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Transmission Dynamics Of Fractional Order Corona Virus Model Using Caputo-Fabrizio Operator

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Abstract

A fractional order mathematical model for COVID-19 Model is discussed. An approximate analytical expression of concentrations of Susceptible individuals ,Infected individuals, Symptomatic individuals, Recovered individuals and Deceased individuals by using q-Homotopy Analysis method. The main objective is to propose an analytical solution to fractional COVID-19 Model. Added with the numerical stimulation carried out using MATLAB .

Keywords: Mathematical Modeling, COVID-19, q-Homotopy Method, Fractional model Corona virus, Epidemic mathematical model.

1 Introduction

The principal purpose of the present work is to analyze and find numerical solution for the fractional mathematical model of the novel Corona Virus (COVID19), which was firstly reported in China and spreader over many countries worldwide. To understand the transmission dynamics of this disease, mathematical models can be very effective.[1] Since Fractional order system is related to memory effects, it is more effective for modeling the epidemic diseases. Motivated by this, we purpose fractional order susceptible individuals, asymptotic infected, symptomatic individuals, recovered and deceased individuals SIERD model for the spread of COVID-19 disease[2]. We consider both classical and fractional order model and estimate the parameters by using the real data of Italy, reported by the World Health Organization.

Fractional Calculus (FC) was introduced by Guillaume de l'Hopital before 300 years ago. Modeling by using the perception of FC penetrates the basic fundamentals for many dynamical systems, since integer order calculus is only a narrow subset of Calculus having fractional order[3]. The biological phenomena modeled through derivatives of arbitrary order carry information of not only present state but also past states. FC considers the system with memory hereditary behaviors. These properties are important for portraying the problems that arose in science and technology. There are several powerful methods are available for us to find the exact solutions for fractional order mathematical systems. [4]In his literature we discuss about a technique q-Homotopy method which was introduced by Singh. This is the graceful algorithm which is the coupling of Laplace transform and q-Homotopy analysis algorithm[5, p.]. The most used mathematical models for the spread of infections are the classical ordinary differential equations, such as SI, SIS, SIR, SEIR, SIRD, and SEIRD models. In these models, each variable represents the number of individuals in different groups. From the discovery of the 2019-nCoV, several models have been proposed to study its dynamics. [6] Zhicheng Du proposed a simple SIR model for predicting novel Corona virus, according to China's first reported data. Yang and Wang presented an extended SEIR model for COVID-19 with time-varying transmission rates by considering the environmental effects. [7] Liang described the growth propagation of three pandemic diseases, COVID-19, SARS, and MERS, by mathematical models and found that the growth rate of COVID-19 is much greater than SARS and MERS. The fractional-order differential equations have been recently used for describing the behavior of the epidemics.

The fractional derivatives are dependent on the historical states, in addition to the current state, and thus have memory properties. Therefore, they are better choice for the epidemic's modeling. Furthermore, in the fractional model, the derivative order provides a degree of freedom in fitting data. Due to these properties, the fractional differential equations have been used for various applications in different fields

González–Parra presented a fractional-order SEIR model for explaining the out- break of influenza A(H1N1). They showed that the fractional model agrees with the real data better than the classical model[8]. Demirci proposed a fractional-order SEIR epidemic model with vertical transmission with considering that the death rate is dependent on the number of the total population. Area analyzed the data of the Ebola outbreak by both integer-order and fractional-order SEIR models[2]. All of the studies in modeling the spread of COVID-19 have considered or dinary differential equations, while there are some claims that the fractional-order models have a better fitting to the real data. the authors [9]have presented a SEIRD model for analyzing and predicting the COVID-19.

SEIRD Model

In our investigation, we use the SEIRD model proposed, which contains five populations of susceptible individuals (S), the infected individuals who are not detected (asymptomatic) (E), the symptomatic (I), recovered (R), and deceased (D) individuals. The describing equations of this Fractional model are as follows [9]:

$$\frac{dS}{dt} = -S(r_1E + r_2I + \eta)$$

$$\frac{dE}{dt} = S(r_1E + r_2I) - (a_1 + c_1 + \eta)E$$

$$\frac{dI}{dt} = c_1E - (a_2 + c_2 + \eta)I$$

$$\frac{dR}{dt} = a_1E + a_2I - \eta R$$

$$\frac{dD}{dt} = c_2I - \eta D \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(1)}$$

where recovering rate of asymptomatic individuals is a_1, transmission rate of getting symptoms of asymptomatic individuals is c_1. Similarly recovering rate of symptomatic individuals is a_2, deceased rate of them is c_2, infection rate of asymptomatic individuals and symptomatic individuals are $r_1$and r_2 respectively. Since detected infected people are isolated, we kept r_2 = 0.Usually at the starting situation of epidemic diseases, initial susceptible individuals are equal to total population. But the impose of social distancing S(0) value is decreased. We take the death rate caused by viral attack η = 0.01.Let the average time of incubation and initial symptoms to death be 5 and 11 and inverse of them be α_1,α_2 .We have $\alpha_1 = a_1 + b_1$, $\alpha_2 = a_2 + b_2$. the mean death rate is 0.02 and is calculated by$m = \frac{c_1c_2}{\alpha_1\alpha_2}$ .Hence arbitrary parametes are $a_1, r_1, S(0)$ and $E(0)$ only.

