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Extension Structure Of Fermatean Fuzzy Soft Subgroup

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Abstract:
In this paper, we introduce the concept of an extension fermatean fuzzy soft subgroup (EFFSG) and have studied their related properties. Also we study the level subset of a fermatean fuzzy soft set and how they are acted in extended group structure.

Keywords: Fuzzy set, soft set, fermatean fuzzy set, fermatean fuzzy soft set, level cut, homomorphism, extension fermatean fuzzy soft group, reta(G,∅).

1.Introduction:

Later the idea of uncertainty collections of Zadeh [20], Lee [10] presented another trend of uncertainty collections called bipolar valued uncertainty sets (BVFS). Bipolar valued uncertainty set defined over the interval [-1, 1] which was to be extended from the ordinary fuzzy set interval [0, 1]. The idea of bipolar parameterized collections and several identification of bipolar parameterized collection were presented by Shabir and Naz [15]. Abdulla et al. [1] studied the idea of bipolar uncertainty parameterized collections by combining parameterized collections and bipolar uncertainty collections sponsored by Zhang [18, 19], and given parametrical ideal identifications of bipolar uncertainty parameterized collections. Naz and Shabir [13] discussed the idea of uncertainty bipolar of parameterized collections, and studied various structures on uncertainty bipolar parameterized collections. Akram et.al. [3] explained the idea of bipolar uncertainty soft sub semi group and bipolar uncertainty soft-ideals in a semi group. The minus membership function and the plus membership function defined in [-1, 0] and [0, 1] in bipolar uncertainty setting. In this bipolar uncertainty setting '0' refers that the elements are subjected to irrelevant. They are familiar representation and down word representation. The familiar forms of bipolar uncertainty collections are used in their representations. In 2011, bipolar valued fuzzy K–sub algebras are analyzed by Farhat Nisar [5]. Inspired by the concepts recently, the result of bipolar valued fuzzy sub algebras/ideals of a BF-algebra [4] has been discussed by applying the notion of bipolar valued uncertainty collection (BVFS) in BF-algebras [4]. Fermatean uncertainty bipolar model as a combination of uncertainty bipolar model and Pythagorean uncertainty bipolar. Group symmetry analyzes a moral character to molecule structures. Isotope molecules decay with a certain rate, so the uncertainty sense comes into it. But till now no algebraic structure is discussed on Fermatean uncertainty situations . Senapti and Yager [17] coined the Fermatean uncertainty set (FFS) with its relational measures. Collections data between parameterized collections were studied by Maji et al. [12]. Ali et al. [2] explained various identifications on the parameterized collections, and Sezgin and Atagun [16] investigated on parameterized set identifications as well. In this paper, we introduce the concept of an extension fermatean fuzzy soft subgroup(EFFSG) and have studied their related properties. Also we study the level subset of a fermatean fuzzy soft set and how they are acted in extended group structure.

2. Preliminaries and Basic Definition:

A non-empty set G together with extension structure operation $\emptyset: G^n \to G$,where n≥ 2 is called an extension groupiod structure and is denoted by (G,$\emptyset$). According to the general convention used in the theory of extension groupiods structure of elements x_i , x_{i+1}, ... x_i is denoted by x^jIn this case, if $j < i$,it denotes the empty symbol. If$x_{i+1} = x_{i+2} = x_{i+3} = \cdots x_{i+i} = x$, then instead of x^{i+i}we write$x^{(p)}$. In this conversion $\emptyset(x_1, x_2, x_3 \dots . x_n) = \emptyset(x^n)$ and $\emptyset(x_1, x_2, x_3, \dots x_i, x, x \dots, x^n) = \emptyset(x^i, x^{(p)}, x^{i+p+1})$. An extension groupiod structure (G,$\emptyset$) is called an (i,j)- associative if $\emptyset(x^{i-1}, f(x^{n+i-1}), x^{2n-1}) = \emptyset(x^{i-1}, f(x^{n+j-1}), x^{2n-1})$ hold for all $x_{11}, x_{22}, \dots x_{2n-1} \in G$. If the identity holds for $i \leq i \leq n$, then we say that the operation $\emptyset$ is associative and $(G, \emptyset)$ is called an extension structure of semigroup. It is clear that an extension groupiod structure in associative if and only is (i,j) associative for all $j = 2,3, \dots n$. If the binary case (where n=2) it is a usual semigroup. If for all $x_0, x_1, \dots x_n \in G$and fixed $i \in \{1,2,3 \dots n\}$ then their exists an element $z \in G$ such that $\emptyset(x^{i-1}, Z, x^n) = x^0 ----(1)$ then we say that this equation(1) is i-solvable or solvable at the place 'i'. If the solution is unique, then we say that equation (1) is uniquely i-solvable. An extension groupiod structure $(G, \emptyset)$ uniquely solvable for all $i = 1,2,3 \dots n$ is called an extension structure of quasi group. An associative extension structure of quasi group is an Extension Structure group.

