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Student Interactions In The Redefinition Of The Concept Of Differential Equation: Modelling Of The Law Of Cooling

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Abstract

The objective is to analyse how modelling favours interactions in the mathematics class and allows the resignification of the first order linear differential equation, considered as a modelling tool for change in the cooling phenomenon. The field work was based on the staging of a didactic sequence in which two groups of students, in a differential equations course, carried out an experimental activity of the cooling phenomenon to build a mathematical model that adjusted to the data collected and in this process the resignification of mathematical knowledge took place. The activity was videotaped and the students' written productions were collected. The results show that the practice of modelling favours not only the motivation of the students in relation to mathematical knowledge, giving it sense and meaning, but also confronts them with real situations, which allows the articulation and integration of this knowledge in other areas of knowledge.

Keywords: Modelling, interactions, re-signification, context, reality.

Introduction

This section presents different arguments in favour of modelling in the teaching and learning of mathematics at different educational levels and from some authors' perspective. Modelling and its applications, its teaching and learning at different educational levels, has gained importance recently, due to the use of mathematical sciences and technology in everyday life [1]. There is a need for a change in mathematics education focused on the development of the competencies needed to use mathematics in solving real-world problems. That is to say, it is required that what is taught in the classroom is not far from reality, that it serves to solve or propose alternative solutions to real-world and everyday problems, in the context of students [2]. In this sense, [3] state that mathematical modelling as a method of teaching mathematics at all levels of schooling, allows students not only to learn mathematics in a way that is applied to other areas of knowledge, but also to improve their ability to read, interpret, formulate and solve problem situations.

Blum and Ferri [4] recognize modelling as an essential tool to help students better understand the contexts in which they develop and to promote the development of certain competencies and appropriate attitudes towards mathematics. In this line, modelling plays a role that gives importance to the development of competencies in students in the process of building models, their interpretation, argumentation and validation with the respective real situations, as supported by [5]. For [6], the use of modelling in teaching leads to the learning of mathematical contents that are connected to other forms of knowledge to develop a particular way of thinking and acting, producing knowledge through abstractions and formalizations, interconnected to empirical phenomena and processes considered as problematic situations.

Likewise, the modelling process, as part of school education even from an early age, allows students to acquire skills to analyse, investigate, support and establish mathematical models, which allows establishing solid cognitive roots to broaden the understanding of some phenomenon or situation of daily life to motivate work with mathematics [7]. In [8]-[9], an activity is designed to address the problems of mixtures in a course of differential equations that allows changes of representation in the different registers.

According to the above, modelling can serve other purposes, since it provides cognitive support to students' conceptualizations, places mathematics in the culture and forms a critical attitude towards pre-established models. In [10], in which a modelling practice was carried out (on the pollution of a river), some factors are established in which the importance of modelling practice and its usefulness in the development of scientific competences are highlighted [11]. Finally, [12], confers importance to modelling for being a strategy par excellence of human beings for the creation of knowledge, since it allows validating and making predictions about the behaviour of the system being modelled and the possibility of controlling it.

Model, reality and context

The term model is presented as a polysemic concept, and there are several meanings found in the literature. The notion of model does not emerge from mathematics itself, but it is a relationship between a phenomenon, material or scheme and a mathematical concept, structure or procedure [13], i.e., a mapping is established between the extra-mathematical world and mathematics [14]. In addition, an important aspect in modelling activities is the concept of reality, since it is assumed as a basis for the identification and selection of phenomena, situations or problems to be considered for the work in the mathematics class. In other words, there is a direct relationship between reality and model mediated by mathematics in the real world [15].

On the other hand, [16], does not refer to reality as such, but rather to the real world, which he considers as an aspect close to the real contexts of students, i.e., the everyday, social, cultural, consumer or other science contexts in which students may be immersed. Therefore, it is relevant that the teaching and learning of mathematics be contextualized, based on real objects and about real objects, and from the student's own reality [17]. Furthermore, it is important to consider that this reality is closely related to the contexts in which it develops and, therefore, rather than speaking of reality and context isolated, it is a binomial in which each of the parts conditions and influences the other and both, in turn, influence the practice of modelling in mathematics education [2].

