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Modeling For The Determination Of The Liquid Level In A Tank. An Application Of The Concept Of Definite Integral

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Abstract

The objective is to analyse how modelling favours interactions in the mathematics class and allows presenting the modelling process by the concept of definite integral in the solution of a real situation in the determination of the liquid level in a tank, in a group of engineering students taking integral calculus. The methodology followed was qualitative, descriptive and interpretative, in a case study that adopted the phenomenological design. The results showed a high correspondence between the real data and those obtained with the modelling, so that implementing these processes in mathematics classes generates motivation and commitment of the students, which favours their interactions in the construction and resignification of mathematical concepts such as the definite integral in a real situation such as the determination of the liquid level in a tank.

Keywords: Mathematical modelling; learning context; reality; mathematical modelling; resignification.

Introduction

One need in the teaching of mathematics is to find ways to intervene and improve their learning processes, so that school mathematical knowledge becomes meaningful and functional, and that through modelling, a contraction of the terms modelling and education [1], can integrate reality to transform it and transform the subject that learns, reconstructing and enriching meanings permanently [2], which shows the importance of articulating and integrating mathematical knowledge with other areas of knowledge, especially in the training of engineers [3]-[4]. [5] state that modelling favours student learning of mathematics in contexts of application in other areas of knowledge, as well as improving their reading and interpretative capacity in the formulation and solution of problems, which awakens motivation, commitment and effort towards mathematic [6].

It is important to clarify that, up to this stage of the study, the number of students who pass or fail is not determined. The impact we are seeking to determine is the perception and level of motivation and interest of students in the implementation of a modelling process in class. Therefore, the objective is to describe the implementation of a modelling process in which students in an integral calculus course, immersed in a real problem such as determining the level of liquid in a tank, interact, conjecture and propose a model that allows them to construct and re-signify the mathematical concept of the definite integral.

Modelling and its importance

Modelling is a process by which a non-mathematical situation or problem is solved using mathematics [7], which facilitates the understanding of phenomena by allowing different systems of representations for them and gives meaning to different mathematical activities [8]. The modelling process requires extracting the problem from a reality, in which there are more elements than necessary to work on in class, identifying the main variables about the problem, establishing relationships between them and mathematizing these relationships to generate new knowledge about the problem [9], which requires the use of mathematical skills to not only arrive at a single answer but a wider range that describes the behaviour of the situation under study and gives the solver participation and control in the solution processes [10]. This makes modelling a learning tool to be considered in classroom work, due to its usefulness not only in real-world applications such as engineering [11], business, social sciences, among others, but also within mathematics education itself [12], since in addition to providing meaning to certain mathematical concepts and integrating different areas of mathematics, it also promotes another way of interpreting, reasoning and understanding mathematical concepts, it also promotes another way of interpreting, reasoning and acting by integrating the abstract and formal part of mathematics to situations and phenomena assumed as problematic in the classroom, which helps students to consolidate solid cognitive bases from which they can develop various mathematical concepts [13].

Blum and Borromeo-Ferri [14] recognize modelling as a tool to help students understand the contexts in which they operate and promote the development of appropriate competences and attitudes towards mathematics. Regarding the development of scientific competences, [15] reaffirm the importance of modelling practice since, by constructing models, whether numerical or graphical, among others, students can argue and establish consensus, thereby contributing to the strengthening of a scientific vision of the world. Finally, [16] states that modelling contributes to changing a vision of mathematics in which one learns by repetition, as it is centred on analytical methods, to one in which one learns by construction and understanding, which is based on qualitative and numerical methods [11].

