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Gsp$\alpha\omega$-Closed Sets In Topological Spaces

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Abstract: The aim of this paper is to introduce a new class of closed sets namely $gsp\alpha\omega$-closed sets which is obtained by generalizing gsp -closed sets via $\alpha\omega$- open sets and investigate some of their basic properties in topological spaces.

Introduction:

LEVINE [5] introduced semi-open sets in 1963. In 1986, **D.ANDRIJIEVIC [1]** introduced the notion of semi-pre- open sets in topological spaces. In 2000, the ω closed sets [9] were introduced and studied by **P.SUNDARAM** and **M.SHRIK JOHN**. **M.PARIMALA [8]** introduced the concept of $\alpha\omega$ -closed sets, and studied their properties in 2017.The aim of this paper is to introduce a new class of closed sets namely gspα $\alpha\omega$ - closed sets and investigate some of their basic properties in topological spaces.

1. PRELIMINARIES:

DEFINITION 1.1:A subset $\mathbf{A}$ of a space$(\mathbf{X}, \boldsymbol{\tau})$is called a

1. semi open set if $\mathbf{A} \subseteq \mathbf{cl}(\mathbf{int}(\mathbf{A}))$
2. $\boldsymbol{\alpha}$-open set if $\mathbf{A} \subseteq \mathbf{int}\left(\mathbf{cl}(\mathbf{int}(\mathbf{A}))\right)$
3. semi pre(=$\boldsymbol{\beta}$)-open set if $\mathbf{A} \subseteq \mathbf{cl}\left(\mathbf{int}(\mathbf{cl}(\mathbf{A}))\right)$
4. b-open set if $\mathbf{A} \subseteq \left(\mathbf{cl}(\mathbf{int}(\mathbf{A}))\right) \cup \left(\mathbf{int}(\mathbf{cl}(\mathbf{A}))\right)$
5. regular-open set if $\mathbf{A} = \mathbf{int}(\mathbf{cl}(\mathbf{A}))$

DEFINITION 1.2: A subset $\mathbf{A}$ of a space $(\mathbf{X}, \boldsymbol{\tau})$ is called

1. generalized closed (briefly g-closed) if $\mathbf{cl}(\mathbf{A}) \subseteq \mathbf{U}$ whenever $\mathbf{A} \subseteq \mathbf{U}$ and $\mathbf{U}$ is open
2. regular-generalized closed(briefly rg-closed)if $\mathbf{cl}(\mathbf{A}) \subseteq \mathbf{U}$ whenever $\mathbf{A} \subseteq \mathbf{U}$ and $\mathbf{U}$ is regularopen.
3. generalized b-closed (briefly gb-closed) if $\mathbf{bcl}(\mathbf{A}) \subseteq \mathbf{U}$ whenever $\mathbf{A} \subseteq \mathbf{U}$ and $\mathbf{U}$ is open
4. regular-generalized b- closed(briefly rgb-closed)if$\mathbf{bcl}(\mathbf{A}) \subseteq \mathbf{U}$whenever$\mathbf{A} \subseteq \mathbf{U}and\mathbf{U}$is regular-open
5. generalized semi-preregular-closed (briefly gspr-closed) if $\mathbf{spcl}(\mathbf{A}) \subseteq \mathbf{U}$whenever $\mathbf{A} \subseteq \mathbf{U}$ and $\mathbf{U}$ is regular open
6. generalized$\boldsymbol{\beta}$-closed (briefly g $\boldsymbol{\beta}$-closed) if $\boldsymbol{\beta}\mathbf{cl}(\mathbf{A}) \subseteq \mathbf{U}$ whenever $\mathbf{A} \subseteq \mathbf{U}$ and $\mathbf{U}$ is open.