Caputo Fabrizio fractional SEIRD model

Inspired by above mention literature we have used Caputo Fabrizio operator of order α such that $\alpha \in [0,1]$ to analyse the system of non-linear differential equation given by Caputo Fabrizio fractional SEIRD model [5]

$$^{CF}D_t^{\alpha}\big(S(t)\big) = -S(r_1E + r_2I + \eta)$$

$$^{CF}D_t^{\alpha}\big(E(t)\big) = S(r_1E + r_2I) - (a_1 + c_1 + \eta)E$$

$$^{CF}D_t^{\alpha}\big(I(t)\big) = c_1E - (a_2 + c_2 + \eta)I$$

$$^{CF}D_t^{\alpha}\big(R(t)\big) = a_1E + a_2I - \eta R$$

$$^{CF}D_t^{\alpha}\big(D(t)\big) = c_2I - \eta D$$

with initial conditions

$S(0) = a_1, E(0) = a_2, I(0) = a_3, R(0) = a_4, D(0) = a_5.$ ___________________(2)

If $\alpha = 1$ we get classical integer order Corono virus model. The numerical simulation are obtained by q-homotopy method. The parameters and initial conditions are arbitrary chosen according to Corono virus spread in Italy to check our results. In section2 , we give basic definitions of Caputo Fabrizo differential operator and Laplace transformation of it. In section 3 , we discuss about the existence and uniqueness of solutions of our model.

In section 4, centre around the stability results of solutions obtained by q -homotopy method and numerical stimulation are displayed graphically in section 5.

section 6 is the part of discussions of numerical simulations. At the end, the conclusion is given in section 7.

2. Preliminaries

Definition 2.1

[10]For at least n-times differentiable function $g: [0,\infty] \to R$,a Caputo fractional derivative of order $\alpha > 0$ is defined as,

$$^{C}D_t^{\alpha}\big(f(t)\big) = \frac{1}{\Gamma(n-\alpha)}\int_0^t (t-\tau)^{n-\alpha-1} f^n(\tau)d\tau \text{ where } n = \lfloor\alpha\rfloor + 1,$$ __________(3)

by changing the kernel $(t-\tau)^{n-\alpha-1}$ by the function $e^{-\left(\frac{\alpha}{1-\alpha}\right)(t-\tau)}$ and $\frac{1}{\Gamma(n-\alpha)}$ by $\frac{\mu(\alpha)}{1-\alpha}$

we get New Caputo –Fabrizio fractional operator of order α.

Definition 2.2

[11]For $0 < \alpha < 1$, $\mathbf{f} \in H'(0,b), b > 0$. The Caputo-Fabrizio fractional derivative is defined by,

$$ {}^{CF}D_t^{\alpha}\big(f(t)\big) = \frac{\mu(\alpha)}{1-\alpha}\int_0^t e^{-\left(\frac{\alpha}{1-\alpha}\right)(t-\tau)} f^{n}(\tau)d\tau, \qquad \text{\_\_\_\_\_\_\_\_\_\_(4)} $$

M(α) is the normalization function depending on $\alpha > 0$ with the condition

M(0) = M(1) = 1.

Let us take M(α) = 1 in our manuscipt.

Definition 2.3

For $0 < \alpha < 1$, and The Caputo-Fabrizio fractional integral operator is defined by,

$$ {}^{CF}J_t^{\alpha}\big(f(t)\big) = (1-\alpha)\{u(t)\} + \alpha\int_0^t u(\tau)d\tau, t \geq 0. \qquad \text{\_\_\_\_\_\_\_\_\_\_(5)} $$

The main advantage of Caputo Fabrizio operator is there is no singularity for $t = s$, but we have it in old Caputo derivative operator.

Definition 2.4

The Laplace transfom for the Caputo Fabrizio 0perator of order $0 < \alpha \leq 1$, $m \in N$ is given by, u

$$ L\left({}^{CF}D_t^{m+\alpha}\big(f(t)\big)\right)(s) = \frac{1}{(1-\alpha)} L\big(f^{m+1}(t)\big) L\left(e^{-\left(\frac{\alpha}{1-\alpha}\right)(t)}\right) $$

$$ = \left\{\frac{s^{m+1}L\big(f(t)\big) - s^{m}f(0) - s^{m-1}f'(0 - \cdots - f^{m}(0))}{s+\alpha(1-s)}\right\}, \qquad \text{\_\_\_\_\_\_\_\_\_\_(6)} $$

In Particular,

$$ L\left({}^{CF}D_t^{\alpha}\big(f(t)\big)\right)(s) = \frac{sL\big(f(t)\big) - f(0)}{s+\alpha(1-s)} $$

$$ L\left({}^{CF}D_t^{\alpha+1}\big(f(t)\big)\right)(s) = \left\{\frac{s^2L\big(f(t)\big) - s\ \ f(0) - f'(0)}{s+\alpha(1-s)}\right\} $$

3.Existence and Uniqueness

By using fixed point hypothesis, we characterize the existence and uniqueness of the solution of the Corono Virus model,

$$ S(t) = S(0) + (1-\alpha)\{-S(t)(r_1E(t) + r_2I(t) + \eta)\} + \alpha\left\{\int_0^t -S(\tau)(r_1E(\tau) + r_2I(\tau) + \eta)d\tau\right\} $$

$$E(t) = E(0) + (1-\alpha)\{S(t)(r_1E(t) + r_2I(t)) - (a_1 + c_1 + \eta)E(t)\} + \alpha\left\{\int_0^t \{S(\tau)(r_1E(\tau) + r_2I(\tau)) - (a_1 + c_1 + \eta)E(\tau)\}d\tau\right\}$$

$$I(t) = I(0) + (1-\alpha)\{c_1E(t) - (a_2 + c_2 + \eta)I(t)\} + \alpha\left\{\int_0^t \{c_1E(\tau) - (a_2 + c_2 + \eta)I(\tau)\}d\tau\right\}$$

$$R(t) = R(0) + (1-\alpha)\{a_1E(t) + a_2I(t) - \eta R(t)\} + \alpha\left\{\int_0^t \{a_1E(\tau) + a_2I(\tau) - \eta R(\tau)\}d\tau\right\}$$

$$D(t) = D(0) + (1-\alpha)\{c_2I(t) - \eta D(t)\} + \alpha\left\{\int_0^t \{c_2I(\tau) - \eta D(\tau)\}d\tau\right\} \quad (7)$$