Finding an extension structure operation $\emptyset$,when n≥ 3, then elements a^{n-2}, we obtain the new binary operation $x * y = \emptyset(x, a^{n-2}, y)$. If $(G, \emptyset)$ is an extension group structure then $(G, °)$ is a group.. choosing different elements a^{n-2} we different groups. So, we consider only the groups of (Dudek and J.Michal ski) the form $ret_a(G, \emptyset) = (G, °)$ where $(x \circ y) = (x, a^{n-2}, y)$. In this group $e = a, x^{-1} = \emptyset(a, a^{n-3}, x, a)$. In the theory of extension group structures, the following theorem plays an important role.

2.1Theorem: For any extension group structure $(G, \emptyset)$ there exist a group $(G, \circ)$ its auto-morphism χ and an element $b \in G$ such that $\emptyset(x^n) = \chi(x_1) \circ \chi(x_2) \circ, \chi(x_3) \dots . \circ \chi^{n-1}(x_n) \circ b --- (2)$ hold for all $x^n \in G$. In what follows, G is a non-empty set and $(G, \emptyset)$ is an extension group structure unless otherwise specified.

2.2 Definition [Senapati and Yager 2019(a)]

Let X be an universe of discovers. A fermatean fuzzy set (FFS) F in X is an object having the form $F = \{< x, m_F(x), n_F(x) > x \epsilon X\}$ where $m_F(x): x \to [0,1] n_F(x): x \to [0,1]$ including the condition $0 \leq ((m_F(x)^3 \leq n_F(x))^3 \leq 1$ for all $x \in X$. The number $m_F(x)$ and $n_F(x)$ denote the degree of membership and degree of non-membership of the element $x \in F$.

For any FFS of F and $x \epsilon X, \Pi_F(x) = \sqrt[3]{1 - m_F(x)_3 - n_F(x)^3}$ is identified as the degree of indeterminacy of X to F. For convenience, Senapati and Yager called $(m_F(x), n_F(x))$ a fermatean fuzzy number(FFN) denoted by $F = (m_F, n_F)$. We shall point out the membership grades (MGS) related to fermatean fuzzy sets as fermatean membership grades (FMGS).

2.3 Theorem: The set of FMGS is greater than the set of Pythagorean membership grades(PMGS) and intuitionistic membership grade(IMGS).This development can be evidently recognized infig-1.

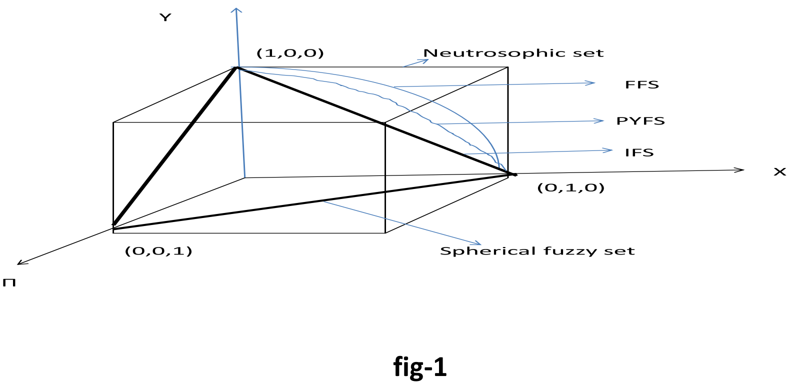

fig-1

Here we notice that IMGs are all points breath terms ≤ 1, the PMGS are all points $x^2 + y^2 \leq 1$
and the FMGS are all the points with $x^2 + y^2 \leq 1$. We see that the FMGS enable the presentation of a bigger body of non-standard membership grades than IMGS and PMGS.

2.4 Definition:
Let $(G, \emptyset)$ be an extension group structure. A fuzzy subset of G is called subgroup of $(G, \emptyset)$ if the following axioms holds:
(EFG_1): $\mu(\emptyset(x^n)) \geq T\{\mu(x_1), \dots \mu(x_n)\}$
(EFG_2): $\mu(x) \geq \mu(x), \forall x \in G$ and $x \in G$,

Note that for n=3 the second condition (EFG_2) of definition (2.4) can be replaced the condition:
(EFG_3): $\mu(x) = \mu(x) \forall x \in G$, because in this case n=3, we have$x = x$. These rwo condition are equivalent for all extension group structures in which for every $x \in G$, then their exits a natural number k such that $x^k = x$ where x^k denotes the elements skew to $x^0 = x$.

3. Extension Structure of Fermatean Fuzzy Soft Subgroups
3.1 Definition: A FFS set F= $(m_F(x), n_F(x))$ in G is called extension structure of fermatean fuzzy soft subgroup (ESFFSG) of $(G, \emptyset)$ if the following axioms hold:
*($EFFSG_1$): $m_F(\emptyset(x_1) \geq T\{m_F(x_1), \dots m_F(x_n)\}$,
*($EFFSG_2$): $n_F(\emptyset(x_1) \leq S\{n_F(x_1) \dots \dots .. n_F(x_n)$,
*($EFFSG_3$): $m_F(x) \geq m_F(x)$,
*($EFFSG_4$): $n_F(x) \leq n_F(x), \forall x_1, x_1 \in G$.

