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Fully Decentralized Optimal Power Flow Of Multi-Area Interconnected Power Systems Based On Distributed Interior Point Method

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ABSTRACT

The Optimal Power Flow **(**OPF**)** model represents the problem of determining the bestoperating levels for electric power plants in order to meet demands given throughout a transmission network, usually with the objective of minimizing operating cost. Traditionally, the optimal power flow (OPF) problem is solved in a centralized manner. However, with continuous expansion of the scale of multi-area interconnected power systems, realisticapplications of the centralized method face additional challenges. To date, a number of decomposition techniques have been proposed to tackle decentralized OPF problems such as Lagrangian relaxation, Auxiliary Problem Principle (APP), the Alternating Direction Multiplier Method (ADMM), Benders decomposition, dual problem formulation etc., In this paper, we propose a fully decentralized OPF algorithm for multi-area interconnected power systems based on the distributed interior point method, where solving the regional correction equation was converted into solving a parametric quadratic programming (QP) problem during each Newton-Raphson iteration. Our proposed method utilizes a unidirectional ring communication graph for information sharing among areas, to eliminate the upward and downward communication among cliques. In addition, our method not only solves loosely coupled problems, but can also solve other complex distributed computing problems ofpower systems (e.g. non-convex optimization problem) in a fully decentralized manner. The proposed approach is fully decentralized without the need of a central authority to compute the cliques or update the multipliers, and no parameter tuning is required. It is robust to network partitioning. Namely, its convergence remains stable regardless of how the system is partitioned. Furthermore, it is applicable to any networks, including real multi-area power systems. This is a novel approach to tackling decentralized OPF problems. Furthermore, this decentralized algorithm enjoys the same convergence performance and accuracy as the centralized interior point method.

Keyword: OPF, APP, QP,ADMM, Lagrangian relaxation, a fully decentralized OPF.

INTRODUCTION

The Optimal Power Flow (OPF) model represents the problem of determining the best operating levels for electric power plants in order to meet demands given throughout a transmission network, usually with the objective of minimizing operating cost. Because electrical power flows according to nonlinear, non-convex functions of the system's physical characteristics, this can be a difficult problem. However, in actual operations, an instance with the entire distribution network must be solved in real time (every five minutes for many Independent System Operators) to ensuredemand is met accurately.

The exhaustive knowledge of optimal power flow (OPF) methods is critical for proper system operation and planning, since OPF methods are utilized for finding the optimal state of any system under system constraint conditions, such as loss minimization, reactive power limits, thermal limits of transmission

lines, and reactive power optimization. The OPF in power system is an optimization problem under various constraints. It is practically significant and well-explored subfield of constrained optimization. The important feature of OPF is the presence of the load flow equations in the set of equality constraints. Carpentier introduced the OPF problem in 1979. Carpentier introduced OPF as an extension to the problem of optimal economic dispatch (ED) of generation in traditional power systems. Carpentier's key contribution was the inclusion of the electric power flow equations in the ED formulation.

Classical mathematical methods (gradient, Newton's, linear, and integer programming, etc.) allow the finding of optimal solutions of real-world problems. In Jan 2004, Sun et al. proposed a penalty-based scheme for OPF with transient stability, in which ad joint equation technique calculates the gradient of the penalty term related to the stability constraints. This ad joint equation approach greatly reduces the computational cost.

The optimal power flow (OPF) problem is one of the most widely studied subjects in power systems, and is researched mainly based on the centralized methodin current practice. However, with continuous expansion of the scale of power systems and increasingly complex interconnections among areas, realistic applications of the centralized method have revealed a number of limitations:

- The accurate collection and processing of massive amounts of data is a daunting challenge in the centralized control centre and can result in communication bottlenecks;
- From considerations of confidentiality, it is impractical for each area to upload all of their information to the centralized control centre;
- Each area must operate independently and make its own decisions. Decentralized algorithms for solving OPF problems present a desirable alternative control scheme.