Modelling in mathematics courses and re-signification

There is a social demand that what is taught at school, and especially mathematical knowledge, should not be disconnected from reality in such a way that it allows students to solve or propose possible solutions to real problems. In this sense, modelling, its learning and teaching allows the use of mathematics in the solution of real-world problems [17], since, in mathematics courses in general, apart from the theoretical and conceptual level, there is no linking of experimental activities, as a step in the modelling process, of real phenomena and close to the students' experiences with mathematical contents. The modelling process, which in numerous instances is hypothetical, is associated with an activity at the end of each chapter of the guide text, the application problems that are related to modelling, but are not a source of knowledge because they are decontextualized and outside reality [2]. In this sense, modelling should be a form of interaction with others, with reality and with events [18]. Furthermore, in the modelling process that students develop, the resignification of a mathematical concept is not simply giving a new meaning to that concept, but rather it is a construction that is made of that knowledge as part of their interactions in a particular context [19]. Finally, the knowledge to be re-signified in this activity is the concept of the definite integral that is used to model a real-life problem, the determination of the level of liquid in a tank, and in this process to give meaning to this concept.

Experimentation in the mathematics classroom, reality and context in the modelling process

Despite the importance of the role of modelling in mathematics curricula [20]-[21], there is no link between experimental activities in the classroom or outside it, which favour the integration and relationship of mathematical concepts with other areas of knowledge and which at the same time allow mathematical knowledge to be given meaning and life. In other words, experimentation is an unusual practice [15], although mathematics has been constructed, to a large extent, as a result of different interactions with situations or phenomena in the real physical world [22], which has led to minimizing the construction and creation of mathematics through experimentation in the laboratory or in the classroom, as it is assumed that this type of activity is exclusive to the natural sciences and not to mathematics.

Reality in the framework of modelling

Reality is assumed as the starting point for the identification and selection of phenomena, situations or problems to be considered for work in the mathematics classroom, since it is related to the context in which it is developed, and therefore they are a binomial in which each of the parts conditions and influences the other and both, in turn, influence the modelling process in school mathematics [2]. For [14], reality is everything else that is outside the world of mathematics and includes aspects such as nature, society, everyday life and other scientific disciplines, which is linked to the contexts of students, i.e. the everyday, social, cultural or other sciences in which they may be immersed [23].

In the framework of realist mathematics education or MRE developed by Hans Freudenthal in the 1970s, he conceives of mathematics as a human activity that consists of mathematicizing, that is, organizing or structuring reality, including mathematics itself [24]. The concept "realistic" refers more to problem situations that students can imagine than to the reality or authenticity of the problems themselves [25]. The latter does not mean that the relation to real life is not relevant, but that the contexts are not necessarily restricted to real-life situations. Thus, the fantasy world of fairy tales and even the formal world of mathematics are suitable contexts for problems, as long as they are real in the minds of students.

For [24], he expresses that reality is what common-sense experiences as real in a certain scenario, which shows the relationship between reality and context, while for [26], it is the web of interrelated facts and phenomena, whether natural, environmental, socio-cultural and emotional, from which people are informed and receive stimuli for action. Finally, it is assumed that reality is everything that is and occurs internally and externally to the school environment and that can be not only perceived but also imagined or represented by a student based on their senses and mental processes, and whose interpretation and analysis is influenced both by their own subjectivity and the context in which they are immersed [2].

Context

Context is a fundamental element in the conception of reality and in the construction of meanings and knowledge based on it, and in this sense it is directly related to the area that gives content to the problem situation posed that must be solved [27]. According to [28], the context of a problem is inherent to the problem itself and not just a mere veneer to be removed that would allow students to imagine the situation posed, represent it schematically by a model and arrive at the result of the problem in question. A necessary, but not sufficient, condition to trigger this process is that the problem situations are familiar and meaningful to students in such a way that the common sense and reasoning used function as strategies for solving and guiding the mathematical task [29], which is necessary for mathematics in context [30]. Therefore, the context must be familiar to the students or at least allow them to make certain representations and create or recreate their own scenarios.

Regardless of the context in which the modelling takes place, contextualizing mathematical knowledge does not mean simulating it in the classroom with any everyday activity, but rather knowing the representations that students make of this knowledge and the meaning of their conceptions, as well as seeing how they make them work in the chosen field [27]. Therefore, reality and context are elements that, in addition to being indissoluble, give meaning to a social practice and give it particular characteristics. Therefore, the context is given by the natural dynamics of a class [2], in this particular case, the determination of the liquid level in a tank through an experimental practice.