3.2 Example: Consider $(Z, \emptyset)$, where $\emptyset: Z^4 \rightarrow Z_4$ is defined by $\emptyset(x_1, x_2, x_3, x_4) = S(x_1, x_2, x_3, x_4)$ clearly $(Z, \emptyset)$ is a four dimension subgroup derived from addition group Z_4. Define FFS set $F = (m_F, n_F)$ is $(Z_4, \emptyset)$ as follows:
$m(x) = 0.6, if\ x = 0; 0.1\ if\ x = 1,2,3$
$n(x) = 0.4, if\ x = 0; 0.8\ if\ x = 1,2,3$
Then it is easy to verify that FFS set $F = (m_F, n_F)$ is four dimensional structure of fuzzy soft subgroup of $(Z_4, \emptyset)$.

3.3 Theorem: If $\{F_i \in \Lambda\}$ is an arbitray family of an extension structure of fermatean fuzzy soft subgroup of (G, ϕ), then F is an extension structure of fermaten fuzzy soft subgroup of (G, ϕ) where $F_i = \{(x_i \wedge m_F(x) \vee n_F(x))/x \in G\}$.

Proof:
The proof is obvious.

3.4 Theorem: If a FFSF=(m_F, n_F) is G is an extension structure of fermatean fuzzy soft subgroup of $(G, \emptyset)$. Then so is *A where *A=$\{x, m_F, 1 - m_F(x)/x \in G\}$

Proof:
It is sufficient to show that $m_F$satisfies the condition ($EFFSG_2$) and ($EFFSG_4$).
Let $x^n \in G$.Then
$m_F(\emptyset(x^n) = 1 - m_f(\emptyset(x^n))$
$\leq 1 - T\{m_F(x_1), m_F(x_2), \dots ., m_F(x_n)\}$
$= S\{m_F(x_1), m_F(x_2), \dots . m_F(x_n)\}$
$m_F(x) = 1 - m_F(x)$
$\leq 1 - m_F(x)$
$= m_F(x)$
Hence *A is EFFSG of $(G, \emptyset)$.

3.5 Definition: Let $F = (m_F, n_F)$ be FFSS in G and $t, s \in [0,1]$, then the set $\cup\ (m_F; t) = \{x \in G/m_F(x) \geq t\}$ and $L(n_F; s) = \{x \in G/n_F(x) \leq s\}$ is called(s,t)-level cut of G are extension sub-group structure of $(G, \emptyset)$ for any $s, t \in [0,1]$.

Proof:
Let F be EFFSG of (G,∅) for any $s, t \in [0,1]$, then $m_F(x_i) \geq t$ and $n_F(x_i) \leq s$ for all
$i = 1,2,3,..n$ thus
$m_F(\emptyset(x^n)) \geq T\{m_F(x_1), m_F(x_2) \ldots, m_F(x_n)\}$
$\geq t$
Which implies $\emptyset(x^n) \in \cup(m_F; t)$
$n_F\big(\emptyset(x^n)\big) \leq S\{n_F(x_1), n_F(x_2), \ldots n_F(x_n)\}$
$\leq s$
Which implies $\emptyset(x^n) \in L(n_F; s)$
Moreover $m_F(x) \geq m_F(x)$
$\geq t$ and
$n_F(x) \leq n_F(x)$
Which implies $x \in \cup(m_F; t)$ and $x \in L(n_F; t)$.

Thus (s,t)-level cut are extension subgroup structure of $(G, \emptyset)$. Conversely, assume that (s,t)-level cut are extension subgroup structure of $(G,\circ)$. Let us define
$t = T\{m_F(x_1), m_F(x_2), \ldots m_F(x_n)\}$ and $s = S\{m_F(x_1), m_F(x_2), \ldots, m_F(x_n)\}$, for some x^nG. Then obviously $x_1 \in\cup\ (m_F\colon t)$ and $x_1 \in L(n_F\colon s)$. Consequently, $\emptyset(x^n) \in \cup(m_F\colon t)$ and $\emptyset(x^n) \in L(m_F\colon t)$. Thus$T\{m_F(x_1), m_F(x_2), \ldots m_F(x_n)\}$ and $n_F\big(\emptyset(x^n)\big) \leq s = S\{n_F(x_1), n_F(x_2), .. n_F(x_n)\}$. Now let $x \in\cup\ (m_F\colon t)$ and $x \in L(n_F\colon s)$. Then $m_F(x) = t \geq t$ and $n_F(x) = s \leq s$ thus $x \in \cup(m_F\colon t)$ and $x \in L(n_F\colon s)$. Since by the assumption $x \in \cup(m_F\colon t)$ and $x \in L(n_F\colon s)$ where $m_F(x) \geq t = m_F(x)$ and $n_F(x) \leq s = n_F(x)$.