To date, a number of decomposition techniques have been proposed to tackle decentralized OPF problems such as Lagrangian relaxation, Auxiliary Problem Principle (APP), the Alternating Direction Multiplier Method (ADMM), Benders decomposition, dual problem formulation etc.,

A new interior point nonlinear programming algorithm for optimal power flow problems (OPF) based on the perturbed KKT conditions of the primal problem. Through the concept of the centering direction, this algorithm is extended to classical power flow (PF) and approximate OPF problems. For the latter, CPU time can be reduced substantially. To efficiently handle functional inequality constraints, a reducedcorrection equation is derived, the size of which depends on that of equality constraints. The proposed method is very promising for large scale application due to its robustness and fast execution time.

INTERIOR POINT METHOD

Interior-point methods (also referred to as barrier methods or IPMs) are a certain class of algorithms that solve linear and non-linear convex optimization problems. An interior point method was discovered by Soviet mathematician I. I. Dikin in1967 and reinvented in the U.S. in the mid-1980s. In 1984, Narendra Karmarkar developed a method for linear programming called Karmarkar's algorithm, which runs in provably polynomial time and is also very efficient in practice. It enabled solutions of linear programming problems that were beyond the capabilities of the simplex method. Contrary to the simplex method, it reaches a best solution by traversing the interior of the feasible region. The method can be generalized to convex programming based on a self-concordant barrier function used to encode the convex set.

Any convex optimization problem can be transformed into minimizing (or maximizing) a linear function over a convex set by converting to the epigraph form. The idea of encoding the feasible set using a barrier and designing barrier methods wasstudied by Anthony V. Yurii Nesterov, and Arkadi Nemirovski came up with a special class of such barriers that can be used to encode any convex set. They guarantee that the number of iterations of the algorithm is bounded by a polynomial in the dimension and accuracy of the solution.

Karmarkar's breakthrough revitalized the study of interior-point methods and barrier problems, showing

that it was possible to create an algorithm for linear programming characterized by polynomial complexity and, moreover, that was competitive with the simplex method. Already Khachiyan's ellipsoid method was apolynomial-time algorithm; however, it was too slow to be of practical interest.

OBJECTIVE

Our objective is to minimize the total generation costs:

$$\min \sum_{i \in \Phi^G} C_i(P_i^G),$$

Where, Φ_G is the set of buses with generators; and function $C_i(P_i^G)$ is the generation cost function of the generator at bus i, which can be expressed as a quadratic function:

$$C_i(P_i^G) = c_{2,i}(P_i^G)^2 + c_{1,i}P_i^G + c_{0,i}$$

Where $c_{2,i}$, $c_{1,i}$ and $c_{0,i}$ are the generation cost coefficients for the generator at bus i respectively

POWER BALANCE EQUATIONS

$$P_i^G - P_i^D = V_i \sum_{j \in \Phi_i} V_j (G_{ij} \cos\delta_{ij} + B_{ij} \sin\delta_{ij}), \forall i \in \Phi,$$

$$Q_i^G - Q_i^D = V_i \sum_{j \in \Phi_i} V_j (G_{ij} \sin\delta_{ij} - B_{ij} \cos\delta_{ij}), \forall i \in \Phi,$$

where Φ denotes the set of buses; Φ_i denotes the set of buses connected to busi; $P_i^G + jQ_i^G$ and $P_i^D + jQ^D$ are the complex powers of the generator and load at bus i, respectively; V_i and δ_i are the voltage magnitude and angle at bus i, and $\delta_i = 0$ if bus i is the slack bus; and $G_{ij}+jB_{ij}$ is the ij-th element of the admittance matrix of the power network.