We now consider the following kernals,

$\varphi_1(t) = -S(t)(r_1E(t) + r_2I(t) + \eta)$

$\varphi_2(t) = S(t)(r_1E(t) + r_2I(t)) - (a_1 + c_1 + \eta)E(t)$

$\varphi_3(t) = c_1E(t) - (a_2 + c_2 + \eta)I(t)$

$\varphi_4(t) = a_1E(t) + a_2I(t) - \eta R(t)$

$$\varphi_5(t) = c_2I(t) - \eta D(t)$$

Lemma 3.1

The kernels $\phi_1, \phi_2, \phi_3, \phi_4$ and ϕ_5 given in 7 satisfy the Lipschitz condition if the following inequality holds,

$0 < \chi_1, \chi_2, \chi_3, \chi_4, \chi_5 < 1$

Proof:

Let S_1 , S_2 for kernal ϕ_1 and E_1, E_2 for the kernal ϕ_2 and I_1, I_2 for the kernal ϕ_3 and R_1, R_2 for the kernal ϕ_4 and D_1, D_2 for the kernal ϕ_5 are respective functions corresponds to the following relations,

$$\left\|\varphi_1(t, S_1(t)) - \varphi_1(t, S_2(t))\right\|$$
$$= \|-S_1(t)(r_1E(t) + r_2I(t) + \eta)$$
$$+ S_2(t)(r_1E(t) + r_2I(t) + \eta)\|$$
$$= \|-\{S_1(t) - S_2(t)\}(r_1E(t) + r_2I(t) + \eta)\|$$
$$\leq (r_1e + r_2i + \eta)\|\{S_1(t) - S_2(t)\}\|$$

where $e = \max_{t\in[0,t]}E(t)$ and $i = \max_{t\in[0,t]}I(t)$

Let $\chi_1(t) = r_1e + r_2i + \eta$

Therefore, $\left\|\varphi_1(t, S_1(t)) - \varphi_1(t, S_2(t))\right\| \leq \chi_1(t)\|\{S_1(t) - S_2(t)\}\|$

Next consider,

$$\left\|\varphi_2(t, E_1(t)) - \varphi_2(t, E_2(t))\right\|$$
$$= \left\|S(t)\big(r_1E_1(t) + r_2I(t)\big) - (a_1 + c_1 + \eta)E_1(t)\right.$$
$$\left. - S(t)\big(r_1E_2(t) + r_2I(t)\big) + (a_1 + c_1 + \eta)E_2(t)\right\|$$
$$= \left\|\{E_1(t) - E_2(t)\}\big(r_1S(t) - (a_1 + c_1 + \eta)\big)\right\|$$

where $s = \max_{t\in[0,t]}S(t)$

Let $\chi_2(t) = r_1s - (a_1 + c_1 + \eta)$

Therefore, $\left\|\varphi_2(t, E_1(t)) - \varphi_2(t, E_2(t))\right\| \leq \chi_2(t)\|\{E_1(t) - E_2(t)\}\|$

Next ,

$$\left\|\varphi_3(t, I_1(t)) - \varphi_3(t, I_2(t))\right\|$$
$$= \|c_1E(t) - (a_2 + c_2 + \eta)I_1(t) - c_1E(t) + (a_2 + c_2 + \eta)I_2(t)\|$$
$$=\|\{I_1(t) - I_2(t)\}(a_2 + c_2 + \eta)\|$$

Let $\chi_3(t) = a_2 + c_2 + \eta$

Therefore, $\left\|\varphi_3(t, I_1(t)) - \varphi_3(t, I_2(t))\right\| \leq \chi_3(t)\|\{I_1(t) - I_2(t)\}\|$

For Recovered individuals ,

$$\left\|\varphi_4(t, R_1(t)) - \varphi_4(t, R_2(t))\right\|$$
$$= \|a_1E(t) + a_2I(t) - \eta R_1(t) - a_1E(t) - a_2I(t) + \eta R_2(t)\|$$
$$=\|R_1(t) - R_2(t)(\eta)\|$$

Let $\chi_4(t) = \eta$

Therefore, $\left\|\varphi_4(t, R_1(t)) - \varphi_4(t, R_2(t))\right\| \leq \chi_4(t)\|R_1(t) - R_2(t)\|$

For deceased individuals ,

$$\|\varphi_5(t, D_1(t)) - \varphi_5(t, D_2(t))\|$$
$$= \|c_2 I(t) - \eta D_1(t) - c_2 I(t) + \eta D_2(t)\|$$
$$= \|D_1(t) - D_2(t)(\eta)\|$$

Let $\chi_5(t) = \eta$

Therefore, $\|\varphi_5(t, D_1(t)) - \varphi_5(t, D_2(t))\| \leq \chi_5(t)\|D_1(t) - D_2(t)\|$

By using recursive formula we get,

$$S_n(t) = (1-\alpha)\{\varphi_1(t, S_{n-1}(t))\} + \alpha \int_0^t \varphi_1(\tau, S_{n-1}(\tau))d\tau$$

$$E_n(t) = (1-\alpha)\{\varphi_1(t, E_{n-1}(t))\} + \alpha \int_0^t \varphi_1(\tau, E_{n-1}(\tau))d\tau$$

$$I_n(t) = (1-\alpha)\{\varphi_1(t, I_{n-1}(t))\} + \alpha \int_0^t \varphi_1(\tau, I_{n-1}(\tau))d\tau$$

$$R_n(t) = (1-\alpha)\{\varphi_1(t, R_{n-1}(t))\} + \alpha \int_0^t \varphi_1(\tau, R_{n-1}(\tau))d\tau$$

$$D_n(t) = (1-\alpha)\{\varphi_1(t, D_{n-1}(t))\} + \alpha \int_0^t \varphi_1(\tau, D_{n-1}(\tau))d\tau \qquad (8)$$

Theorem 3.2

If $[(1-\alpha)\psi_i + \alpha\psi_i t] < 1$ for $t \in [0, T]$ and some $\psi_i > 0, \forall i = 1, 2, 3, 4, 5.$
Then (2) has a solution which is zero solution.