The proof is complete, by using the above theorem, we can prove the following characteristion of EFFSG.

3.6 Theorem: A FFSS A in G is EFFSG of $(G, \emptyset)$ if and only if the (s,t) −level cut of G are the extension of subgroup structure of $(G, \emptyset)$ for all $i = 1,2,3 \ldots n$. and all $x^n \in G, F$ satifies the following conditions.
(1) $m_F(\emptyset(x^n) \geq\ T\{m_F(x_1), m_F(x_2), \ldots m_F(x_n)\}$
(2) $n_F\big(\emptyset(x^n)\big) \leq S\{n_F(x_1), n_F(x_2), .. n_F(x_n)\}$
(3) $m_F(\emptyset(x^n) \geq\ T\{m_F(x_1), m_F(x_2), \ldots m_F(x_{i-1}), m_F(\emptyset(x^n), m_F(x_{i-1}), .. m_F(x_n)\}$
(4) $n_F\big(\emptyset(x^n)\big) \leq S\big\{n_F(x_1), n_F(x_2), .. n_F\big(x_{i-1}), n_F\big(\emptyset(x^n)\big), n_F(x_n)\big)\big\}$

Proof:
Assume that F is EFFSG of $(G, \emptyset)$. Similiary as in the proof of the theorem 3.5, we can proof each non-empty level subset $\cup(m_F; t)$ and $L(n_F\colon s)$ are closure under the operation ∅,
That is $x^n \in \cup(m_F\colon t)$ and $x_1 \in L(n_F\colon s)$ implies $\emptyset(x^n) \in \cup(m_F\colon t)$ and $\emptyset(x^n) \in L(n_F\colon s)$.

Now let x_0, x^{i-1}, x^n where $x_0 = \emptyset(x_0, x^{i-1}, Z, x^n)$ for some $i = 1,2,3,..n$.and $z \in G$ which implies $x_0 \in \cup(m_F\colon t)$ and $x_0 \in\ L(n_F\colon s)$.Then, according to (3) and (4), we have $m_F(z) \geq t$ and $n_F(z) \leq s$. So the equation (1) has a solution $z \in m_F(t)$ and $z \in n_F(s)$. Then mean (s,t) −level cut are extension subgroup structure. Conversely assume that (s,t)-level cut are extension subgroup structures. Then it is easy to provide the condition (1) and (2). For $x^n \in G$,
We define $t_0 = T\{m_F(x_1), m_F(x_2), .. m_F(x_{i-1}), m_F\big(\emptyset(x^n)\big), m_F(x_{i-1}), \ldots m_F(x_n)\}$ and
$s_0 = S\{n_F(x_1), n_F(x_2), n_F(x_3), .. n_F(x_{i-1}), n_F\big(\emptyset(x^n)\big), n_F(x_{i-1}), \ldots n_F(x_n)\}$ then $x_{i-1} \in n$,
$\emptyset(x^n) \in\cup\ (m_F\colon t_0)$ and $x^{i-1}, x^n and\ \ \emptyset(x^n) \in L(n_F\colon s_0)$ thus $m_F(x_i) \geq t_0$ and $n_F(x_i) \leq s_0$.
This proves the conditions (3) and (4).

3.7 Definition: Let $(G, \emptyset)$ and $(G', \emptyset)$ be an extension group structure. A mapping $\alpha\colon G \to G'$ is called an extension homomorphism if $\alpha\big(\emptyset(x^n)\big) = \emptyset\big(\alpha^n(x^n)\big)$, where $\alpha^n(x^n) = (\alpha^n(x_1), \alpha^n(x_2), \ldots \alpha^n(x_n)$ for all

$x^n \in G$. For any FFSS A in G', we define the pre-image of A under α, denoted by $\alpha^{-1}(A)$ is an FFSS in G defined by $\alpha^{-1}(A) = \left(m_{F\alpha} - 1(F), n_{F\alpha} - 1(F)\right)$, where $m_{F\alpha} - 1(F) = (m_{F\alpha(x)})$ and $n_{F\alpha} - 1(F) = \left(n_{F\alpha(x)}\right), \forall x \in G$. For any FFSS F in G, we define the image of F under α, denoted by $\alpha(F)$, is FFSS is G^j define by $\alpha(F) = \left(\alpha_{sup}, (m_F), \alpha_{\inf}\ (n_F)\right)$, where

$$\begin{cases} \alpha_{sup}(m_F(\mathrm{x}) = \underset{x \in \alpha^{-1}(y)}{Sup} m_F(x)\,,\ if\ \alpha^{-1}(y) \neq 0; \\ 0 \quad , \text{ otherwise.} \end{cases}$$

$$\begin{cases} \alpha_{inf}(n_F(\mathrm{x}) = \underset{x \in \alpha^{-1}(y)}{inf} n_F(x),\ if\ \alpha^{-1}(y) \neq 0; \\ 0\ , \quad \text{otherwise} \end{cases}$$

for all $x \in G\ and\ y \in G'$.

