)]$ for some $p < \infty$.

Proof.

To prove this theorem we will suggest that we can obtain a contradiction by continuing the proposition (1.11). Let Ψ is a non-injective continuous homomorphism from $L(X)$ onto $L(Y)$ for some nonzero Banach space Y .Where theproperty (⁎) of (E_n) is proved and for $F \subset N$,Thebasis projection of $\{E_n : n \in F\}$ is denoted by P_F.We suggest that if a contradiction exists, it is sufficient to show that the subspace Y_0 is complemented in .In fact, if Y_0 is completed in Y, then $L(Y_0)$ is symmetric as Banach's algebra to the sub-algebra of $L(Y)$.However, when defining $Y_\tau = \Psi(P_{N_\tau})Y$for $\tau < \mathcal{C}$, we know that $(Y_\tau)_{\tau<\mathcal{C}}$is USD of Y_0.Thus $L(Y)$ cannot be a continuous image of $L(X)$since X is separable and has an unconditionalFDDthen the density character of $L(X)$ is equal to c .Thus the theorem ends.

To prove that Y_0 must complete in Y, we use Proposition (1.11) we have $\left\|\sum_{\tau \in F} \Psi(P_{N_\tau})\right\|_{L(Y)} \leq C$ and $[\text{Therem } (1.8)]$That is, we only need to find the constant C to approve $(Y_\tau)_{\tau < C}$ has a discrete lower p estimate so If $F_1, \cdots\cdots\cdots, F_n$ are a discrete finite subset and y in Y ,then

$$\|y\| \geq \frac{1}{C}\left(\sum_{j=1}^{m}\left\|\sum_{\tau \in F} \Psi(P_{N_\tau})y\right\|^p\right)^{\frac{1}{p}} \tag{1}$$

As in Proof Proposition (1.11), we can write $\Psi(P_{N_\tau}) = \Psi(\mathcal{S}_j)$ with $\mathcal{S}_j$ for $1 \leq j \leq m$, So (1) can be rewritten as

$$\|y\| \geq \frac{1}{C}\left(\sum_{j=1}^{m}\left\|\Psi(\mathcal{S}_j)y\right\|^p\right)^{\frac{1}{p}} \tag{2}$$

From Therem (1.8) for any $K_1, \cdots\cdots\cdots, K_m$ in L(Y)we have

$$\left\|\sum_{i=1}^{m} K_j \Psi(\mathcal{S}_j)\right\| \leq C\left(\sum_{j=1}^{m}\left\|K_j\right\|^q\right)^{\frac{1}{q}}, \qquad \frac{1}{p} + \frac{1}{q} = 1 \tag{3}$$

Where C depends only on p Take any $y \in Y$ and take $\lambda_j \geq 0$ with

$$\sum_{i=1}^{m} \lambda_j^q = 1 \quad \text{and} \quad \sum_{j=1}^{m} \lambda_j \left\|\Psi(\mathcal{S}_j)y\right\| = \left(\sum_{j=1}^{m}\left\|\Psi(\mathcal{S}_j)y\right\|^p\right)^{\frac{1}{p}}$$

Let $Y_0 \in Y$ be any unit vector and let K_j be $\Psi(\mathcal{S}_j)$ followed by $\Psi(\mathcal{S}_j)y \longrightarrow \lambda_j \left\|\Psi(\mathcal{S}_j)y\right\| y_0$.Then by (3),

$$\left(\sum_{j=1}^{m}\left\|\Psi(\mathcal{S}_j)y\right\|^p\right)^{\frac{1}{p}} = \sum_{j=1}^{m} \lambda_j \left\|\Psi(\mathcal{S}_j)y\right\| = \left\|\sum_{i=1}^{m} K_j \Psi(\mathcal{S}_j)y\right\|$$

$$\leq C\left(\sum_{j=1}^{m}\left\|K_j\right\|^q\right)^{\frac{1}{q}} \|y\| \leq C\|y\| \text{ , which is (2)}$$

(1.13) Corollary

A banach space X has a Wiza property if has a finite cotybe and subsymmetric basis.