NETWORK SECURITY CONSTRAINTS

$$-P_{ij}^{\max} \le V_i V_j (G_{ij} \cos\delta_{ij} + B_{ij} \sin\delta_{ij}) - V_i^2 G_{ij} \le P_{ij}^{\max}, \forall ij \in \Gamma,$$

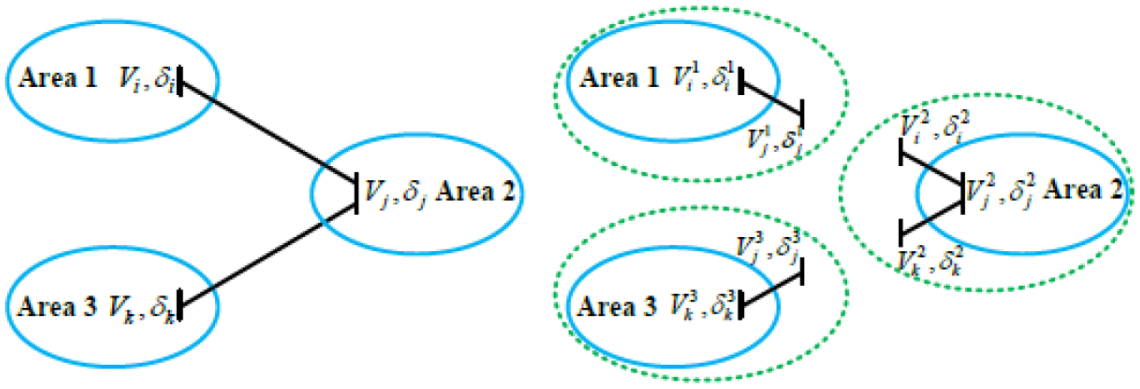
where Γ denotes the set of lines; and P_{ij}^{max} denotes the maximum capacity of line ij.

LIMITS OF VARIABLES

$$\boldsymbol{P}_{\min}^G \le \boldsymbol{P}^G \le \boldsymbol{P}_{\max}^G,$$
$$\boldsymbol{Q}_{\min}^G \le \boldsymbol{Q}^G \le \boldsymbol{Q}_{\min}^G,$$
$$\boldsymbol{V}_{\min} \le \boldsymbol{V} \le \boldsymbol{V}_{\max},$$

where $\mathbf{P}_G$, $\mathbf{Q}_G$ and **V** represent the vector of active and reactive power outputs of generators, and bus voltage magnitude, respectively; the subscripts max and min represent the upper and lower limits of variables respectively.

PARTITIONING OF SYSTEM INTO AREAS



(a) Before decomposition **(b) After decomposition**

A 3-area interconnected power system shown in Fig.1.1 is used as an example to illustrate the decomposition principle. Buses i, j and k are called boundary buses andthe lines among them are called boundary lines. When dividing the overall system according to the geographical partitioning, the boundary lines are duplicated and the voltage magnitude and angle at boundary buses are added to the corresponding areas, respectively, as shown in Fig.1.1 (b). By copying boundary lines, we only need to equivalence of the networks before and after decomposition:

$$V_i^1 = V_i^2, \delta_i^1 = \delta_i^2$$
$$V_j^1 = V_j^2 = V_j^3, \delta_j^1 = \delta_j^2 = \delta_j^3$$
$$V_k^2 = V_k^3, \delta_k^2 = \delta_k^3$$

In this paper, we took the boundary variables in different areas as the same variables, e.g., V_i^1 and V_i^2 were taken as the same variable V_i, thus eliminatingconsistency constraints (2.6) in our decentralized model. Define $x_a^{(I)}$ as the internal variables of area a that are contained in area a only, and $x_a^{(B)}$ as the boundary variables of area a representing the shared variables between area a and other areas.

Then, the variables in area 1 are divided into 2 categories: internal variables $x1^{(I)}$: = ($P1^G$; $Q1^G$; $V1^I$; $\delta1^I$) and boundary variables $x1^{(B)}$: = (V^B ; $\delta1^B$), where $V1^B$ = $(Vi, Vj)^T$ and $\delta1^B$ = $(\delta i, \delta j)^T$.