Method

Design

The methodology was qualitative, descriptive and interpretative, in a case study that adopted a phenomenological design, with the purpose of exploring, describing and understanding the experiences of the students who arose from a process of modelling in the school environment. The information was obtained from the students' productions, short video recordings and audio recordings of some of the discussions that arose during the teamwork.

Setting, actors and materials

The context of the study was an Integral Calculus course of 30 students (18 females and 12 males), who were voluntarily divided into teams of 5 students. The selection of the working group was not random, but by convenience. The experimentation was carried out in and around the classroom during two-hour sessions, following the concept of the definite integral, constructed in class by the students.

The materials used were of common use: an empty bottle, bucket, funnel, rod to measure the height of the water level, activity guide and other everyday materials. In this case, the activity was carried out to scale using a water bottle as a physical model, to take real data and then compare them with the data obtained using the mathematical model found. For this purpose, students were given a guide with indications and orientations, but at all times the students' autonomous work and their own ways of approaching the problem were privileged.

Staging of activities

An experimental activity related to the cooling phenomenon of a staging of the sequence was carried out, each group was divided into four moments [2]: Moment 1. Previous ideas and conjectures about the study phenomenon; Moment 2. Execution of the experimental activity and data taking; Moment 3. Data manipulation; and Moment 4. Discussion of results and verification of conjectures.

Each of which had a defined intentionality, and which are articulated in a coherent way to give some order and consistency to the modelling practice as such. In addition, there are guiding questions that favour discussion in the teams. This type of activity favoured the students' interactions in which they put into play arguments, examples, counterexamples, among others, when working with the real situation.

Results

The following section presents some of the most relevant and representative results of the findings. Excerpts from the written productions that report on the findings are presented.

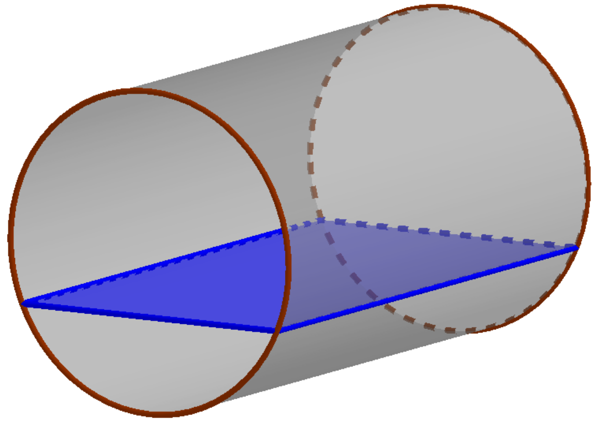
Moment 1. Motivation and prior knowledge

At this point, the aim is to mobilize the students' prior knowledge so that they dare to formulate conjectures about how the problem can be approached and possible alternative solutions.

The initiative for this particular practice arose from a real need in a fuel storage plant. The situation is as follows:

For fuel storage, there is a tank in a roughly cylindrical shape (made of metal), in a horizontal position. The fuel from the tank is distributed through pipes to the different equipment. The supervisor determines when to order fuel from the refinery to refuel the tank (refuelling is done once a month), but does so based on experience as he has no way of knowing the actual volume of fuel in the tank at any given time and thus runs the risk of running out of fuel in the tank. It is intended to determine the volume in the tank by simply reading the height of the fuel marked on a stick inserted through the top of the tank. Figure X shows the shape of the tank (assumed for the activity).