Proof:

By the application of triangular inequality, We get,

$$\|A_n(t)\| = \|S_n(t) - S_{n-1}(t)\|$$
$$\leq (1-\alpha)\|\varphi_1(t, S_{n-1}(t)) - \varphi_1(t, S_{n-2}(t))\|$$
$$+ \alpha \left\| \int_0^t \varphi_1(\tau, S_{n-1}(\tau)) - \varphi_1(\tau, S_{n-2}(\tau))d\tau \right\|$$

$$\| B_n(t)\| = \|E_n(t) - E_{n-1}(t)\|$$
$$\leq (1-\alpha)\|\varphi_1(t, E_{n-1}(t)) - \varphi_1(t, E_{n-2}(t))\|$$
$$+ \alpha \left\| \int_0^t \varphi_1(\tau, E_{n-1}(\tau)) - \varphi_1(\tau, E_{n-2}(\tau))d\tau \right\|$$

$$\|C_n(t)\| = \|I_n(t) - I_{n-1}(t)\|$$
$$\leq (1-\alpha)\left\|\varphi_1\big(t, I_{n-1}(t)\big) - \varphi_1\big(t, I_{n-2}(t)\big)\right\|$$
$$+ \alpha\left\|\int_0^t \varphi_1\big(\tau, I_{n-1}(\tau)\big) - \varphi_1\big(\tau, I_{n-2}(\tau)\big)d\tau\right\|$$
$$\|D_n(t)\| = \|R_n(t) - R_{n-1}(t)\|$$
$$\leq (1-\alpha)\left\|\varphi_1(t, R_{n-1}(t)) - \varphi_1(t, R_{n-2}(t))\right\|$$
$$+ \alpha\left\|\int_0^t \varphi_1(\tau, R_{n-1}(\tau)) - \varphi_1(\tau, R_{n-2}(\tau))d\tau\right\|$$
$$\|F_n(t)\| = \|D_n(t) - D_{n-1}(t)\|$$
$$\leq (1-\alpha)\left\|\varphi_1(t, D_{n-1}(t)) - \varphi_1(t, D_{n-2}(t))\right\|$$
$$+ \alpha\left\|\int_0^t \varphi_1(\tau, D_{n-1}(\tau)) - \varphi_1(\tau, D_{n-2}(\tau))d\tau\right\|$$

Since the kernals satisfy the Lipschitz condition,

$$\|A_n(t)\| = \|S_n(t) - S_{n-1}(t)\|$$
$$\leq (1-\alpha)\chi_1\|S_{n-1}(t) - S_{n-2}(t)\| + \alpha\chi_1 \int_0^t \left\|\big(S_{n-1}(\tau) - S_{n-2}(\tau)\big)d\tau\right\|$$
$$\|B_n(t)\| = \|E_n(t) - E_{n-1}(t)\|$$
$$\leq (1-\alpha)\chi_2\|E_{n-1}(t) - E_{n-2}(t)\| + \alpha\chi_2 \int_0^t \left\|\big(E_{n-1}(\tau) - E_{n-2}(\tau)\big)d\tau\right\|$$
$$\|C_n(t)\| = \|I_n(t) - I_{n-1}(t)\|$$
$$\leq (1-\alpha)\chi_3\|I_{n-1}(t) - I_{n-2}(t)\| + \alpha\chi_3 \int_0^t \left\|\big(I_{n-1}(\tau) - I_{n-2}(\tau)\big)d\tau\right\|$$
$$\|D_n(t)\| = \|R_n(t) - R_{n-1}(t)\|$$
$$\leq (1-\alpha)\chi_4\|R_{n-1}(t) - R_{n-2}(t)\| + \alpha\chi_4 \int_0^t \left\|\big(R_{n-1}(\tau) - R_{n-2}(\tau)\big)d\tau\right\|$$

$$\|F_n(t)\| = \|D_n(t) - D_{n-1}(t)\|$$
$$\leq (1-\alpha)\chi_5\|D_{n-1}(t) - D_{n-2}(t)\| + \alpha\chi_5 \int_0^t \left\|\big(D_{n-1}(\tau) - D_{n-2}(\tau)\big)d\tau\right\|$$

_____________(9)

By using Recursive formula successfully we get,

$\|A_n(t)\| \leq \left((1-\alpha)\chi_1 + \alpha\chi_1 t\right)^n \|S_0(t)\|$ which proves that the solution exists and is continuous. Similar proof for $B_n(t), C_n(t), D_n(t), F_n(t)$

Let $S(t) = S_n(t) + \Delta_{1(n)}(t)$

$E(t) = E_n(t) + \Delta_{2(n)}(t)$

$I(t) = I_n(t) + \Delta_{3(n)}(t)$

$R(t) = R_n(t) + \Delta_{4(n)}(t)$

$D(t) = D_n(t) + \Delta_{5(n)}(t)$

where $\Delta_{1(n)}(t), \Delta_{2(n)}(t), \Delta_{3(n)}(t), \Delta_{4(n)}(t), \Delta_{5(n)}(t)$ are remaining terms of solutions.

$$\|S(t) - S_{n+1}(t)\| = (1-\alpha)\left\|\varphi_1\left(t, S(t)\right) - \varphi_1\left(t, S_n(t)\right)\right\| + \alpha\int_0^t\left\|\varphi_1(\tau, S(\tau)) - \varphi_1(\tau, S_n(\tau))\right\|d\tau$$

$$\leq (1-\alpha)\chi_1\|S(t) - S_n(t)\| + \alpha\chi_1\int_0^t\left\|\left(S(\tau) - S_n(\tau)\right)d\tau\right\|$$

$$\leq (1-\alpha)\chi_1\left\|\Delta_{1(n)}(t)\right\| + \alpha\chi_1\int_0^t\left\|\Delta_{1(n)}(\tau)d\tau\right\|$$

Since $\left||\Delta_{1(n)}(t)|\right| \to 0$ when $n \to \infty$ we have $\|S(t) - S_{n+1}(t)\| \leq 0$.