(1.14) Corollary

The Haar basis $(h_i)_0^\infty$ is an unconditional basis of L^p then has a Wiza property for $1 < p < \infty$.

Proof.

From Theorem (1.12) and definition (1.10),we show that the Haar basis of L^p has a property (⁎). Define for $\tau < \mathcal{C}$an unconditional haarbasis for $L^p(0.1)$ as follows

$$X_\tau = \overline{\text{span}}\{h_{n,i}: n \in N_\tau, 1 \le i \le 2^n\}$$

So that $(h_{n,i})$ is the set of functions of sub-intervals of $(0,1)$ that have length 2^{-n}, X_τ is symmetric to L^p with the isomorphism constant depending only on p by theorem in[6].

2. Examples and properties

We show some of examples of spaces with a property (⁎) and with a Wiza property and prove some properties ofdefinition (1.10).

$(\mathbf{2.1})$**Definition**

Let$(E_n)_{n=1}^{\infty}$is an unconditional FDD for X,We say that (E_n) has property (⁎) with , there is $\{N_\tau: \tau < \mathcal{C}\}$of infinite sets of $\mathbb{N}$for each$\tau < \mathcal{C}$, X is , such that K is positive constant and symmetric to the closed linear span of $\{E_n: n \in N_\tau\}$.However, we need this quantitative idea to fully generalize Theorem(2.5).

(2.3) Definition

We define the subspace of $\left(\otimes_{n=1}^{i} X_i\right)_y$ of all sequences of the form $(0, \dots, 0, x_i, 0, \dots)$ by($X_i \otimes e_i$).for $= 1, 2, \dots$, where (e_i) is an unconditional basis for Y and X_i and $\|\bar{x}\| = \|\sum_{i=1}^{\infty} \|x_i\|. e_i\|_Y$ is finite.

(2.4) Theorem

Let$\left(E_n^i\right)_{n=1}^{\infty}$is an unconditional FDD forX_i,satisfying property (⁎) and definition (1.6) with a constant , for $i = 1, 2, \dots$, we say that $\left(\otimes_{n=1}^{i} X_i\right)_y$ has a Wiza property if an unconditional FDD$\left(E_n^i \otimes e_i\right)_{i,n=1}^{\infty}$ of $\left(\otimes_{n=1}^{i} X_i\right)_y$satisfies (⁎)for each subsymmetric basis (e_i) of Y, and (e_i) has such an estimate.

Proof.

By definitions (1.10) and (2.1).Let$\{N_\tau^i: \tau < \mathcal{C}\}$, it is sufficient to prove that theWiza property follows theorem (1.12) then

$$\{(i, n): i \in N_\tau \text{ and } n \in N_\tau^i\}$$

is an almost discrete continuum of subsets $\mathbb{N} \times \mathbb{N}$. if$(E_n)_{n=1}^{\infty}$satisfy theorem (1.12) have discrete lower p estimates and (e_i) has such estimate then the unconditional FDD $\left(E_n^i \otimes e_i\right)_{i,n=1}^{\infty}$ satisfy definition (2.1).

(2.5) proposition

X_pSatisfies theproperty (⁎) and Wiza property if $p \in (1, +\infty)\backslash\{2\}$

Proof.

Let $p > 2$. Assume that$\mathbb{N}$ as a discrete union of finite subsets ζ_j for $j = 1, 2, \ldots$, with$|\zeta_j| \to \infty$. for$i \in \zeta_j$let $\zeta_i = |\zeta_j|^{\frac{2-p}{2p}}$, so $\eta_i \to 0$ and for every j,$\sum_{i \in \zeta_j} \eta_i^{\frac{2-p}{2p}} = 1$.let$E_j = \text{span}\,(e_i \oplus \zeta_i f_i)_{i \in \zeta_j}$for any infinite sub-sequence of unconditional FDD (E_j), the closed span of this subsequence is similar to X_p. FDD is unconditional because it liein L^p, it has a lower p estimate. Consequently the result follows theorem (1.12). Trace case $1 < p < 2$ given the dual FDD.