Figure 1. Schematic of the tank and water level inside the tank

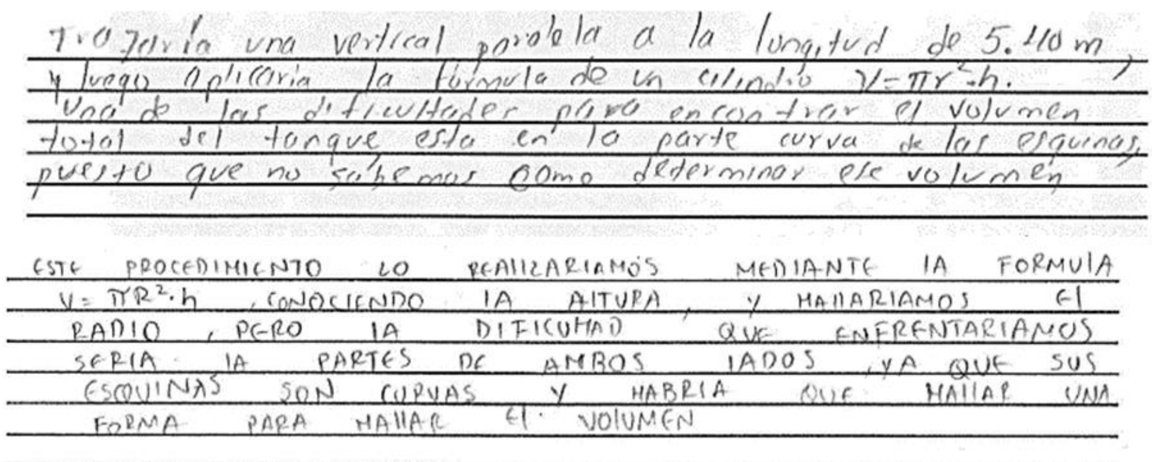


The initial question is then posed:

Suppose you were asked to determine the volume of fuel in the tank at a given time, how would you do this, what procedure do you think could be followed? Also note any difficulties you might have.

Some of the responses are shown in Figure 2:

Figure 2. Responses of two teams to the initial question



Trazaría una vertical paralela a la longitud de 5.40 m, y luego aplicaría la fórmula de un cilindro $V = \pi r^2 \cdot h$. Una de las dificultades para encontrar el volumen total del tanque está en la parte curva de las esquinas, puesto que no sabemos como determinar ese volumen

ESTE PROCEDIMIENTO LO REALIZARIAMOS MEDIANTE LA FORMULA $V = \pi R^2 \cdot h$, CONOCIENDO LA ALTURA, Y HALLARIAMOS EL RADIO, PERO LA DIFICULTAD QUE ENFRENTARIAMOS SERIA LA PARTES DE AMBOS LADOS, YA QUE SUS ESQUINAS SON CURVAS Y HABRIA QUE HALLAR UNA FORMA PARA HALLAR EL VOLUMEN

It can be seen from the answers that, to calculate the volume, the students consider that it is the same to have the tank in a vertical position as in a horizontal position. This is an indication that, for some of them, the formulas applied are independent of the specific problems. This is a situation that occurs frequently in the mathematics class and should be evaluated, since the cases that are generally worked on in class do not always correspond to reality. In the second answer, however, it can be seen that there is greater detail when visualizing the shape of the tank and that this situation makes the situation more difficult.

Moment 2. Experimentation and Data Collection

At this stage, they are asked to take data on the height marked on the rod when it is introduced into the tank for different known volumes of water. To complete this, they use a plastic water bottle (approximately 22 litres), a funnel, water and a rod to measure the height. They then proceed to fill in a data table from the measurements. Some of the teams pointed out possible errors in the measurements, not only because of the shape of the bottle, but also because they assumed that the tank was flat when in fact it was not. These observations are important, as it makes students question how to approach a problem, its limitations and possible alternative solutions. In this exercise, the shape of the tank was not considered, but the situation was left open. Once the information has been collected, the model is set up to determine the volume of liquid in the tank with a single measurement of the height of the rod.

Moment 3: Data handling and mathematical procedures

The students, in teams, discuss what is set out in the guide and begin to build the mathematical model. It is at this point that different visions, prior knowledge and problem-solving strategies come into play.