In [13], three classroom contexts in which modelling can be performed are pointed out. The first one refers to solving problems in which mathematical operations arise as generalizations of real actions. In the second context, the student takes a real-life problem, organizes it, structures it, and then determines the relevant mathematics needed and, finally, solves the problem, and in the third context, the starting point is a real-life problem for which new mathematical concepts are introduced and developed.

Modelling in mathematics classes

Before starting about mathematical modelling in the classroom, it is necessary to differentiate between the term modelling, as a scientific activity, and modelling, as a tool to build mathematical concepts in the classroom. Mathematical modelling is used by mathematicians as a dynamic process that allows them to understand various problems or a situation of specific importance in physics, chemistry, biology or other science, while mathematical modelling (contraction of the terms modeling and education) is conceived as a method in teaching-learning that is used in the scientific activity of this process [16], [18], which is performed in the mathematics class about real or contextual situations and involves students with different social, mathematical and communication processes, which gives them an extensive range of learning possibilities, and therefore, motivates them to learn mathematics [19].

According to the above, one of the characteristics of mathematics courses at different educational levels is the scarce or insufficient linkage with experimental activities that manage to articulate mathematical contents with real situations or phenomena close to the daily life and experiences of students, so that mathematical knowledge is placed in a different plane from the theoretical and conceptual and emerges as an important tool and support in other areas of knowledge. It is common for modelling to be associated as a final activity. This is evident in the texts when the application or modelling problems are presented at the end of the chapter. Thus, it is assumed that these application problems could only be performed when the mechanization of processes has been previously acquired, i.e., it is considered an application of the mechanized [20], but not a source that allows the resignification of school mathematical knowledge or that can be performed at the same time as the development of the courses.

Although mathematical modelling is considered in the curricular designs of mathematics courses in general [21], it must be linked to experimental activities, in the classroom or outside it, that promote the articulation with other fields of knowledge and at the same time allow giving meaning and life to mathematical knowledge [22]. The importance of experimental activity in modelling practices in mathematics teaching has already been considered by some authors [23]-[25].

Despite the importance that has been conferred to experimental activities, in modelling practices, in the mathematics classroom there is still no linkage and integration of these in mathematics curricula, since, in school contexts, experimentation is an unusual practice [26]. In this same sense, for [27] the school has minimized mathematical creation from experimentation in the laboratory, and it is discouraged in the classroom, so modelling can be seen through the linking of school practices with its environment. The practice of modelling in this sense, should start, as far as possible and according to the available resources, from a real activity close to the students who allow them to be actively involved and to live and experience the mathematical activity.

Resignification through modelling

To re-signify mathematical knowledge is not to give it a new meaning, but it is the transformation of knowledge into functional knowledge in and with human activities that transform and that is regulated by institutional and cultural aspects in a particular context and that is manifested in the use of knowledge within a specific situation [28].

Thus, modelling in a variety of daily, professional or scientific practices [29], when exercised by human groups (students and teachers) in a particular context, and under certain circumstances, promotes interactions, negotiations and consensus that give sense and meaning to school mathematical knowledge, which is constructed and reconstructed in the situations of interaction that occur in the classroom, producing resignifications of meanings in a process of negotiation [30].

This is how resignifications are produced through modelling to the extent that, in the exercise of this practice, there is a specific, situated and intentional use of the differential equation, in this particular case, in a situation of change as it occurs with the phenomenon of cooling. The way in which this change occurs is what gives added value to the differential equation and its use, understood as a mathematical object to model various phenomena [31], is no longer isolated and decontextualized from reality but linked to it in an interactive process in which the students, through their own experience, via the modelling practice, are the ones who will find new uses and meanings to a school mathematical knowledge, thus enriching meanings at the same time.

This resignification is achieved from the confrontation between what students already know or think they know, between what they intuit or what their common sense suggests, and the results obtained from a real situation in which they have been the main actors and have had the possibility of interacting with their peers and the teacher. It is in this sense that the resignification gives value, interest, new questions and new visions to mathematical knowledge, which in this particular case is the differential equation that models the change or variation, enriching its meaning when a group of students in a particular context exercises the practice of modelling on the phenomenon of cooling of a body.