7. $\boldsymbol{\psi\hat{g}}$-closedif$\boldsymbol{\psi cl(A)} \subseteq \mathbf{U}$whenever$\mathbf{A} \subseteq \mathbf{U}$andUis$\hat{\mathbf{g}}$-open
8. $\boldsymbol{\psi g}$-closed if$\boldsymbol{\psi cl(A)} \subseteq \mathbf{U}$ whenever $\mathbf{A} \subseteq \mathbf{U}$ and $\mathbf{U}$ is open
9. generalized semi-closed (briefly gs-closed) if $\mathbf{scl(A)} \subseteq \mathbf{U}$ whenever $\mathbf{A} \subseteq \mathbf{U}$ and U is open
10. $\hat{\eta}$*-closed if $\mathbf{spcl(A)} \subseteq \mathbf{U}$ whenever $\mathbf{A} \subseteq \mathbf{U}$ and $\mathbf{U}$ is $\boldsymbol{\omega}$-open
11. $\boldsymbol{\psi}$-closed if $\mathbf{scl(A)} \subseteq \mathbf{U}$ whenever $\mathbf{A} \subseteq \mathbf{U}$ and $\mathbf{U}$ is sg-open
12. $\boldsymbol{\omega}$(or$\hat{\mathbf{g}}$) closed if $\mathbf{cl(A)} \subseteq \mathbf{U}$ whenever $\mathbf{A} \subseteq \mathbf{U}$ and $\mathbf{U}$ is semi-open
13. $\boldsymbol{\alpha\omega}$-closed if$\boldsymbol{\omega}\mathbf{cl(A)} \subseteq \mathbf{U}$ whenever $\mathbf{A} \subseteq \mathbf{U}$ and $\mathbf{U}$ is $\boldsymbol{\alpha}$-open
14. $\mathbf{g}\boldsymbol{\alpha\omega}$ -closed if $\mathbf{cl(A)} \subseteq \mathbf{U}$ whenever $\mathbf{A} \subseteq \mathbf{U}$ and $\mathbf{U}$ is $\boldsymbol{\alpha\omega}$-open

2. gspαω -CLOSED SET

DEFINITION 2.1:
A subset A of (X, τ) is called a **gspαω-closed set** if $\mathrm{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is αω-open in (X,, τ). The complement of **gspαω -closed set** is **gspαω –open set.**

EXAMPLE2.2:
Let X = {a, b, c}; τ= {X,φ, {c}, {a, c}}. Closed sets are {X,φ, {b}, {a, b}}
semi-pre closed sets are (X,φ , {a}, {b}, {a, b}}. αω-open sets are {X,φ , {c}, {a, c}}
gspαω -closed sets are {X,φ , {a}, {b}, {a, b}, {b, c}}

THEOREM 2.3:
Every closed set is gspαω -closed set

PROOF:
Let A be a closed set, $\mathrm{cl}(A) = A$. Let $A \subseteq U, U$ be αω-open
We've $\mathrm{spcl}(A) \subseteq \mathrm{cl}(A) \subseteq U \Rightarrow \mathrm{spcl}(A) \subseteq U$. Hence A is gspαω -closed set.
The converse of the above theorem need not be true as seen from the following example.

EXAMPLE 2.4: Let X = {a, b, c},**τ**= {X,**φ** , {c}, {a, c}}
Closed sets are {X, φ, {b}, {a, b}. gspαω-closed sets are {X,φ, {a}, {b}, {a, b}, {b, c}}
Let A ={a}. Here, {a} is gspαω-closed set but not closed set in (X, τ)

THEOREM 2.5:
Every regular-closed set is gspαω-closed set

PROOF:
Let A be a regular-closed set. Let $A \subseteq U, U$ be αω-open
But, every regular closed set is closed set∴ $\mathrm{cl}(A) = A$.
We've $\mathrm{spcl}(A) \subseteq \mathrm{cl}(A) = A \subseteq U \Rightarrow \mathrm{spcl}(A) \subseteq U$. Hence A is gspαω-closed set
The converse of the above theorem need not be true as seen from the following example.

EXAMPLE 2.6:
Let X = {a, b, c}, τ= {X,φ , {b}, {c}, {b, c}}, Closed sets are {X,φ , {a}, {a, b}, {a, c}}
regular-closed sets are {X,φ , {a, b}, {a, c}}, gspαω-closed sets are {X,φ , {a}, {b}, {c}, {a, b}, {a, c}}. Let

A ={b}. Here, {b} is gspαω-closed set but not regular-closed set in (X, τ).

THEOREM 2.7:
Every pre-closed set is gspαω-closed set

PROOF:
Let A be a pre-closed set. Let $A \subseteq U, U$ be αω-open
Since A is pre-closed, $pcl(A) = A$. We've$spcl(A) \subseteq pcl(A) = A \subseteq U \Rightarrow spcl(A) \subseteq U$
Hence A is gspαω-closed set
The converse of the above theorem need not be true as seen from the following example.