The following is one of the students' productions, based on what was proposed in the guide (Figure 3):

Figure 3. Mathematical procedure to find the model, assuming that the base of the tank is circular

1. Para simplificar el trabajo, ubica el centro del círculo en el punto (0,0) (figura 6)

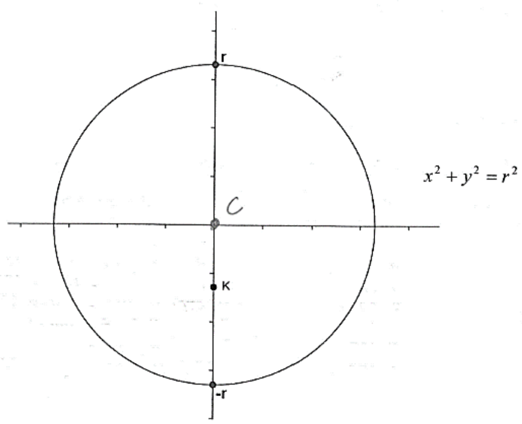


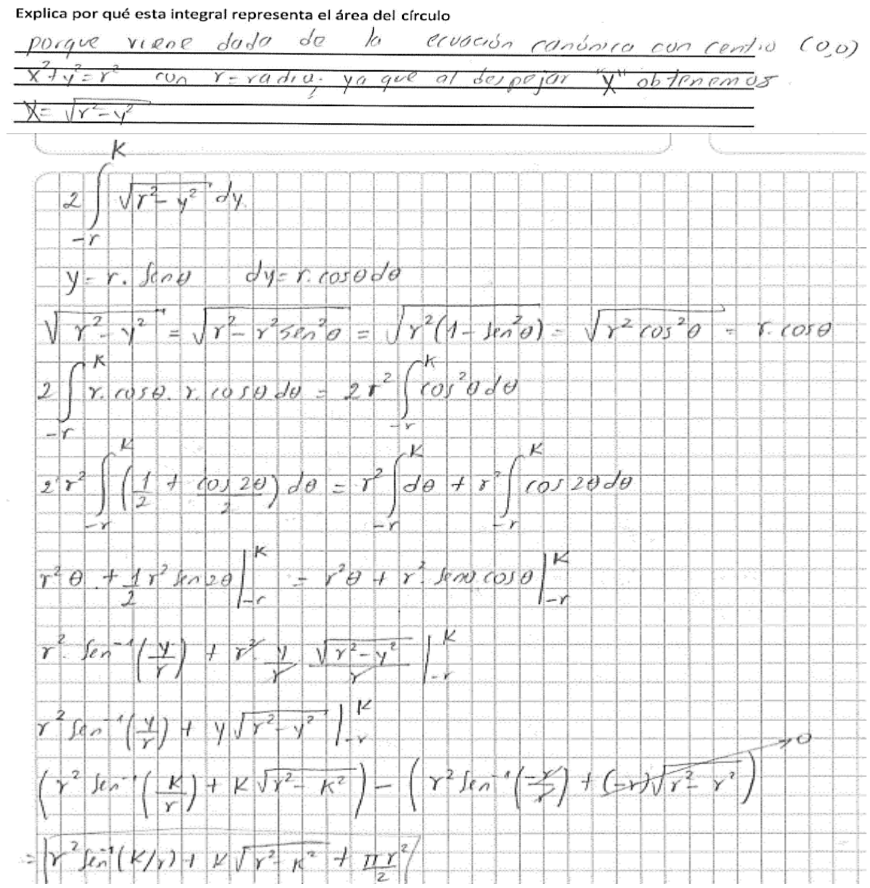
Figura 6.

Encuentra el área entre **–r** y un valor **K** (medido sobre el eje y), de la siguiente forma:

$$\int_{-r}^{K}\left(\sqrt{r^2 - y^2} - \left(-\sqrt{r^2 - y^2}\right)\right)dy = 2\int_{-r}^{K}\sqrt{r^2 - y^2}\,dy$$

Figure 4 shows the procedure followed by one of the teams to find the model to determine the area of the section formed on the tank bases:

Figure 4. Construction of the mathematical model for finding the area of the section at the base as a function of the radius of the base and the height K of the liquid level above the rod