Interactions in the mathematics classroom

Classroom interaction corresponds to the representations and ways in which different elements that make up the teaching and learning process relate and communicate with each other (teacher, student and knowledge) in addition to the socio-educational environment in which they participate [32]. This means that knowledge, in this case school mathematics, as a collective construction occurs in the interaction from the teacher to the student, generates modifications in the cognitive structure, provided that these interactions meet certain criteria, mentioning among them intentionality and reciprocity, meaning and transcendence [33]. [34], affirm that the teacher, the student, the object of knowledge and the teaching objectives are the elements of any educational practice, but it is the interaction between them that determines that practice and is at the same time the intrinsic element of the effectiveness of any educational environment, especially the mathematics classroom.

In social interactions, the daily experience of the social world is revealed, distinguished and perceived in a given scenario, in this case the mathematics classroom, precisely because it is in these relationships, where behaviours are produced that are related to the demands, conflicts and influences of society and culture [35]. Therefore, those that contribute to the construction and resignification of school mathematical knowledge in the context of the mathematics classroom are of interest, such as the processes of association of students and teachers, among which there is an exchange, an orientation and an affectation of behaviour. These processes of interaction between the members of a group generate a network of relationships that constitute a learning space, in which, and through which, the co-organized subjects support each other [36] and a space of exchange facilitation is provided [37].

In classroom interactions, discourse plays a central role, constituted by communicative practices that generate the production and transaction of intentions and meanings in socially and culturally situated interactions [38]. Therefore, it makes sense to analyse interactions through the discourse assumed in its various expressions or manifestations whether verbal or written, each of which in turn allows us to analyse and account for the nature of the arguments and knowledge used not only in the construction of school mathematical knowledge [39], but also in its resignification. In this experience, the interactions are sought to be promoted in the exercise of a modelling practice and more specifically in the exercise of the modelling practice of the cooling phenomenon. In this way, oral and written productions are considered forms of expression of these interactions and therefore susceptible of being analysed.

Method

Design

The chosen method is given from a qualitative perspective, since it implies a thorough observation in the natural context where these interactions and behaviours take place, and makes possible a permanent communication with the subjects to be studied, directly or indirectly, to know their perceptions and conceptions about the phenomenon or event and to achieve the most detailed and complete descriptions possible of the situation, in this case, the resignification of school mathematical knowledge in the exercise of the modelling practice, which is oriented towards the ethnographic method.

The aim is to record and analyse in as much detail as possible the interactions that occur (within teams, between teams and the whole group) and how they favour the resignification of mathematical knowledge, in this case of the differential equation that models the cooling phenomenon and not simply a description of these interactions and events through an argumentative and discursive scenario (in the mathematics class) in which students and teachers, as a human group, interactively construct arguments, establish conjectures, explanations and create tools (the models) and meanings from the interaction with a real-world phenomenon.

Setting, actors and materials

In the realization of the experimental activity, the scenario for its development was the laboratory. The subject chosen was Differential Equations for students studying Engineering programs. Two groups of differential equations students from two different institutions were selected for the modelling practice. The first group was a private university with engineering programs. A total of 21 students between 18 and 22 years of age attended. The second group, an official university institution, consisted of 10 students from different engineering and technology programs between 20 and 27 years of age. All the students had already taken courses in Differential Calculus and Integral Calculus. In both cases, the groups were divided into teams of three or four students to carry out the activity and the respective analyses. In both cases, the duration of the work session was four hours.

Staging of the work sequence

An experimental activity related to the cooling phenomenon of a material (silicon) was carried out with engineering students of 2 differential equations courses. The two groups were divided into teams of three and four students, on a voluntary basis.

Moments of the staging of the working sequence

The staging of the sequence was divided into four moments, each of which had a defined intentionality, and which are articulated in a coherent whole to give some order and consistency to the modelling practice as such.
Moment 1. Previous ideas and conjectures about the study phenomenon.
Moment 2. Execution of the experimental activity and data collection.
Moment 3. Data manipulation.
Moment 4. Discussion of results and verification of conjectures.