3.8 Theorem: Let α be an extension homomorphism mapping grom G to G^j with $\alpha(x) = \alpha(x)$ for all $x^n \in G$ and F is an extension fermatean fuzzy soft subgroup of $\left(G^j, \emptyset\right)$.Then $\alpha^{-1}(F)$ is an extension fermatean fuzzy soft subgroup of $(G, \emptyset)$.

Proof:
Let $x^n \in G$, we have

$$\begin{aligned} m_\alpha(F)(\emptyset(x_1) &= m_F(\alpha(\emptyset(x_1) \\ &= m_F(\alpha^n(\emptyset(x^n)) \\ &\geq T\{m_F\alpha(x_1), m_F\alpha(x_2), \dots . m_F\alpha(x_n)\} \\ &= T\{m_\alpha - 1F(x_1), m_\alpha - 1F(x_2), \dots . m_\alpha - 1F(x_n)\} \end{aligned}$$

$$\begin{aligned} n_\alpha - 1F\left(\emptyset(x_1)\right) &= n_F(\alpha(\emptyset(x_1)) \\ &= n_F(\alpha^n(\emptyset(x^n)) \\ &= S\left\{n_F\left(\alpha(x_1)\right), n_F\left(\alpha(x_2)\right), \dots . n_F\left(\alpha(x_n)\right)\right\}. \\ &= S\{n_\alpha - 1F(x_1), n_\alpha F(x_2), \dots n_\alpha - 1F(x_n)\} \end{aligned}$$

$$\begin{aligned} m_\alpha - 1F(x) &= m_F\alpha(x) \\ &\geq m_F\alpha(x) \\ &= m_\alpha - 1F(x) \end{aligned}$$

$$\begin{aligned} n_\alpha - 1F(x) &= n_F\alpha(x) \\ &\geq n_F\alpha(x) \\ &= n_\alpha - 1F(x) \end{aligned}$$

This proof is completed . If we strengthen the condition, then we can construct the converse of the Theorem (3.9) as follows.

3.9 Theorem: Let α be an extension homomorphism mapping from G to G^j with $\alpha^{-1}(F)$ is an extension fermatean fuzzy soft subgroup of $(G, \emptyset)$. Then F is an extension fermatean fuzzy soft subgroup of $(G. \emptyset)$.

Proof:
For any $x \in G^j$, there exits $a \in G$, such that $\alpha(a_1) \epsilon x_1$ and for any $\emptyset(x^n) \in \left(G^j, \emptyset\right)$, then there exist $\emptyset(a^n) \in (G, \emptyset)$ such that $\alpha(\varphi(a^n) = \emptyset(x^n)$.

Then,
$$\begin{aligned} m_F\left(\emptyset(x^n\ )\right) &= m_F(\alpha(\varphi(x^n)) \\ &= m_\alpha 1 - F(\varphi(a_1)^n \\ &= T\{m_\alpha - 1F(a_1), m_\alpha - 1F(a_2), \dots, m_\alpha - 1F(a_n)\} \\ &= T\{m_F(x_1), m_1(x_2), \dots, m_F(x_n)\} \end{aligned}$$

$$\begin{aligned} n_F\left(\emptyset(x_n)\right) &= n_F(\alpha(\varphi(a^n)) \\ &= n_\alpha - 1F\left(\varphi(a^n)\right) \\ &\geq S\{n_\alpha - 1F(a_1\ ), n_\alpha - 1F(a_2), \dots . n_\alpha - 1F(a_n)\} \end{aligned}$$

$$= S\{n_F(\alpha(a_1)), n_F(\alpha(a_2)), \ldots\ldots, n_F(\alpha(a_n))\}$$
$$= S\{n_F(x_1), n_F(x_2), \ldots., n_F(x_n)\}$$

For any $x \in G$ there exist $a \in G$ such that $\alpha(a) = x$, we have

$$m_F(x) = m_F(\alpha(a))$$
$$= m_F - 1F(a)$$
$$\geq m_F(\alpha) - 1F(a)$$
$$= m_F(\alpha(a))$$
$$= m_F(x)$$
$$n_F(x) = n_F(\alpha(a))$$
$$= n_F - 1F(a)$$
$$\geq n_F(\alpha) - 1F(a)$$
$$= n_F(\alpha(a))$$
$$= n_F(x)$$

This complete the proof.

3.10Theorem: Let αbe a mapping from G to G'. If F is an EFFSG of $(G, \emptyset)$,Then $x, \alpha_{\sup}\ \ (m_F), \alpha_{\inf}\ \ (n_F))$ is an EFFSG of $(G^j, \emptyset)$.

Proof:

Let α be a mapping from G in to G^j and let $x^n \in G$ and $y^n \in G^j$. Nothing that $\{x_i(i = 1,2,3, \ldots n)/\alpha^{-1}((y^n))\} \subseteq \{\emptyset(x^n) \in G/x_1 \ \in \alpha^{-1}(\varphi(y_1)), \ldots. x_n \in \alpha^{-1}(\varphi(y^n))\}$.