In [7] we find a few more is omorphically distinct spaces that are isomorphic to complemented subspaces of L^p when $p \in (1, \infty) \setminus \{2\}$Based on X_p and the classical complementary subspaces of L^p,can show that they all have the (⁎) and Wiza property.from theorem (2.5) and Based on X_pThis space is denoted by B_p in [7].The ℓ^psum of spaces $X_i$each having a one symmetric basiswith uniform constant.ifX_i is isomorphic to ℓ^2 the isomorphism constant tends to (∞). $B_p$has (⁎) and the wiza property.

Also in [8] for $p \in (1, \infty)\backslash\{2\}$ the first infinite family of non-isomorphic complemented subspaces of L^p is generated .generally, if $\left(E_n^i\right)_{n=1}^{\infty}$is an unconditional FDD forX_i , $X_1, X_2 Y_1, Y_2$ subspaces of $L^p(\Omega)$and $T_i \in (X_i, Y_i)$Then $\left(E_n^1 \otimes_p E_m^1\right)_{n,m=1}^{\infty}$is an unconditional FDD such that $T_1 \otimes_p T_2 \in L\left(X_1 \otimes_p X_2, Y_1 \otimes_p Y_2\right)$. (This was done in [8]), that the$\left(X \otimes_p Y\right)$isomorphism class depends only on the isomorphism classes X and Y and that if X and Y are complemented in $L^p(\Omega)$, then $\left(X \otimes_p Y\right)$ is complemented in $L^p\left(\Omega^2\right)$.Also we define by X_p some isomorph of X_p that is complemented in $L^p[0, 1]$. let$Y_1 = X_p$, and for $n = 2, 3, \ldots$, let $Y_n = Y_{n-1} \otimes_p X_p$. it is clear that the spaces Y_n are complemented in some L^pspace isometric to $L^p[0, 1]$.

(2.6) Theorem

Let's say $X_1, \ldots, X_n$ are Banach spaces, each of which has an unconditional FDD with property (⁎). Suppose $Y_1 \otimes \cdots \cdots \otimes Y_n$ denotes the tensor product with norm in some n classes with the following properties:

I. for $j = 1, \ldots, n$, If$T_j \in \left(Y_j, Q_j\right)$then

$$T_1 \otimes \cdots \cdots \otimes T_n : Y_1 \otimes \cdots \cdots \otimes Y_n \longrightarrow Q_1 \otimes \cdots \cdots \otimes Q_n$$

Is bounded.

II. If Y_j has an unconditional FDD $\left(F_n^j\right)_{n=1}^{\infty}$, then $(F_{n1}^1 \otimes \cdots \cdots \otimes F_{nm}^j)_{n1,\ldots\ldots,nm=1}^{\infty}$ is an unconditional FDD for the completion of $(Y_1 \otimes \cdots \cdots \otimes_n Y_n)$.

Then, if $(X_1, \ldots, X_n)$, the completion of $(X_1 \otimes \cdots \cdots \otimes X_n)$ has an unconditional FDD with property (⁎).

Proof.

Let$\left(E_n^i\right)_{n=1}^{\infty}$is an unconditional FDD forX_i, for each $= 1, \ldots, m$. By definition (1.10)$\{N_\tau^i: \tau < C, n \in N_\tau^i\}$. Consider

$$\{N_\tau^1 \times \cdots \times N_\tau^m: \tau < C\}$$

The continuum of subsets of $\mathbb{N}^m$. This is an almost discretecollectionwhose origin is considered a continuum.

Property (II): tensor norms $(E_{n1}^1 \otimes \cdots \cdots \otimes E_{nm}^j)_{n1,\ldots\ldots,nm=1}^{\infty}$is an unconditional FDD for the completion of $(X_1 \otimes \cdots \cdots \otimes X_m)$.