Explica por qué esta integral representa el área del círculo

porque viene dado de la ecuación canónica con centro (0,0) $x^2+y^2=r^2$ con r = radio. ya que al despejar "x" obtenemos $x=\sqrt{r^2-y^2}$

$$2\int_{-r}^{K}\sqrt{r^2-y^2}\,dy$$

$$y=r.\,sen\theta \qquad dy=r.\cos\theta\,d\theta$$

$$\sqrt{r^2-y^2}=\sqrt{r^2-r^2sen^2\theta}=\sqrt{r^2(1-sen^2\theta)}=\sqrt{r^2\cos^2\theta}=r.\cos\theta$$

$$2\int_{-r}^{K}r.\cos\theta.\,r.\cos\theta\,d\theta=2r^2\int_{-r}^{K}\cos^2\theta\,d\theta$$

$$2r^2\int_{-r}^{K}\left(\frac{1}{2}+\frac{\cos2\theta}{2}\right)d\theta=r^2\int_{-r}^{K}d\theta+r^2\int_{-r}^{K}\cos2\theta\,d\theta$$

$$r^2\theta+\frac{1}{2}r^2sen2\theta\Big|_{-r}^{K}=r^2\theta+r^2sen\theta\cos\theta\Big|_{-r}^{K}$$

$$r^2 sen^{-1}\left(\frac{y}{r}\right)+r^2\frac{y}{r}\frac{\sqrt{r^2-y^2}}{r}\Big|_{-r}^{K}$$

$$r^2 sen^{-1}\left(\frac{y}{r}\right)+y\sqrt{r^2-y^2}\Big|_{-r}^{K}$$

$$\left(r^2 sen^{-1}\left(\frac{K}{r}\right)+K\sqrt{r^2-K^2}\right)-\left(r^2 sen^{-1}\left(\frac{-r}{r}\right)+(-r)\sqrt{r^2-r^2}\right)$$

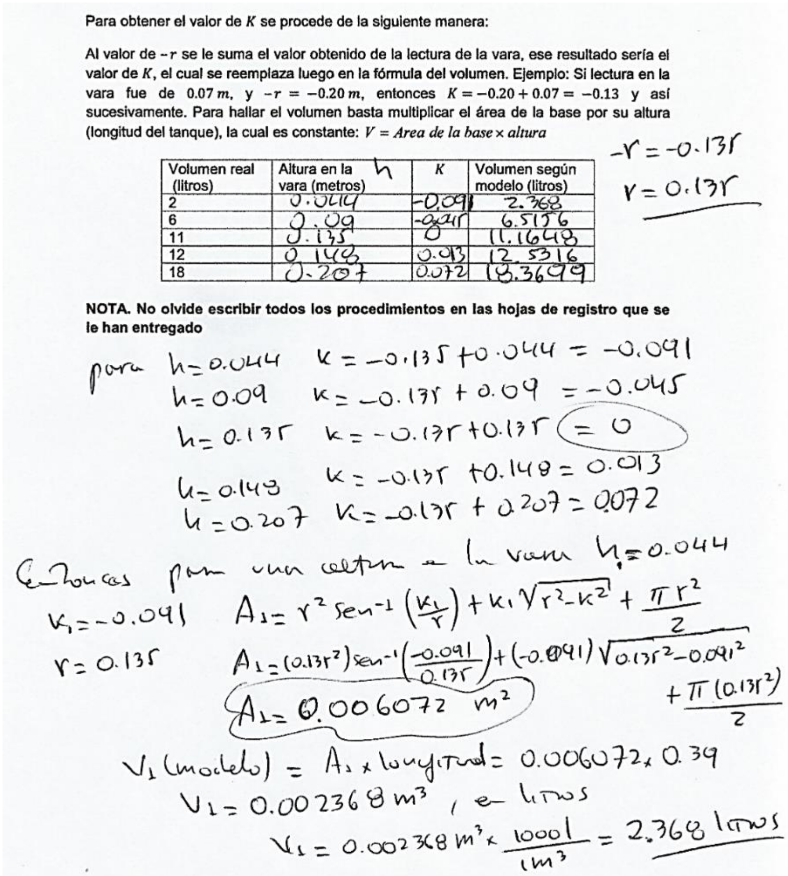
$$=r^2 sen^{-1}(K/r)+K\sqrt{r^2-K^2}+\frac{\pi r^2}{2}$$

In general, none of the teams had major difficulties in the solution of the integral, although a couple of teams expressed some doubts about the trigonometric substitution to solve it, this was clarified as part of the development of the activity.