We have $\alpha_{sup}(m_F)(\ \emptyset(y^n)) = \sup\{m_F(x^n)/x_i \in \alpha^{-1}(\emptyset(y^n))\}$

$$\leq \sup\{m_F(\emptyset(x^n))/x_1 \in \alpha^{-1}(\emptyset(y_1)), x_2 \in \alpha^{-1}(\emptyset(y_2)), \ldots x_n\alpha^{-1}(\emptyset(y_n)),$$
$$\geq \sup\{T\{m_F(x_1), \ldots. m_F(x_n)\}/x_1 \in \alpha^{-1}(\emptyset(y_1)), \ldots x_n \in \alpha^{-1}(\emptyset(y_n))\}$$
$$= T\{\alpha_{sup}\ (m_F)y_1\ , \alpha_{sup}\ (m_F)y_{2\ldots,}\alpha_{sup}\ (m_F)y_n\ \}$$
$$= \alpha_{\inf}\quad (n_F)(\ \emptyset(y^n)\ ) = \ \inf(\ n_F(y^n)/x_i\ \in \alpha -^{1}\ (y^n)\}$$
$$\leq \inf\{(n_F)(\ \emptyset(y^n)\ )/x_1 \in \alpha^{-1}(\emptyset(y_1)), x_2 \in \alpha^{-1}(\emptyset(y_2)), \ldots x_n\alpha^{-1}(\emptyset(y_n)),\}$$
$$= \ S\{\inf\{n_F(\ x_1\ )/x_1 \in \alpha_1(y_1), \ldots\ \inf\{n_F(\ x_n)/x_n \in \alpha_n(y_n), \qquad = S(\alpha_{\inf}\ (m_F(y_1), \alpha_{\inf}\ (m_F(y_2), \ldots \alpha_{\inf}\ (m_F(y_n)$$

$$\alpha_{sup}\left(m_F(x)\right) = \sup\{(m_F(x)/x \in \alpha^{-1}(\emptyset(y)\},$$
$$\geq \{\ sup\ (m_F(x)/x \in \alpha^{-1}(\emptyset(y)\},$$
$$= \alpha_{sup}\left(m_F(x)\right)$$

and

$$\alpha_{inf}\ \left(\ n_F(x)\right) = \ \ \inf\{(n_F(x)/x \in \alpha^{-1}(\emptyset(y)\},$$
$$\geq \{\ inf\ (n_F(x)/x \in \alpha^{-1}(\emptyset(y)\},$$
$$= \alpha_{inf}\ \left(\ n_F(x)\right)$$

This complete the proof.

3.11 Corollary: Let F be EFFSG of (G,∅),if their exists an elements a$\in G$such that $m_F(a) > m_F(x)$ and $n_F(x) \leq n_F(x)$ for every $x \in G$. Then F is an EFFSG of a group ∅(G, ∅).

Proof: For all $x, y, a \in G$, we have

$$m_F(x \circ y) = m_F(\emptyset(x, a^{n-2}, y))$$
$$\geq T\{m_F(x), m_F(a), m_F(y)\}$$
$$= T\{m_F(x), m_F(y)\}$$
$$n_F(x \circ y) = n_F(\emptyset(x, a^{n-2}, y))$$
$$\geq S\{n_F(x), n_F(a), n_F(y)\}$$
$$\geq S\{n_F(x), n_F(y)\}$$
$$m_F(x^{-1}) = m_F\left(\emptyset\left(x, a^{(n-2)}, y\right)\right)$$
$$\geq \mathrm{T}\{m_F(x), m_F(x), m_F(a), m_F(a)\}$$
$$= m_F(x)$$

$$n_F(x^{-1}) = n_F\left(\emptyset\left(x, a^{(n-3)}, a\right)\right)$$
$$\geq \text{S}\{n_F(x), n_F(x), n_F(a), n_F(a)\}$$
$$= n_F(x)$$

This completes the proof. In the above theorem, the assumption that $m_F(a) \geq m_F(x)$ and $n_F(a) \leq n_F(x)$ can not be omitted.

3.12 Example: Consider $(Z_4, \emptyset)$ where $\emptyset: Z^3 \to Z_4$ is defined by $\emptyset(x_1, x_2, x_3) = \max\{x_1, x_2, x_3\}$. Clearly $(Z_4, \emptyset)$ iis a ternary subgroup defined from Z_4. Define FFSSF $F = (m_F, n_F)$ as follows$m_F(x) = \begin{cases} 0.2 & if\ x = 0 \\ 0 & if\ x = 1,2,3 \end{cases}$

Clearly F is fermatean fuzzy soft ternary subgroup of $(Z_4, \emptyset)$. For $\theta(Z_4, \emptyset)$, we have $m_F(0,0) = m_F(\emptyset(0,1,0)) = m_F(1) = 0.1 \ngtr T\{m_F(0), m_F(0)\} = 0.2.$ $n_F(0,0) = n_F(\emptyset(0,1,0) = n_F(1) = 0.4 \nless S\{n_F(0), n_F(0)\} = 0.4$. Hence the assumpation $m_F(a) \geq m_F(x)$ and $n_F(x) \leq n_F(x)$ for all $a, x \in G$, then F is EFFSG of $(G, \emptyset)$.