Results

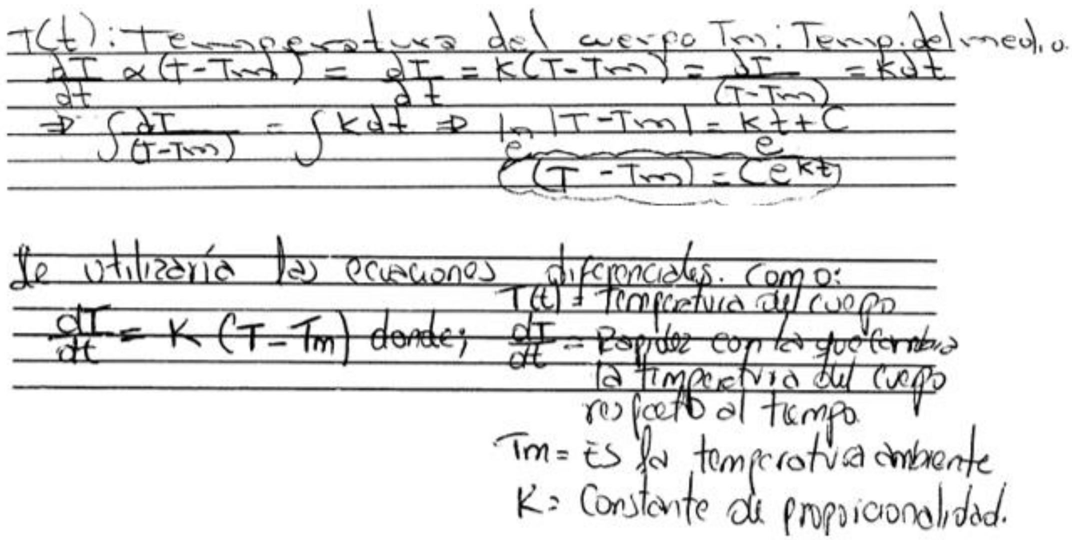
Moment 1. Motivation and previous knowledge

This is a first exercise that promotes interaction among students, argumentation, consensus building and communication, but also serves to awaken motivation and prepare the ground for the other activities, since their prior knowledge is mobilized so that they can make conjectures, establish links with other real-world situations and propose mental models about the cooling phenomenon. Some of the questions and their respective answers from this moment are analysed below:

Question 1: What mathematical tools or mathematical concepts would you employ to determine or identify that trend or behaviour?

Answer:

Figure 1. Response to question 1, moment 1



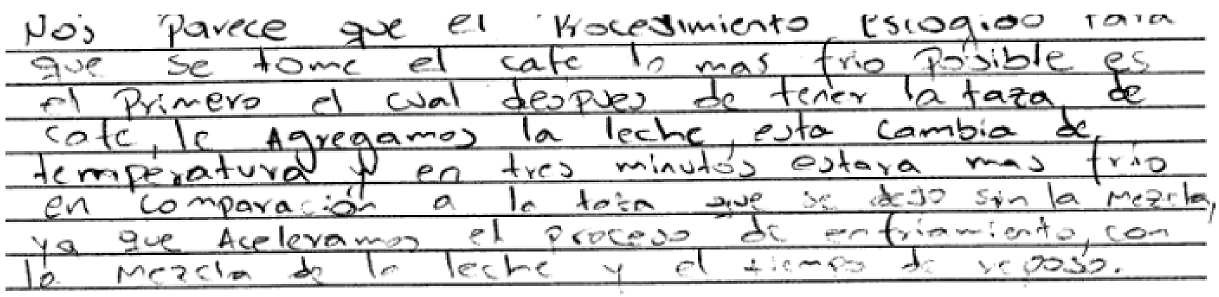
It is evident that the situation related to cooling is clear and in fact they know what the solution is, which assumes that there will be no difficulties in the solution of the differential equation or application of mathematics to a real situation. However, this was not the case because at the moment of using this knowledge in a real situation, the students did not manage to make an adequate articulation (See the following question).