Moment 4: Validation and discussion of results

Using the model for the base area as a function of r and K, the volume of water in the tank is determined, considering the height marked on the rod (Figure 5).

Figure 4. Calculation of volumes using the found model



Para obtener el valor de K se procede de la siguiente manera:

Al valor de $-r$ se le suma el valor obtenido de la lectura de la vara, ese resultado sería el valor de K, el cual se reemplaza luego en la fórmula del volumen. Ejemplo: Si lectura en la vara fue de $0.07\,m$, y $-r = -0.20\,m$, entonces $K = -0.20 + 0.07 = -0.13$ y así sucesivamente. Para hallar el volumen basta multiplicar el área de la base por su altura (longitud del tanque), la cual es constante: $V = Area\ de\ la\ base \times altura$

$-r = -0.135$
$r = 0.135$

Volumen real (litros)	Altura en la vara (metros) h	K	Volumen según modelo (litros)
2	0.044	-0.091	2.368
6	0.09	-0.045	6.5156
11	0.135	0	11.1648
12	0.148	0.013	12.5316
18	0.207	0.072	18.3699

NOTA. No olvide escribir todos los procedimientos en las hojas de registro que se le han entregado

para $h = 0.044$ $\quad k = -0.135 + 0.044 = -0.091$
$h = 0.09$ $\quad k = -0.135 + 0.09 = -0.045$
$h = 0.135$ $\quad k = -0.135 + 0.135 = 0$
$h = 0.148$ $\quad k = -0.135 + 0.148 = 0.013$
$h = 0.207$ $\quad k = -0.135 + 0.207 = 0.072$

Entonces para una altura en la vara $h_1 = 0.044$

$k_1 = -0.091$ $\quad A_1 = r^2 \operatorname{sen}^{-1}\left(\frac{k_1}{r}\right) + k_1\sqrt{r^2 - k^2} + \frac{\pi r^2}{2}$

$r = 0.135$ $\quad A_1 = (0.135^2)\operatorname{sen}^{-1}\left(\frac{-0.091}{0.135}\right) + (-0.091)\sqrt{0.135^2 - 0.091^2} + \frac{\pi(0.135^2)}{2}$

$A_1 = 0.006072\ m^2$

$V_1(\text{modelo}) = A_1 \times \text{longitud} = 0.006072 \times 0.39$

$V_1 = 0.002368\ m^3$, en litros

$V_1 = 0.002368\ m^3 \times \frac{1000\,l}{1\,m^3} = 2.368$ litros

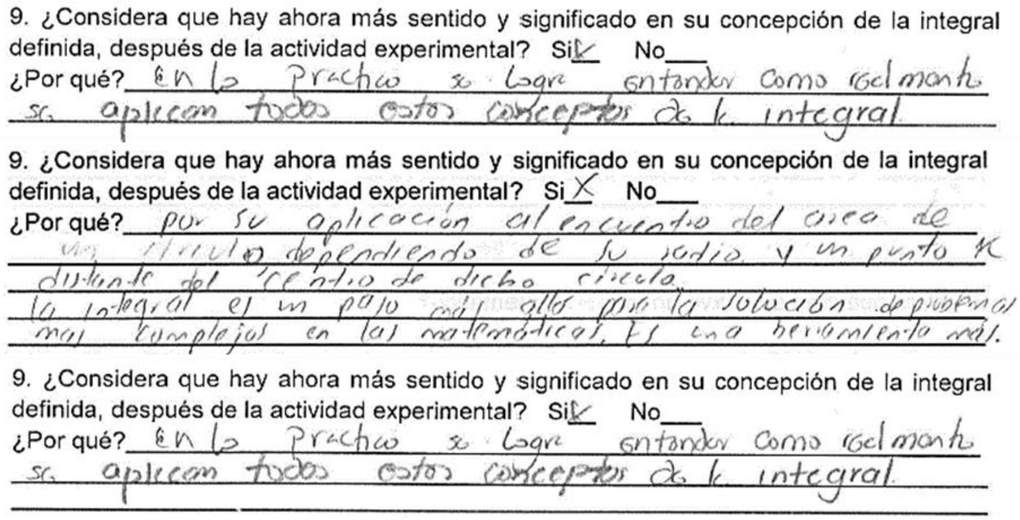
Once the teams made the calculations, they arrived at similar results, with some differences due to the quality of the measurements, but they realized that the data provided by the model coincided with the real data, which generated a positive attitude towards the work developed. In addition, in the midst of the discussions, they came to formulate predictive hypotheses in terms of the fact that the larger the volume of liquid in the tank, the better the model fits, one of the objectives of the modelling.