Proof

By theorem (2.1) any extension group structure can be represented of the form equation(2), we have $(G,\circ) = \theta(G, \varphi), \psi_a(a, x, x^{n-2})$ and $b = \varphi$(a,a,….a). Then we have

$$m_F(\psi(x)) = m_F\left(\emptyset\left(x, x, x^{(n-2)}\right)\right)$$
$$\geq \text{T}\{m_F(m_F(x), m_F(x), m_F(a), m_F(a))\}$$
$$= m_F(x)$$
$$m_F(\psi^2(x)) = m_F\left(\emptyset\left(a, (\psi(x), x^{(n-2)}\right)\right)$$
$$\geq \text{T}\{m_F(m_F(a), m_F\psi(x),, m_F(a))\}$$
$$= m_F(\psi(x)$$
$$\geq m_F(x)$$

Consequently, $m_F\left(\psi^k(x)\right) \geq \psi_F(x) \forall x \in G\ and\ k \in N$

and

$$n_F(\psi(x)) = n_F\left(\emptyset\left(a, x, x^{(n-2)}\right)\right)$$
$$\geq \text{S}\{n_F(n_F(a), n_F(x), n_F(a))\}$$
$$= n_F(x)$$
$$n_F(\psi^2(x)) = n_F\left(\emptyset\left(a, (\psi(x), x^{(n-2)}\right)\right)$$
$$\leq \text{S}\{m_F(m_F(a), m_F\psi(x),, m_F(a))\}$$
$$= n_F(\psi(x)$$
$$\geq n_F(x)$$

Consequently, $n\left(\psi^k(x)\right) \leq \psi_F(x) \forall\ x \in G\ and\ k \in N$

Similarly, for all $x \in G$. We have

$$m_F(b) = m_F\left(\emptyset(\ a, a, \dots.a)\right)$$
$$\geq m_F(a)$$
$$\geq m_F(x)$$
$$n_F(b) = n_F\left(\emptyset(\ a, a, \dots.a)\right)$$
$$\geq n_F(a)$$
$$\geq n_F(x)$$

Thus, $m_F((\psi^n(x)) = m_F(x_1 \circ \psi(x_2) \circ \psi^2(x_3) \circ, \dots \dots, \circ \psi^{n-2}(x_n) \circ \text{b})$

$$\geq \text{T}\{m_F(x_1), m_F\psi(x_2), m_F\psi^2(x_3), \dots, m_F\psi^{n-2}(x_n), m_F(b))\}$$
$$\geq \text{T}\{m_F(x_1), m_F(x_2), m_F(x_3), \dots., m_F(x_n), m_F(b)\}$$
$$\geq \text{T}\{m_F(x_1), m_F(x_2), m_F(x_3), \dots., m_F(x_n)\}$$
$$n_F((\psi(x^n) = n_F(x_1 \circ \psi(x_2) \circ \psi^2(x_3) \circ, \dots \dots, \circ \psi^{n-2}(x_n) \circ \text{b})$$
$$\geq \text{S}\{n_F(x_1), n_F\psi(x_2), n_F\psi^2(x_3), \dots, n_F\psi^{n-2}(x_n), n_F(b))\}$$

$\geq$ S$\{n_F(x_1), n_F(x_2), n_F(x_3), \ldots., n_F(x_n), n_F(b)\}$
$\geq$ T$\{n_F(x_1), n_F(x_2), n_F(x_3), \ldots., n_F(x_n)$

We have $x = (\psi(x) \circ \psi^2(x) \circ, \ldots \circ \psi^{n-2}(x) \circ b)^{-1}$

Thus, $m_F(x) = m_F(\psi(x) \circ \psi^2(x) \circ, \ldots \circ \psi^{n-2}(x) \circ b)^{-1}$
$\geq T\{m_F(\psi(x) m_F \psi^2(x), \ldots . m_F \psi^{n-2}(x) \circ b)^{-1}$
$\geq T\{m_F(x), m_F(b)\}$
$= m_F(x)$

Also, $n_F(x) = n_F(\psi(x) \circ \psi^2(x) \circ, \ldots \circ \psi^{n-2}(x) \circ b)^{-1}$
$\geq S\{n_F(\psi(x) m_F \psi^2(x), \ldots . m_F \psi^{n-2}(x) \circ b)$
$\geq S\{n_F(x), n_F(b)\}$
$= n_F(x)$

Hence the proof.