EXAMPLE 2.8:
Let X = {a, b, c}, τ= {X, φ, {a}, {a, c}}. Closed sets are {X,φ , {b}, {b, c}}
pre-closed sets are {X,φ , {b}, {c}, {b, c}}
gspαω -closed sets are {X,φ , {b}, {c}, {a, b}, {b, c}} Let A = {a,b}
Here, {a, b}isgspαω-closed set but not pre-closed set in (X, τ)

THEOREM 2.9:
Every α-closed set is gspαω-closed set

PROOF:
Let A be a α-closed set. Let $A \subseteq U, U$ be αω-open
Since A is α-closed, $cl\left(int\left(cl(A)\right)\right) = A$
We've $A \subseteq cl(A) \Rightarrow cl(int(cl(A))) \subseteq cl(A)$. Also, $cl(A) = A$
$$\Rightarrow cl(int(A)) \subseteq A \Rightarrow int\left(cl\left(int(A)\right)\right) \subseteq int(A) \subseteq A, [\text{since } int(A) \subseteq A]$$
$\Rightarrow int\left(cl\left(int(A)\right)\right) \subseteq A \subseteq U \Rightarrow spcl(A) \subseteq U$. Hence A is gspαω -closed set
The converse of the above theorem need not be true as seen from the following example.

EXAMPLE 2.10:
Let X = {a, b, c}, τ= {X,φ , {c}, {a, c}}, Closed sets are {X,φ , {b}, {a, b}}
α-closed sets are {X,φ , {a}, {b}, {a, b}}
gspαω -closed sets are {X,φ , {a}, {b}, {a, b}, {b, c}}. Let A = {b,c}
Here, {b, c} is gspαω-closed set but not α-closed set in (X, τ).

THEOREM 2.11:
Every semi-pre-closed set is gspαω -closed set

PROOF:
Let A be a semi-pre-closed set. Let $A \subseteq U, U$ be αω-open
Since A is semi-pre-closed, $spcl(A) = A \Rightarrow spcl(A) = A \subseteq U \therefore spcl(A) \subseteq U$
Hence A is gspαω-closed set
The converse of the above theorem need not be true as seen from the following example.

EXAMPLE 2.12:

Let X = {a, b, c}, τ= {X,φ , {c}, {a, c}}, Closed sets are {X,φ, {b}, {a, b}}
semi-pre-closed sets are {X,φ , {a}, {b}, {a, b}}
gspαω-closed sets are {X,φ, {a}, {b}, {a, b}, {b, c}} Let A = {b,c}
Here, {b, c} is gspαω -closed set but not semi-pre-closed set in (X, τ)

THEOREM 2.13:
Every gαω-closed set is gspαω-closed set

PROOF:
Let A be a gαω -closed set Let$A \subseteq U, U$ be αω-open
Since A is gαω-closed, $cl(A) \subseteq U, U$ isαω-open
We've $spcl(A) \subseteq cl(A) \subseteq U \Rightarrow spcl(A) \subseteq U.$
Hence A is gspαω-closed set
The converse of the above theorem need not be true as seen from the following example.

EXAMPLE 2.14:
Let X = {a, b, c}, τ= {X,φ , {b}, {a, b}}, Closed sets are {X,φ , {c}, {a, c}}
αω-open sets are {X,φ , {b}, {a, b}}, gαω -closed sets are {X,φ , {c}, {b, c}, {a, c}}
gspαω-closed sets are {X,φ , {a}, {c}, {b, c}, {a, c}}. Let A ={a}
Here, {a} is gspαω -closed set but not gαω-closed set in (X, τ)

THEOREM 2.15:
Every gpαω -closed set is gspαω-closed set

PROOF:
Let A be a gspαω-closed set. Let $A \subseteq U, U$ be αω-open
Since A is gpαω -closed, $pcl(A) \subseteq U, U$ is αω-open
We've $spcl(A) \subseteq pcl(A) \subseteq U \Rightarrow spcl(A) \subseteq U.$
Hence A is gspαω -closed set
The converse of the above theorem need not be true as seen from the following example.

EXAMPLE-2.16:
Let X = {a, b, c}, τ= {X, , {b}, {c}, {b, c}}, Closed sets are {X,φ , {a}, {a, b}, {a, c}}
pre-closed sets are {X, , {a}, {a, b}, {a, c}}, αω-open sets are {X,φ , {b}, {c}, {b, c}}
gpαω-closed sets are {X,φ , {a}, {a, b}, {a, c}}
gspαω -closed sets are {X, , {a}, {b}, {c}, {a, b}, {a, c}}. Let A ={b}
Here, {b} is gspαω-closed set but not gpαω-closed set in (X, τ)

THEOREM-2.17:
If A is αω-open and gspαω-closed, then A is semi-pre-closed.