Similarly, the variables in area 2 are also divided into 2 categories: internal variables $x2^{(I)}$: = ($P2^G$; $Q2^G$; $V2^I$; $\delta2^I$) and boundary variables $x2^{(B)}$: = ($V2^B$; δ^B), where $V2^B$ = $(Vi, Vj, Vk)^T$ and $\delta2^B$ = $(\delta i, \delta j, \delta k)^T$.

Finally, the variables in area 3 are also divided into 2 categories: internal variables $x3^{(I)}$: = ($P3^G$; $Q3^G$; $V3^I$; $\delta3^I$) and boundary variables $x3^{(B)}$: = ($V3^B$; δ^B), where $V3^B$ = (Vk, Vj)T and $\delta3^B$ = $(\delta k, \delta j)^T$.

Hence, we can formulate the self-governing objective functions and constraints related to each independent system. Accordingly, the D-OPF model of the 3-area system can be written in the following compact form:

The OPF model of area a, a{1, 2, 3}:

$$\begin{aligned}
&\min f_a(\boldsymbol{x}^{a(\mathrm{I})}) \\
&\text{s.t.}\quad \boldsymbol{h}_i(\boldsymbol{x}^{a(\mathrm{I})}, \boldsymbol{x}^{a(\mathrm{B})}) = 0,\ i \in \Phi^a \\
&\quad \underline{g}_{1,k} \le g_{1,k}(\boldsymbol{x}^{a(\mathrm{I})}, \boldsymbol{x}^{a(\mathrm{B})}) \le \overline{g}_{1,k},\ k \in \Gamma^{a(\mathrm{I})} \\
&\quad \underline{g}_{2,k} \le g_{2,k}(\boldsymbol{x}^{a(\mathrm{B})}) \le \overline{g}_{2,k},\ k \in \Gamma^{a(\mathrm{B})} \\
&\quad \boldsymbol{x}_{\min}^{a(\mathrm{I})} \le \boldsymbol{x}^{a(\mathrm{I})} \le \boldsymbol{x}_{\max}^{a(\mathrm{I})} \\
&\quad \boldsymbol{x}_{\min}^{a(\mathrm{B})} \le \boldsymbol{x}^{a(\mathrm{B})} \le \boldsymbol{x}_{\max}^{a(\mathrm{B})}
\end{aligned}$$

where $f_a(x_a)$ represents the objective function of area a.

$h_i(x_a^{(I)}, x_a^{(B)})$ represents the active and reactive power balance equations at bus i of area a;

$\Gamma^{a(I)}$ and $\Gamma^{a(B)}$ denote the set of internal and boundary lines of area a, respectively;.

The subscripts max and min represent the upper and lower limits of variables, respectively.

we observe that there are no coupling objectives or coupling constraints occurring in regional sub-problems; the only contact of area a's sub-problem with other areas' sub-problems is the shared boundary variables between area a and other areas.

DISTRIBUTED INTERIOR POINT METHOD

The C-OPF problem can be rewritten in the following compact form:

$$\begin{aligned}
&\min f(\boldsymbol{x}) \\
&s.t.\quad \boldsymbol{h}(\boldsymbol{x}) = 0 \\
&\quad \underline{\boldsymbol{g}} \le \boldsymbol{g}(\boldsymbol{x}) \le \overline{\boldsymbol{g}}\ .
\end{aligned}$$

We adopt the interior point method to solve this OPF problem. Afterintroducing slack variables u and l (u, l > 0), Lagrange multipliers λ, w and z (w < 0, z> 0), we can construct the following Lagrange function:

$$\begin{aligned}
L = {} & f(\boldsymbol{x}) - \boldsymbol{\lambda}^T \boldsymbol{h}(\boldsymbol{x}) - \boldsymbol{w}^T [\boldsymbol{g}(\boldsymbol{x}) + \boldsymbol{u} - \overline{\boldsymbol{g}}] \\
& - \boldsymbol{z}^T [\boldsymbol{g}(\boldsymbol{x}) - \boldsymbol{l} - \underline{\boldsymbol{g}}] - \mu(\sum_{i=1}^{r} \ln u_i + \sum_{i=1}^{r} \ln l_i)
\end{aligned}$$

where λ is the barrier parameter, and r denotes the number of inequality constraints.