3.13 Corollary: If $(G, \emptyset)$ is a ternary group structure, then any EFFSG of $\theta_a(G, \varphi)$ is an EFFSTG of (G, φ).

Proof:

Since a is a neutral elements of a group $\theta_a(G, \varphi)$ then $m_F(a) \geq m_F(x)$ and $n_F(a) \leq n_F(x) \forall x \in G$. Thus $m_F(a) \geq m_F(x)$ and $n_F(a) \leq n_F(x)$. But in ternary group a=a for any $a \in G, where\ \ m_F(a) = m_F(x) \geq$ $\mathrm{m_F(a) \geq m_F(x)}$ and $n_F(a) = n_F(a) \leq n_F(x)$. So, $m_F(a) = m_F(a) \geq m_F(x)$ and $n_F(a) = n_F(a) \leq n_F(x). \forall x \in G$. This means that the assumptions of the theorem(3.15) are satisfied.

3.14 Example: Consider the ternary group $Z_{12}, \varphi)$, where $\varphi: Z^3 \rightarrow Z_{12}$ is defined by $\varphi(x_1, x_2, x_3) =$ $\max(x_1, x_2, x_3)$, dervied from the addition group Z_{12}. Let F be an EFFSG of $\theta_1\varphi(G, \varphi)$ induced by subgroups $S_1 = \{1\}, S_2 = \{5,11\},\ S_3 = \{1,3,5,7,9,11\}$. Define EFFSF as follows

$$(m_F(x)) = \begin{cases} 0.8\ if\ x = 11 \\ 0.6\ if\ x = 5 \\ 0.4\ if\ x = 1,3,7,9\ if\ x \in S_3 \end{cases}$$

and

$$(n_F(x)) = \begin{cases} 0.1\ if\ x = 11 \\ 0.2\ \ if\ x = 5 \\ 0.5\ \ if\ x = 1,3,7,9 \\ 0.9\ \ if\ x \in S_3 \end{cases}$$

Then $m_F(5) = m_F(7) = 0.4 \ngtr 0.6 = m_F(5), n_F(5) = n_F(7) = 0.5 \nless 0.3 = n_F(5)$. Hence F is not an EGGSGTG of (G, φ).

3.15 Theorem: Let (G, φ) be an extension group structure of l –dervied from the group$(G,\circ)$,any EFFSGF of $(G,\circ)$ such that $m_F(l) \geq m_F(x)$ and $n_F(l) \leq n_F(x)$ for every $x \in G$ is an EFFSG of $(G,\circ)$.

Proof:

The condition (EFFSG_1) $and\ (EFFSG_2)$ of Definition 3.1 are obvious. To prove (EFFSG_3) $and\ (EFFFSG_4)$, we have an extension group structure $(G, \varphi)\ l$ –from the group $(G,\circ)$ which implies $x = \left(x^{n-2} \circ l\right)^{-1}$, Where x^{n-2} is the power of x in $(G,\circ)$. Thus, for all $x \in G$.

$$m_F(x) = m_F((x^{n-2} \circ l)$$
$$\geq m_F((x^{n-2} \circ l)$$
$$\geq T\{m_F(x^{n-2}), m_F(l)\}$$
$$= m_F(x)$$
$$n_F(x) = n_F((x^{n-2} \circ l)$$
$$\geq n_F((x^{n-2} \circ l)$$
$$\geq S\{n_F(x^{n-2}), n_F(l)\}$$
$$= n_F(x)$$

Hence the proof.

3.16 Theorem: Any FFSSG of a group, $(G,\circ)$ is an EFFSSG of an group extension (G,φ) dervied from $(G,\circ)$.

Proof:
If an extension group structure (G,φ) is derived from the group $(G,\circ)$, then $l = e$. Thus $m_F(e) \geq m_F(x) and\ n_F(e) \leq n_F(x) \forall x \in G$.

Observations:

From the example (3.16) it follows that:
1.There are EFSSG of $\theta_a(G,\varphi)$ which are not EFFSSG of (G,φ).
2.If theorem(3.15), the assumption $m_F(a) \geq m_F(x) and n_F(a) \leq n_F(x)$ can not be omitted. In example(3.14), we have $m_F(1) = 0.4 \ngtr 0.6 = m_F(5) and\ n_F(1) = 0.5 \nless 0.3 = m_F(5)$.
3.The assumption $m_F(a) \geq m_F(x)\ and\ n_F(a) \leq n_F(x)$ can not be replaced by the natural assumption $m_F(a) \geq m_F(x) and n_F(a) \leq n_F(x)$.
In example(3.16), T=11, then $m_F(11) \geq m_F(x)\ and\ n_F(11) \leq n_F(11) \forall z \in Z_{12}$.

Conclusion:

The concept of an extension fermatean fuzzy soft subgroup(EFFSG) and have studied their related properties. we study the level subset of a fermatean fuzzy soft set and how they are acted in extended group structure. One can obtain the extension structures based Neutrosophic soft Environment and Pythagorean fuzzy soft sets.

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