$S(t) = \lim_{n\to\infty} S_n(t)$

Similarly,

$E(t) = \lim_{n\to\infty} E_n(t)$,

$I(t) = \lim_{n\to\infty} I_n(t)$,

$R(t) = \lim_{n\to\infty} R_n(t)$

$D(t) = \lim_{n\to\infty} D_n(t)$, ___________(10)

are the solutions of system equations in (2).

Theorem 3.3

There exists a unique solution of the system given by, (2) .

Proof:

Let there is another solution of the system (2), say

$S^*(t)$, $E^*(t)$, $I^*(t)$, $R^*(t)$, $D^*(t)$. let us consider,

$$\|S(t) - S^*(t)\| \leq (1-\alpha)\left\|\varphi_1\big(t, S(t)\big) - \varphi_1\big(t, S^*(t)\big)\right\| + \alpha\int_0^t \left\|\varphi_1\big(\tau, S(\tau)\big) - \varphi_1\big(\tau, S^*(\tau)\big)\right\| d\tau$$

$$\|E(t) - E^*(t)\|$$
$$\leq (1-\alpha)\left\|\varphi_2(t, E(t)) - \varphi_2(t, E^*(t))\right\| + \alpha\left\|\int_0^t \varphi_2(\tau, E(\tau)) - \varphi_2(\tau, E^*(\tau)) d\tau\right\|$$

$$\|I(t) - I^*(t)\|$$
$$\leq (1-\alpha)\left\|\varphi_3(t, I(t)) - \varphi_3(t, I^*(t))\right\| + \alpha\left\|\int_0^t \varphi_3(\tau, I(\tau)) - \varphi_3(\tau, I^*(\tau)) d\tau\right\|$$

$$\|R(t) - R^*(t)\|$$
$$\leq (1-\alpha)\left\|\varphi_4(t, R(t)) - \varphi_4(t, R^*(t))\right\| + \alpha\left\|\int_0^t \varphi_4(\tau, R(\tau)) - \varphi_4(\tau, R^*(\tau)) d\tau\right\|$$

$$\|D(t) - D^*(t)\|$$
$$\leq (1-\alpha)\left\|\varphi_5(t, D(t)) - \varphi_5(t, D^*(t))\right\| + \alpha\left\|\int_0^t \varphi_5(\tau, D(\tau)) - \varphi_5(\tau, D^*(\tau)) d\tau\right\|$$

Now, $\|S(t) - S^*(t)\| \leq (1-\alpha)\chi_1\|S(t) - S^*(t)\| + \alpha\chi_1 t\|S(t) - S^*(t)\|$

$$\|E(t) - E^*(t)\| \leq (1-\alpha)\chi_2\|E(t) - E^*(t)\| + \alpha\chi_2 t\|E(t) - E^*(t)\|$$

$$\|I(t) - I^*(t)\| \leq (1-\alpha)\chi_3\|I(t) - I^*(t)\| + \alpha\chi_3 t\|I(t) - I^*(t)\|$$

$$\|R(t) - R^*(t)\| \leq (1-\alpha)\chi_4\|R(t) - R^*(t)\| + \alpha\chi_4 t\|R(t) - R^*(t)\|$$

$$\|D(t) - D^*(t)\| \leq (1-\alpha)\chi_5\|D(t) - D^*(t)\| + \alpha\chi_5 t\|D(t) - D^*(t)\|$$

This implies that

$$\|S(t) - S^*(t)\|\left\{1 - \{(1-\alpha)\chi_1 + \alpha\chi_1 t\}\right\} \leq 0$$

$$\|E(t) - E^*(t)\|\left\{1 - \{(1-\alpha)\chi_2 + \alpha\chi_2 t\}\right\} \leq 0$$

$$\|I(t) - I^*(t)\|\left\{1 - \{(1-\alpha)\chi_3 + \alpha\chi_3 t\}\right\} \leq 0$$

$$\|R(t) - R^*(t)\|\left\{1 - \{(1-\alpha)\chi_4 + \alpha\chi_4 t\}\right\} \leq 0$$

$$\|D(t) - D^*(t)\| \left\{1 - \{(1-\alpha)\chi_5 + \alpha\chi_5 t\}\right\} \le 0$$

Hence, $S(t) = S^*(t), E(t) = E^*(t), I(t) = I^*(t), R(t) = R^*(t), D(t) = D^*(t)$ __________(11)

4. q- Homotopy Analysis Method

We consider the Caputo-Fabrizio fractional derivative non-linear equation as ${}^{CF}D^{\alpha}(u(t)) + R(u(t)) + N(u(t)) = f(t)$,where ${}^{CF}D^{\alpha}(u(t))$ is Caputo-Fabrizio derivative of $u(t)$, R is linear operator, N is nonlinear operator , $f(t)$ is known function.