By applying the Newton-Raphson method into the KKT optimality condition Newton directions Δx and Δλ as well as Δu, Δl, Δw and Δz can be calculatedby solving the following correction equations: where H and J denote the Hessianmatrix and Jacobian matrix respectively; L_x, $L\lambda$, L_u, L_l, L_w and L_z denote the residuals of the perturbed KKT condition.

$$\begin{bmatrix} \boldsymbol{H} & (\boldsymbol{J})^T \\ \boldsymbol{J} & 0 \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{x} \\ \Delta \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}_x \\ \boldsymbol{L}_\lambda \end{bmatrix},$$

$$\Delta \boldsymbol{u} = -\boldsymbol{L}_w - \nabla_x^T \boldsymbol{g}(\boldsymbol{x}) \Delta \boldsymbol{x},$$

$$\Delta \boldsymbol{l} = \boldsymbol{L}_z + \nabla_x^T \boldsymbol{g}(\boldsymbol{x}) \Delta \boldsymbol{x},$$

$$\Delta \boldsymbol{w} = -\boldsymbol{U}^{-1} \boldsymbol{L}_u' - \boldsymbol{U}^{-1} \boldsymbol{W} \Delta \boldsymbol{u},$$

$$\Delta \boldsymbol{z} = -\boldsymbol{L}^{-1} \boldsymbol{L}_l' - \boldsymbol{L}^{-1} \boldsymbol{Z} \Delta \boldsymbol{l},$$

$$L_x \equiv \nabla f(x) - \nabla h(x) y - \nabla g(x)(z + w) = 0$$

$$L_y \equiv h(x) = 0$$

$$L_z \equiv g(x) - l - \underline{g} = 0$$

$$L_w \equiv g(x) + u - \overline{g} = 0$$

$$L_l \equiv LZe = 0$$

$$L_u \equiv UWe = 0$$

$$(l,\ u;\ z) \geq 0,\quad w \leq 0,\quad y \neq 0$$

where (L, U, Z, W) ϵ $R^{(r*r)}$ are all diagonal matrices, y ϵ $R^{(m)}$ and (Z, W) ϵ $R^{(r)}$ are Lagrangian multipliers and e = [1 ,...,1]' ϵ $R^{(r)}$

If $l_i^{(k)}$ happens to become zero at the k-th iteration, it is obvious from the above equation that $\Delta l_i^{(k)}$ is equal to zero and hence $l_i^{(k+1)} = l_i^{(k)} + \Delta l_i^{(k)} = 0$. That is, once l_i falls on the boundary of a feasible region, it is stuck at that point on the boundary. The same situation may occur for variable z_i. Such an undesirable attribute clearly precludes the convergence of the algorithm. In order to overcome this difficulty, we introduce a perturbed factor μ > 0 torelax (3.14) and (3.15) as:

$$L_l^\mu \equiv LZe - \mu e = 0 \tag{3.17}$$

$$L_u^\mu \equiv UWe + \mu e = 0 \tag{3.18}$$

SIMULATION OUTPUTS

A simple 3-bus system shown in fig 5.1 was used to intuitively illustrate the proposed algorithm. The 3 bus test system, which was divided into 3 areas, is depicted in Fig.5.2. Bus 3 was treated as the slack bus. The generating unit data are given in Table 5.1. The limits of bus voltage magnitudes were between 0.95 p.u. and 1.052 p.u..