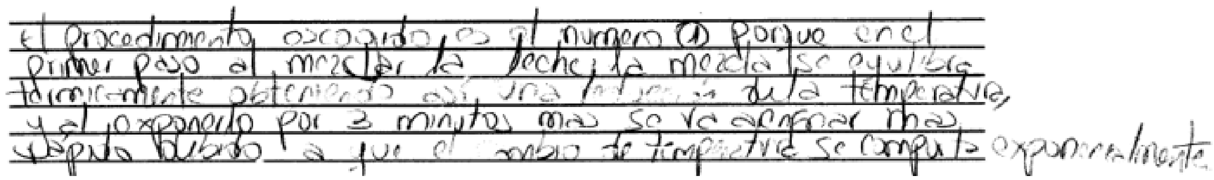
Question 2: Analyse the following situation and try to justify the chosen procedure. Suppose you are served a coffee, half of a cup, at a very high temperature, say 85 °C. You are rushing to drink it, but you want it mixed with milk. Next to you, you have a small jug with milk at room temperature, say, about 20 °C. To cool the coffee, you can follow two procedures. One, you can add the milk to the coffee until the cup is full and let it cool down again for, say, three minutes. Two, you can let the coffee only in the cup cool down during the same three minutes, add the milk and drink the coffee with milk obtained. How should you proceed to get the coffee as cold as possible in the same three minutes and drink it without burning yourself? (Taken from: Physics for inquiring minds. Coffee cooling).

The previous situation, close to the students' reality, has the purpose of promoting discussion and observing if in the solution the students' knowledge of the cooling phenomenon reflected in their answers is useful to solve it.

Answer: In this case, the students selected the first procedure: "you can add the milk to the coffee until the cup is full and let it cool a little more for, say, three minutes".

Figure 2. Response to question 2, moment 1


Nos parece que el procedimiento escogido para que se tome el cafe lo mas frio posible es el primero el cual despues de tener la taza de cafe, le agregamos la leche, esta cambia de temperatura y en tres minutos estara mas frio en comparación a la taza que se dejo sin la mezcla, ya que aceleramos el proceso de enfriamiento, con la mezcla de la leche y el tiempo de reposo.


El procedimiento escogido, es el numero (1) porque en el primer paso al mezclar la leche, la mezcla se equilibra térmicamente obteniendo así, una [illegible] de la temperatura, y al exponerlo por 3 minutos mas se va enfriar mas rápido debido a que el cambio de temperatura se comporta exponencialmente

From the previous answers, it is observed that the phenomenon of cooling in its physical interpretation is not fully understood by the students, there is no clear meaning of the behaviour of the phenomenon. In this case, the fact of knowing the mathematical formulation is not a guarantee to know how to explain the phenomenon or its behaviour. It can be said that there is no significant and meaningful understanding of the mathematical knowledge related to the cooling phenomenon, although mathematically they show that they know how to interpret it.

Moment 2. Experimentation and data collection

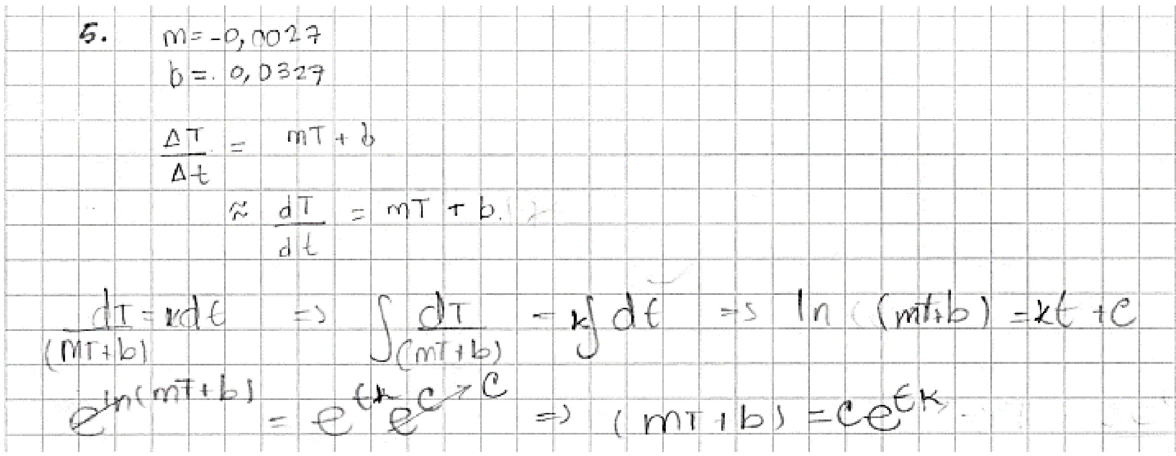
Presently, the teacher gave a general explanation of the procedure and data recording. Each team was given the respective materials and the workshop to carry out the experimental activity. Here, the active participation of the student is significant, since they are in front of a real situation, and through experimentation they put the modelling into practice.