By taking Laplace transform we get,

$$\frac{sL(u(t)) - u(0)}{s + \alpha(1-s)} + L(R(u(t))) + L(N(u(t))) = L(f(t))$$

$$sL(u(t)) - u(0) + (s + \alpha(1-s))(L(R(u(t))) + L(N(u(t))) - L(f(t))) = 0$$

$$L(u(t)) - \frac{u(0)}{s} + \frac{(s + \alpha(1-s))}{s}(L(R(u(t))) + L(N(u(t))) - L(f(t))) = 0$$

Non- linear operator defined in Homotopy method is,

$$N(\varphi(t;q)) = L(u(t;q)) - \frac{1}{s}(u(t;q))(0) + \frac{(s + \alpha(1-s))}{s}(L(R(u(t;q))) + L(N(u(t;q))) - L(f(t;q))) = 0$$

where $q \in \left[0, \frac{1}{n}\right]$ and $n - 1 < \alpha \le n$. _____________(12)

We construct a Homotopy deformation equation,

$$(1 - nq)L(\varphi(t;q) - u_0(t)) = hqH(t)N(\varphi(t;q)) \qquad \text{_____________(13)}$$

If q=$\frac{1}{n}$, then we get $N\left(\varphi\left(t;\frac{1}{n}\right)\right)$=0, i.e. $\varphi\left(t;\frac{1}{n}\right)$ =$u(t)$,

If q=0, then we get $\varphi(t;0) = u_o(t)$

As we vary q from 0 to $\frac{1}{n}$, $\varphi(t;q)$ converge to $u(t)$ from $u_o(t)$.

We can expand φ (t; q) by using Taylor's series expansion as,

$$\varphi(t;q)=u_o(t)+\sum_{m=1}^{\infty}u_m(t)q^m \text{ where } u_m(t)=\frac{1}{m!}\frac{\partial^m\varphi(t;q)}{\partial q^m} \text{ at q=0.}$$

The solution of nonlinear system is given by

$$u(t)=u_o(t)+\sum_{m=1}^{\infty}u_m\left(t;\frac{1}{n}\right)\left(\frac{1}{n}\right)^m \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_(14)}$$

Equating power of q on both side we get the q-homotopy recursive equation as,

$$L(u_m(t)-k_m u_{m-1}(t))=hH(x,t)R_m(\vec{u}_{m-1}(t)) \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_(15)}$$

where

$$R_m(\vec{u}_{m-1}(t))=\frac{1}{(m-1)!}\frac{\partial^{m-1}N(\varphi(x,t;q))}{\partial q^{m-1}} \text{ at q=0}$$

Therefore we get, $u_m(t)-k_m u_{m-1}(t)=hH(x,t)L^{-1}(R_m(\vec{u}_{m-1}(x,t)))$ _________(16)

where $k_m=\begin{cases}0, m\le 1\\ n, m>1\end{cases}$

By solving this kind of recursive equations, we get the components of components of

q-Homotopy series solutions.

(i) $\dfrac{sL(S(t)-S(0))}{(s+\alpha(1-s))}+r_1L(SE)+r_2L(SI)+\eta L(S)=0$

$$sL(S(t))-S(0)+(s+\alpha(1-s))(r_1L(SE)+r_2L(SI)+\eta L(S))=0$$

$$L(S(t))-\frac{S(0)}{s}+\frac{(s+\alpha(1-s))}{s}(r_1L(SE)+r_2L(SI)+\eta L(S))=0$$

(ii) $\dfrac{sL(E(t)-E(0))}{(s+\alpha(1-s))}-r_1L(SE)-r_2L(SI)+a_1L(E)+c_1L(E)+\eta L(E)=0$,

$$sL(E(t)-E(0)+(s+\alpha(1-s))(-r_1L(SE)-r_2L(SI)+a_1L(E)+c_1L(E)+\eta L(E))=0$$

$$L(E(t)-\frac{E(0)}{s}+\frac{(s+\alpha(1-s))}{s}(-r_1L(SE)-r_2L(SI)+a_1L(E)+c_1L(E)+\eta L(E))=0$$

(iii) $\dfrac{sL(I(t)-I(0)}{(s+\alpha(1-s))}-c_1L(E)+a_2L(I)+c_2L(I)+\eta L(I)=0$

$$sL(I(t)-I(0)+(s+\alpha(1-s))(-c_1L(E)+a_2L(I)+c_2L(I)+\eta L(I))=0$$

$$L(I(t)-\frac{I(0)}{s}+\frac{(s+\alpha(1-s))}{s}\left(-c_1L(E)+a_2L(I)+c_2L(I)+\eta L(I)\right)=0$$

(iv) $$\frac{sL(R(t)-R(0)}{(s+\alpha(1-s))}-\left(a_1L(E)+a_2L(I)\right)+\eta L(R)=0$$

$$sL(R(t)-R(0)+(s+\alpha(1-s))\left(-\left(a_1L(E)+a_2L(I)\right)+\eta L(R)\right)=0$$

$$L(R(t)-\frac{R(0)}{s}+\frac{(s+\alpha(1-s))}{s}\left(-\left(a_1L(E)+a_2L(I)\right)+\eta L(R)\right)=0$$

(v) $$\frac{sL(D(t)-D(0)}{(s+\alpha(1-s))}-c_2L(I)-\eta L(D)=0$$

$$sL(D(t)-D(0)+(s+\alpha(1-s))\left(-c_2L(I)-\eta L(D)\right)=0$$

$$L(D(t)-\frac{D(0)}{s}+\frac{(s+\alpha(1-s))}{s}\left(-c_2L(I)-\eta L(D)\right)=0$$