The qualitative re-signification of the concept of integral

Finally, it can be said that, in this particular case, students were able to give meaning and significance to the concept of the definite integral, and its use in a concrete situation.

Some of the students' responses are shown below (Figure 5):

Figure 5. Meaning attributed to the definite integral



9. ¿Considera que hay ahora más sentido y significado en su concepción de la integral definida, después de la actividad experimental? Si X No___
¿Por qué? En la práctica se logró entender como realmente se aplican todos estos conceptos de la integral

9. ¿Considera que hay ahora más sentido y significado en su concepción de la integral definida, después de la actividad experimental? Si X No___
¿Por qué? por su aplicación al encuentro del área de un círculo dependiendo de su radio y un punto K distante del centro de dicho círculo. la integral es un paso más allá para la solución de problemas más complejos en las matemáticas, es una herramienta más.

9. ¿Considera que hay ahora más sentido y significado en su concepción de la integral definida, después de la actividad experimental? Si X No___
¿Por qué? En la práctica se logró entender como realmente se aplican todos estos conceptos de la integral

In all the teams, the answers show the importance of showing the functional part of mathematics, the concept of the definite integral, from an aspect of attributing meaning and value to the concepts in their use in real life.

Students' evaluation of the activity

In general, the assessment of the activity was very positive, since ultimately, it was not about finding the best model but about all the cognitive and collaborative processes that were put into play when trying to respond to the situation formulated.

Discussion

Modelling as a process that favours interaction, resignification and the construction of mathematical knowledge that occurs when students experiment collectively and in an environment of permanent discussion, confrontation and consensus-building in the mathematics class [31], and not simply when modelling is taught as another content or simply as a strategy for solving problems. When they established the model, they realized that what they had done in reality corresponded to what they had found mathematically, thus verifying that mathematics is not alien to contexts and that it is part of everyday life. The above shows the use of modelling in the solution of a real problem in the mathematics class, the determination of the level of liquid in a tank, close to other similar problems such as those posed by [32]-[34], among others, or other examples of application such as those of [35]-[36], which have applications in the industrial sector [37] and are useful in the training of engineers [11], [38].

Finally, thinking of the mathematics class as a scenario in which the resignification of mathematical knowledge is the main activity and that this is the result of interaction between students and with the teacher as mediator, is what makes the practice of modelling a highly significant and motivating alternative for students in their learning process [2]. In this sense, it is necessary to incorporate modelling in the curricular and micro-curricular designs of the different mathematics subjects, as established in the guiding guidelines [20]-[21], since students show interest and motivation, which is why it is necessary to homogenize its practice in the classroom [39].

Conclusions

The answers given by the students allow us to conclude that mathematical knowledge can be understood differently to the traditional one, no longer as mathematical objects out of context, but as an articulated whole in which the situated and intentional exercise of modelling practice favours the process of resignification of this knowledge via the dynamics of the interactions that emerge from this situation. Modelling as a practice, in engineering students, is the means that brings them closer to the professional reality in which interactions play a fundamental role by allowing the development of processes of resignification of school mathematical knowledge, no longer isolated and disconnected but integrated and functional in that reality.

The experimental modelling work generated interest, motivation and interaction among the students, since for most of them it was their first time doing an experimental activity in mathematics, which generated a positive attitude towards the work, favoured teamwork, allowed the identification of learning weaknesses and helped argumentation as an element of peer learning. The assessment of the activity was positive since, according to the students, this type of activity is better than traditional classes.

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