Moment 3: Data manipulation and mathematical procedures

Currently, the results obtained in the experimental activity are mathematized to try to build a mathematical model that fits the data and allows predicting future states. What is important here is not that the model fits the data exactly or that it is about finding the "best model", but the arguments, explanations, conjectures, consensus and tools (models) are key aspects of the interactions that emerge as the discussion develops in the activity itself, which provide evidence of the resignification of school mathematical knowledge. Presently, there is a first construction or elaboration that the students have to do, that is, they must try to arrive at a model that fits the data and that is also a possible solution to the differential equation posed by Newton's cooling law. In this case, first a linear adjustment is made, and then the differential equation is solved, using the method of separation of variables.

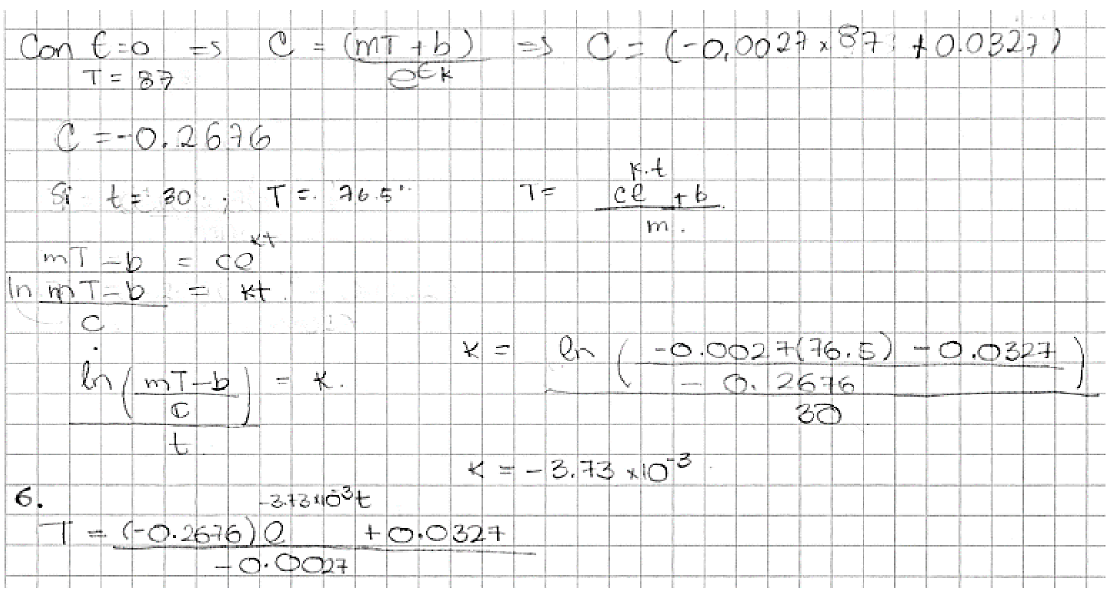
Below is the response from a team of students:

Figure 3. Response time 3



It is observed that they introduce a constant in their approach, without justification, and then what they do is to use two conditions to obtain the values of the constants k and C :

Figure 4. Complementar response time 3



This approach is different from the others, and a correct handling of the exponential and the initial conditions in the differential equation is noted. Although this procedure is not the one commonly used, the students were able to solve the differential equation and arrive at the solution, and the model was quite tight.

Moment 4: Validation and discussion of results

At this stage, the models obtained are compared with the real data taken from the experimental activity, which allows identifying weaknesses and learning needs, correct or incorrect ideas and previous concepts, strengths and successes in the development of the practice. Here, students realize the importance of mathematics in other areas of knowledge and how school mathematical knowledge can be re-signified from a real modelling situation. This is the moment of group discussion in which consensus is established, and some concepts are institutionalized.

Evaluation of the activity by the students.