We define nonlinear operator,

$$N^1(\phi_1(t;q),\phi_2(t;q),\phi_3(t;q),\phi_4(t;q),\phi_5(t;q))=$$
$$L(\phi_1(t;q))-\frac{1}{s}S(0)+\frac{(s+\alpha(1-s))}{s}\left(r_1L(\phi_1(t;q)\phi_2(t;q))+r_2L(\phi_1(t;q)\phi_3(t;q)_3)\right)+\eta L(\phi_1(t;q))$$

$$N^2(\phi_1(t;q),\phi_2(t;q),\phi_3(t;q),\phi_4(t;q),\phi_5(t;q))=$$
$$L(\phi_2(t;q))-\frac{1}{s}E(0)+\frac{(s+\alpha(1-s))}{s}\left(\left(-r_1L(\phi_1(t;q)\phi_2(t;q))-r_2L(\phi_1(t;q)\phi_3(t;q))+(a_1+c_1+\eta)L(\phi_2(t;q))\right)\right)$$

$$N^3(\phi_1(t;q),\phi_2(t;q),\phi_3(t;q),\phi_4(t;q),\phi_5(t;q))=$$
$$L(\phi_3(t;q))-\frac{1}{s}I(0)+\frac{(s+\alpha(1-s))}{s}\left(-c_1L(\phi_2(t;q))+(a_2+c_2+\eta)L(\phi_3(t;q))\right)$$

$$N^4(\phi_1(t;q),\phi_2(t;q),\phi_3(t;q),\phi_4(t;q),\phi_5(t;q))=$$
$$L(\phi_4(t;q))-\frac{1}{s}R(0)+\frac{(s+\alpha(1-s))}{s}\left(-\left(a_1L(\phi_2(t;q))+a_2L(\phi_3(t;q))\right)+\eta L(\phi_4(t;q))\right)$$

$$N^5(\phi_1(t;q),\phi_2(t;q),\phi_3(t;q),\phi_4(t;q),\phi_5(t;q))=$$
$$L(\phi_5(t;q))-\frac{1}{s}D(0)+\frac{(s+\alpha(1-s))}{s}\left(-c_2L(\phi_3(t;q))-\eta L(\phi_5(t;q))\right)$$

Homotopy equation becomes,

$$S_m(t)-k_mS_{m-1}(t)=hH(t)L^{-1}\left(R_{1m}\left(\overline{S}_{m-1},\overline{E}_{m-1},\overline{I}_{m-1},\overline{R}_{m-1},\overline{D}_{m-1}\right)\right)$$

$$E_m(t)-k_mE_{m-1}(t)=hH(t)L^{-1}\left(R_{2m}\left(\overline{S}_{m-1},\overline{E}_{m-1},\overline{I}_{m-1},\overline{R}_{m-1},\overline{D}_{m-1}\right)\right)$$

$$I_m(t)-k_m I_{m-1}(t)=hH(t)L^{-1}\left(R_{3m}\left(\bar{S}_{m-1},\bar{E}_{m-1},\bar{I}_{m-1},\bar{R}_{m-1},\bar{D}_{m-1}\right)\right)$$

$$R_m(t)-k_m R_{m-1}(t)=hH(t)L^{-1}\left(R_{4m}\left(\bar{S}_{m-1},\bar{E}_{m-1},\bar{I}_{m-1},\bar{R}_{m-1},\bar{D}_{m-1}\right)\right)$$

$$D_m(t)-k_m D_{m-1}(t)=hH(t)L^{-1}\left(R_{5m}\left(\bar{S}_{m-1},\bar{E}_{m-1},\bar{I}_{m-1},\bar{R}_{m-1},\bar{D}_{m-1}\right)\right)$$

$$R_{1m}\left(\bar{S}_{m-1},\bar{E}_{m-1},\bar{I}_{m-1},\bar{R}_{m-1},\bar{D}_{m-1}\right)$$
$$=L(S_{m-1}(t))-\left(1-\frac{k_m}{n}\right)\frac{1}{s}S_0+\frac{(s+\alpha(1-s))}{s}\left(r_1L\left(\sum_{i=0}^{m-1}S_iE_{m-1-i}\right)+r_2L\left(\sum_{i=0}^{m-1}S_iI_{m-1-i}\right)+\eta L(S_{m-1}(t))\right)$$

$$R_{2m}\left(\bar{S}_{m-1},\bar{E}_{m-1},\bar{I}_{m-1},\bar{R}_{m-1},\bar{D}_{m-1}\right)=L(E_{m-1}(t))-\left(1-\frac{k_m}{n}\right)\frac{1}{s}E_0-\frac{(s+\alpha(1-s))}{s}\begin{pmatrix}r_1L\left(\sum_{i=0}^{m-1}S_iE_{m-1-i}\right)-r_2L\left(\sum_{i=0}^{m-1}S_iI_{m-1-i}\right)\\+a_1L(E_{m-1})+c_1L(E_{m-1})+\eta L(E_{m-1})\end{pmatrix}$$

$$R_{3m}\left(\bar{S}_{m-1},\bar{E}_{m-1},\bar{I}_{m-1},\bar{R}_{m-1},\bar{D}_{m-1}\right)$$
$$=L(I_{m-1}(t))-\frac{1}{s}\left(1-\frac{k_m}{n}\right)I_0-\frac{(s+\alpha(1-s))}{s}\left(c_1L(E_{m-1})+a_2L(I_{m-1})+c_2L(I_{m-1})+\eta L(I_{m-1})\right)$$

$$R_{4m}\left(\bar{S}_{m-1},\bar{E}_{m-1},\bar{I}_{m-1},\bar{R}_{m-1},\bar{D}_{m-1}\right)=$$
$$L(R_{m-1}(t))-\frac{1}{s}\left(1-\frac{k_m}{n}\right)R_o-\frac{(s+\alpha(1-s))}{s}\left(a_1L(E_{m-1})+a_2L(I_{m-1})-\eta L(E_{m-1})\right)$$

$$R_{5m}\left(\bar{S}_{m-1},\bar{E}_{m-1},\bar{I}_{m-1},\bar{R}_{m-1},\bar{D}_{m-1}\right)=L(D_{m-1}(t))-\frac{1}{s}\left(1-\frac{k_m}{n}\right)D_0-\frac{(s+\alpha(1-s))}{s}\left(c_2L(I_{m-1})-\eta L(D_{m-1})\right)$$

By replacing m values as 1,2,3…we get numerical solution for the system.