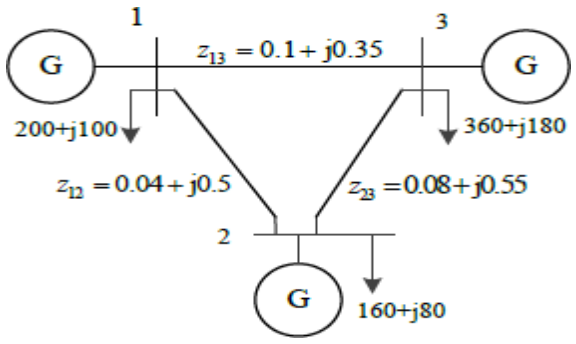


Fig.1.2 Three-bus interconnected test system before decomposition.

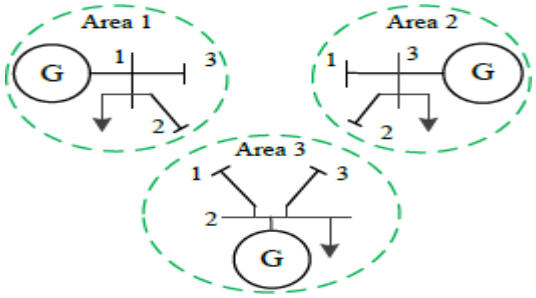


Fig.1.3 Three-bus interconnected test system after decomposition.

TABLE 1.1 LOAD DATA FOR 3-BUS SYSTEM

LOAD DATA					
Bus	Type of bus	Voltage		Load	
		|V| (p.u)	δ (θ)	P (MW)	Q (MW)
1	PQ	1.0000	0	200	100
2	PQ	1.0000	0	160	80
3	Slack	1.0000	0	360	180

TABLE 1.2 LINE DATA FOR 3-BUS SYSTEM

LINE DATA			
Bus no.	Bus no.	R (p.u.)	X (p.u.)
1	2	0.04	0.5
1	3	0.01	0.35
2	3	0.08	0.55

TABLE 1.3 GENERATING UNIT DATA FOR 3-BUS SYSTEM

GENERATING UNIT DATA			
Parameters	Generators		
	1	2	3
$P^G_{i,max}$ (MW)	400	100	300
$P^G_{i,min}$ (MW)	100	50	100
$Q^G_{i,max}$ (M Var)	180	80	230
$Q^G_{i,min}$ (M Var)	-90	-40	-115
$C_{2,i}$ ($/MW^2)	0.04	0.05	0.06
$C_{1,i}$ ($/MW)	30	15	30
$C_{0,i}$ ($)	200	80	100