According to the students' answers, most of them agree that there is a better understanding, and it creates in the students more confidence that makes them see mathematical knowledge as something close to them, since the environment created is favourable and conducive to the development of a really effective and dynamic teaching and learning process.

The qualitative resignification of school mathematical knowledge

In the resignification, besides the main knowledge, other annexed and complementary knowledge is used to solve the proposed problem. This justifies the school mathematical knowledge and acquires meaning for the student in the intentional and situated use. Although the procedural aspect is important in the sense that students were able to identify, for example, that they had weaknesses when solving an integral, this aspect is not fundamental because it can be easily corrected. What is important is that students realize that school mathematical knowledge has other scopes and that mathematical knowledge can be built in interaction.

This gives validity to this modelling experience, as a promoter of varied and significant interactions that allow re-signifying previous knowledge, strengthening current knowledge and preparing the ground for future knowledge in a dynamic of construction, argumentation and collective discussion.

Discussion

According to the results of the pedagogical experience, Newton's Cooling Law and application of the differential equation concept, and other similar activities [40]-[42], it can be said that modelling is not only the formulation of mathematical symbols about a phenomenon, it is also a way of identifying learning difficulties, of relating to knowledge, to others and to the environment, and with this, other forms of interaction and of re-signification and collective construction of mathematical knowledge are promoted. Social interaction in the classroom is fundamental in educational practices where behaviours are produced, the attitude towards the subject, cognitive and affective development based on learning experiences and relationships in the institutional context [43].

Modelling as an educational practice in mathematics favours interaction, resignification and construction of mathematical knowledge that occurs when students do it collaboratively, in an environment of permanent discussion, confrontation and consensus building, and not simply when modelling is taught as another content or simply as a strategy to solve problems. In this sense, modelling mathematizes the reality of the given situation through the construction and development of a model [13] that is integrated into problem-solving as a didactic strategy applied to situations associated with real, physical, social and cultural environments [44].

The students' answers allow us to conclude that mathematical knowledge can be understood differently from the traditional one, as a set of mathematical objects out of context and lifeless, as it is usually done in Engineering faculties [45], but as an articulation in which the practice of modelling favours the process of resignification of that knowledge, and through interactions, mathematics can be mathematized as a tool that allows articulating and giving meaning to the mathematics taught, through the generation of problematic questions [46] to give life and dynamism to school mathematical knowledge, no longer isolated and detached from reality, but integrated and functional in that reality.

To identify the learning needs of students around the practice of modelling revolve around the fact that it is difficult for them to build the mathematical model [17] and this generally goes hand in hand because traditional teaching schemes prevail [14], [47] that move away from the reality and context of the student, where this is a curricular organizer, which considers the physical, natural and social, to propose situations close to the student [48].

Finally, new ways of teaching mathematics in engineering are needed, applying field or laboratory activities, which are effective for students to obtain a better understanding of the different phenomena and processes that are objects of mathematical modelling; achieved a conceptual increase in terms of interpretation, formulation and solution of problems, in addition to allowing a rapprochement between the student and mathematics as a useful tool in the exercise of engineering [49].

Conclusion

Beyond teaching modelling as a content, it was intended to study the phenomena that occur around this practice, the interactions that emerge from the discussions in front of a problem, the previous and current knowledge when these are presented, the different solution alternatives proposed by the students, their arguments and justifications, their exemplifications, their assumptions and certainties, their learning needs and the strategies they seek to meet those needs, the teacher's orientations, his discourse and the management of the situation, the way in which the situation is handled, and the way in which they seek to meet those needs, their assumptions and certainties, their learning needs and the strategies they seek to meet those needs, the teacher's orientations, his discourse and management of the situation, the form of institutionalization of the school mathematical discourse, everything that comes together in the modelling situation and the reasons that justify the actions of others. The modelling practices themselves are the pretext that allows us to identify and discover other elements consubstantial to the resignification and construction of mathematical knowledge and its functional character.

Thinking the mathematics class as a scenario in which the resignification of mathematical knowledge is the main activity and that this is the result of the interaction among students and with the teacher as mediator, is what makes the modelling practice a highly significant and motivating alternative for students in their learning process. The modelling practice in this sense should start, as far as possible and according to the available resources, from a real activity close to the students that allow them to be actively involved and to live and experience the mathematical activity.

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