5. Illustration

In this section, we find the numerical simulations of Caputo Fabrizio Corona virus model with initial conditions $S_0 = a_1 = 3.6\times10^6$, $E_0 = a_2 = 1.24\times10^4$, $I_0 = a_3 = 221$

$R_0 = a_4 = 1$, $D_0 = a_5 = 7$,for various values of $0 < \alpha < 1$.The corresponding parameters with the values are given in table 1.By using q-Homotopy method ,we get three terms of approximate solution of fractional order Corona virus model

Parameter	Initial values
r1	9.3*10^̃8
r2	0
A1	0.17
M	0.02
alpha2	1/11
alpha1	1/5
n	2.08547*10^(-5)

Table-1

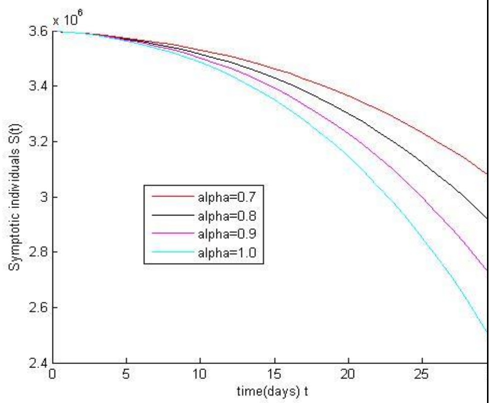

Figure -1

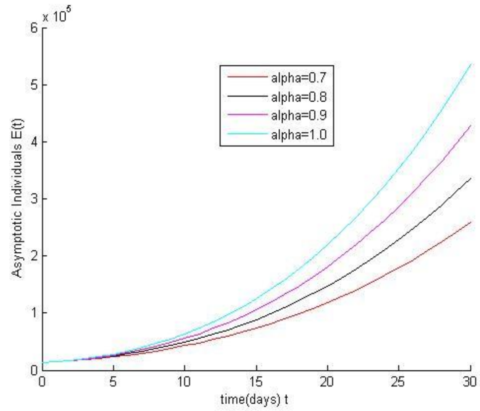

Figure -2

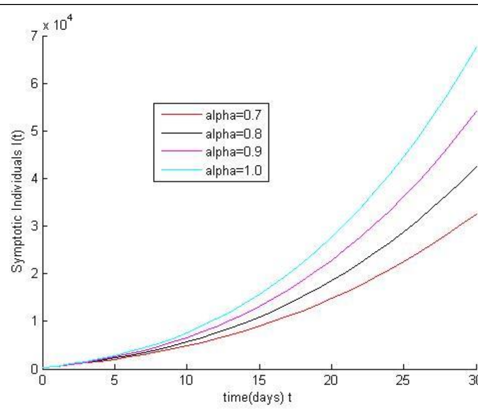

Figure -3

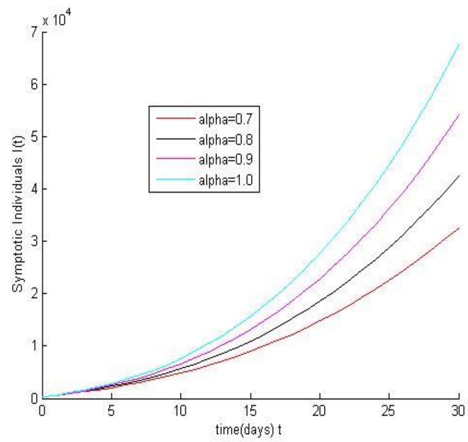

Figure -4

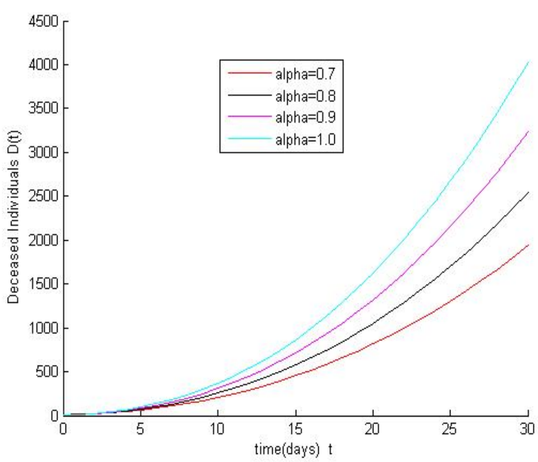

Figure -5

6.Numerical Results and Discussion

Figure 1 to 5 shows the graphical representation of numerical simulation for suscepti- ble individuals S(t), Asymptotic individuals E(t),Symptotic individuals I(t),Recovered individuals R(t) and deceased individuals D(t) for distinct values of α= 0.7, 0.8,0.9,and 1.0 using q-Homotopy method. For the period of 30 days ,figure 1 shows that susceptible individuals S(t) gradually decreases as time increases but Asymptomatic individuals E(t),Symptomatic individuals I(t), Recovered individuals R(t)and Deceased D(t) are increase monotonically as time increases.

It is observed that fractional order alpha also is directly proportional to number of people in the susceptible class, and the same alpha is indirectly proportional to number of people in the Asymptotic individuals E(t), Symptotic individuals I(t), Recovered individuals R(t) and deceased individuals D(t) classes. After 15 days, number of peoples E(t) and R(t) approximately same. This shows that people can recovered their normal life gradually.

7.Conclusion

In this work, we analyzed the transmission dynamics of Caputo-Fabrizio Corono virus model. We discuss about Existence, uniqueness and numerical simulations of Corona virus model. By using the numerical series solution obtained by q-Homotopy method ,we can predict the future performance viral transmission. Here we used a set of parameters from the country Italy. In the future, we can simulate the same technique to other part of world and also we can use the same methodology to other epidemic spreading diseases.

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