Property (1) for each $< C$, the closed linear span of

$$(E_{n1}^1 \otimes \cdots \cdots \otimes E_{nm}^j)_{(n1,\ldots\ldots,nm=1) \in N_\tau^1 \times \cdots \times N_\tau^m}$$

is isomorphic to the completion of $(X_1 \otimes \cdots \cdots \otimes X_m)$.

it is clear from definition(2.3) that if $X_1, \ldots, X_m$ are subspaces of L^p for $1 \le p < 2$ that have subsymmetric bases, then $(X_1 \otimes \cdots \cdots \otimes X_m)$ has property (⁎) and the Wiza property.(For $p > 2$, the isomorphism includes only $\ell^p$and ℓ^2).

(2.7)Problem

Let$\in$ $(1, \infty) \setminus \{2\}$,suppose thatX a complementary subspace of L^p.Does X have a Wizaproperty? When X has an unconditional basis and itis one of the$\aleph_1$spaces was done in [9].

We finalize this part by discussing another class of classic Banach spaces that has the property (⁎)and Wiza property; That is, the NeumannC_p representations of compact operators Ton ℓ^2 of $(T * T)^{1/2}$ eigenvalues are p summable. WeHandle case 1< p < ∞ but then note how one can prove that C_1 (the operators of the trace class on ℓ^2) have a Wizaproperty. Neither C_1 nor its pre dualC_∞ has an unconditional FDD [10] and therefore these spaces do not have a property (⁎).In the complement, let$p = 2$because C_2, being isomorphic with ℓ^2, has already been discussed.

First, assumeT_p subspace of C_p formed by the lower trigonometric matrices of C_p.Here we exclude $p = 1, p = \infty$ and p = 2. There is no unconditional basis for T_p and C_p [10], but T_p has a unconditional FDD (E_n) ; which is $E_n = \mathrm{span}_{1\leq j\leq n} e_i \otimes e_j$; That is, the matrix is in E_n if and only if the only nonzero terms are in the first n entries of the n-throw. Since multiplying all entries in a row with the same standard order of magnitude is equal scale on C_p, (E_n) up to 1-unconditional. If M is an infinite subset of N,let $T_p(M)$ be the closed span in T_p of $(E_n)_{n\in M}$ and is norm one complemented in T_p Since (E_n) is 1-unconditional We claim that T_p is isometric to a K_p-complemented subspace of $T_p(M)$ with K_p independent of M. The space T_p is isomorphic to $\ell^p(T_p)$ [11, p. 85], thus the decomposition in [2, Theorem 2.2.3] shows that T_p is isomorphic to $T_p(M)$. Thus T_p has property (⁎).

Theorem (1.12) applies because C_p has finite cotype when $p < \infty$, so T_p has the Wiza property when $1 < p < \infty$. Now for $1 < p < \infty$, T_p is complemented in $C_p$through the projection that zeroes out the inputs that lie above the diagonal [12], [13], it follows [11] that T_p is isomorphic to C_p. Furthermore, for M an infinite set of N, there is subspace Y of C_p that is isometric to $T_p(M)$ such that $T_p \subset Y$, which is required.

(2.8)Theorem

The space T_p has the property (⁎)for $1 < p < \infty$. Then, the space C_p has the Wiza property for $1 < p < \infty$.

As stated earlier, it can be shown that C_1 has the Wiza property which does not have unconditional FDD. despite this, the C_p norm for $1 \leq p \leq \infty$ in [10] is called the unconditional matrix norm; i.e., the norm of a linear combination of the natural basis elementsis equivalent to the norm of sequences of $(\varepsilon_i)_{i=1}^{\infty}$ and $(\delta_j)_{j=1}^{\infty}$. One can determine the property variance (⁎) of bases with this unconditional property, and check that the normal bases for C_p, It fulfills this characteristic, and proves a copy of Theorem (1.12). This shows that C_1 has a Wiza property .This difference from Theorem (1.12) does not apply toC_∞, which does not have ainfinite cotype defined in (1.7), and we do not know if C_∞ has a Wiza property. The main reason for bringing $C_p$is to explain why the property (⁎) for FDD unconditionalrather than just unconditional bases.

Acknowledgment:

The authors would like to thanks the Deanship of Scientific Research at Prince Sattam Bin Abdulaziz University, Alkharj, Saudi Arabia for the assistance.

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