PROOF:
Let $A \subseteq U, U$ beαω–open. Since A is αω -open, take $A = U$ ___(1)
Also, A is gspαω-closedand open, then $A \subseteq A$and $spcl(A) \subseteq U = A$ (by (1))
$\Rightarrow A \subseteq A$ and $spcl(A) \subseteq A$. $spcl(A) = A$. Hence A is semi-pre-closed

THEOREM 2.18:
Union of two gspαω-closed sets is gspαω -closed set

PROOF:
Let A and B be two gspαω -closed sets in (X, τ)
Let G be any αω-open set in (X, τ) such that $A \cup B \subseteq G$, then$A \subseteq G$and$B \subseteq G$
Since A and B are gspαω-closed sets, then $spcl(A) \subseteq G$ and $spcl(B) \subseteq G$
But, $spcl(A \cup B) = spcl(A) \cup spcl(B) \subseteq G$
$\Rightarrow spcl(A \cup B) \subseteq G$, Gis αω-open Hence $A \cup B$ is gspαω-closedset

REMARK 2.19:
Intersection of two gspαω-closed sets need not be gspαω-closed sets
For example, X = {a, b, c}, τ= {X,φ , {c}}, Closed sets are {X,φ , {a, b}}
gspαω-closed sets are {X, φ, {b}, {c}, {a, b}, {b, c}, {a, c}}
Here, {a, b} and {a, c} are gspαω-closed sets
But {a, b} ∩{a, c} = {a} is not a gspαω -closed set

THEOREM 2.20:
If A is gspαω-closed set in X and $A \subseteq B \subseteq spcl(A)$, then B is also gspαω-closed set in X.

PROOF:
Let A be gspαω-closed set in X and $A \subseteq B \subseteq spcl(A)$. Let $B \subseteq U$ and U be αω-open set in X. Since $A \subseteq B$, then $A \subseteq U$ and A is gspαω-closed set, $spcl(A) \subseteq U$
Given $B \subseteq spcl(A) \Rightarrow spcl(B) \subseteq spcl(spcl(A)) \Rightarrow spcl(B) \subseteq spcl(A) \subseteq U$
$\therefore spcl(B) \subseteq U$. Hence B is gspαω -closed set in X.

THEOREM 2.21:
Let $A \subseteq Y \subseteq X$ and suppose that A is gspαω-closedsetin X. Then A is gspαω-closed set relative toY.

PROOF:
Let $A \subseteq Y \cap G$, G be αω–open. Since A is gspαω-closed set, then $spcl(A) \subseteq G$, whenever $A \subseteq G$, G is αω-open$\Rightarrow Y \cap spcl(A) \subseteq Y \cap G$.
Hence A is gspαω-closed set relative to Y.

REMARK 2.22:
The set gp*-closed set and gspαω -closed set are independent and this can be seen from the following example.

2.23 EXAMPLE:
Let X = {a, b,c}, τ= {X, , {b}, {a, b}}
Closed sets are {X, φ ,{c}, {a, c}}
gp-open sets are {X,φ , {a}, {b}, {b, c}, {a, c}}
semi-pre closed sets are {X,φ , {a}, {c}, {b, c}, {a, c}}
αω-open sets are {X,φ , {b}, {a, b}}
Let $A = \{a, b\} \subseteq X, cl(A) = X \subseteq X$.

Let $= \{a\} \subseteq \{a, b\}, spcl(A) = \{a\} \subseteq \{a, b\}$.
gp*-closed sets are {X,φ , {c}, {a, b}, {a, c}}
gspαω-closed sets are {X, , {a}, {c}, {b, c}, {a, c}} .
Here, the sets {a} and {b, c} are gspαω-closed set but not gp*closed set.
Also, the set {a, b} is gp*-closed set but not gspαω-closed set.

CONCLUSION

In this paper we have introduced gspαω-closed set and studied some of their properties in topological spaces. Also we can extend the study to gspαω-continuous maps, gspαω-irresolute maps .This study can be extended to the concept of compactness, connectedness and separation axioms. Also it can be extended to spaces like Bitopology, Fuzzy and Ideal topological spaces.

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