SIMULATION ON IEEE 9-BUS SYSTEM

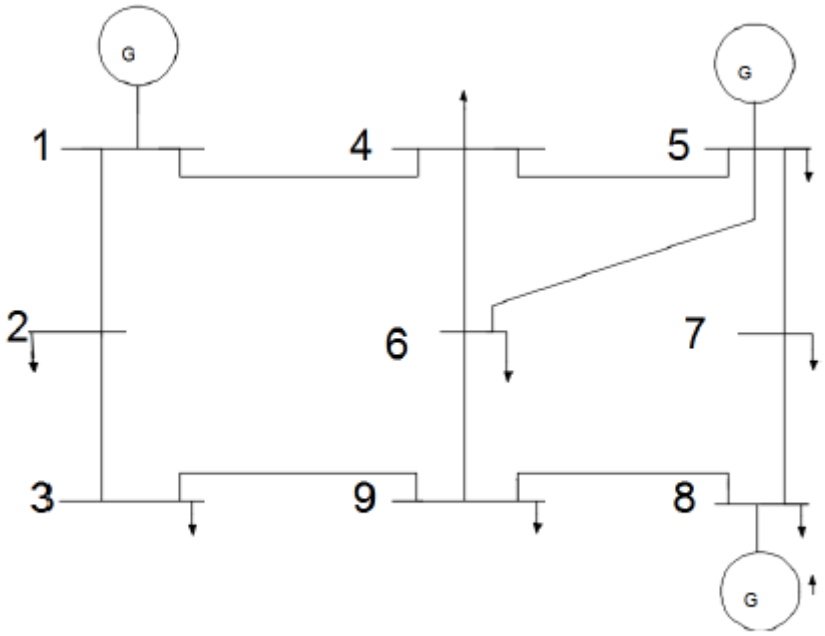


Fig.1.3 One line diagram of IEEE 9-bus system

TABLE 1.4 NUMBER OF P,V,Q,δ CALCULATIONS

Area 1	1,4,5,7
Area 2	2,3,6,9,8
Generator bus	1,5,8
No, of PV bus	3
No. of PQ bus	6
No. of P, δ calculations	8
No. of Q,V calculations	6
Dimensions of Jacobian	8+6=14*14

TABLE 1.5 LOAD DATA OF IEEE 9-BUS SYSTEM

LOAD DATA							
Bus	Type of bus	Voltage		Load		Generation	
		|V| (p.u)	δ (θ)	P (MW)	Q (MW)	P (MW)	Q (MW)
1	Slack	1.0300	0	0	0		
2	PQ	1.0000	0	10	5		
3	PQ	1.0000	0	25	15		
4	PQ	1.0000	0	60	40		
5	PQ	1.0600	0	10	5	80	
6	PV	1.0000	0	100	80		
7	PQ	1.0000	0	80	60		
8	PV	1.0100	0	40	20	120	
9	PQ	1.0000	0	20	10		

TABLE 1.6 LINE DATA OF IEEE 9-BUS SYSTEM

LINE DATA					
Bus no.	Bus no.	R (p.u.)	X (p.u.)	½ B (p.u.)	Transformer tap
1	2	0.0180	0.0540	0.0045	1
1	4	0.0150	0.0450	0.0038	1
2	3	0.0180	0.0560	0	1
3	9	0.0200	0.0600	0	1
4	5	0.0130	0.0360	0.0030	1
4	6	0.0200	0.0660	0	1
5	6	0.0600	0.030	0.0028	1
5	7	0.0140	0.0360	0.0030	1
6	9	0.0100	0.0500	0	1
7	8	0.0320	0.0760	0	1
8	9	0.0220	0.0650	0	1

TABLE 1.7 GENERATOR LIMITS & COST CO-EFFICIENTS OF IEEE 9-BUS SYSTEM

Unit number	Pgi (min) (MW)	Pgi (max) (MW)	Qgi (min) (MW)	Qgi (max) (MW)	a ($/h)	b ($/Mwh)	c ($/Mwh$^{2)}$
1	10	300	-20	200	150	2	0.1100
5	30	300	-20	300	600	1.2	0.0850
8	25	250	-20	300	355	1	0.1225

TABLE 1.8 SIMULATION RESULTS OF 3-BUS SYSTEM

Bus number	Active power P (MW)		Reactive power Q (MW)	
1	P_1^G (MW)	342.1609	Q_1^G (MW)	71.9953
5	P_2^G (MW)	98.0964	Q_2^G (MW)	42.4530
8	P_3^G (MW)	286.1425	Q_3^G (MW)	168.7293

TABLE 1.9 SIMULATION RESULTS OF IEEE 9-BUS SYSTEM

Bus number	Active power P (MW)		Reactive power Q (MW)	
1	P_1^G (MW)	150.8330	Q_1^G (MW)	-19.5170
5	P_5^G (MW)	79.5460	Q_5^G (MW)	241.1660
8	P_8^G (MW)	119.4190	Q_8^G (MW)	26.6130

TABLE 1.10 GENERATION, DEMAND, LOSSES AND COST

Parameter	3-bus system	IEEE 9-bus system
Total demand P_D (MW)	720	345
Total generation P_G (MW)	726.3998	349.7980
Total loss P_L (MW)	6.3998	4.7980
Total cost ($)	31,4576.3980	6408.9487

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