

# Graphic Methods Of Image And Mathematical Description Of Lobe Closed Helical Surfaces

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**Annotation:** The article proposes geometrical and mathematical models of petal closed helical surfaces, to allow a method for constructing a torus mesh for performing images of a given model according to specified parameters and to find characteristic key determinants that make it possible to design flexible links of various mechanisms in CAD.

**Keywords:** a closed helical surface, an enveloping torus, a petal, an open torus, a torus with a pinhole, sphere, parallels, meridians, cell, right turn, natural value. characteristic drawing elements, closed curve, Mobius strip.

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## Introduction

One of the most important directions in the development of scientific research in the field of natural and technical sciences is the expansion of theoretical and applied research, in particular, applied mathematics, mechanical engineering and machine parts, aimed at improving and efficiently using material resources in the national economy.

## Literature review

A number of scientists in foreign countries and in our country have conducted research on the surface and its structure, including G.Monj, M.Ya.Gromov, S.V.Mikhaylenko, E.E.Manasheroov, A.I.Volkov, V.I. Yakunin, D.Kuchkarova, Sh.Murodov, B.Khaitov, D.Achilova, A.Khamrakulov, A.Kakhkharov and others gave scientifically based recommendations and suggestions on teaching methods.

## Analysis and results

A significant acceleration in the development of mechanical engineering is facilitated by the creation of more efficient technologies that reduce material consumption and energy consumption. Some of the main characteristics of machines are: productivity, efficiency, weight, dimensions and cost [1]. It is especially important, without changing the weight, overall dimensions and cost of machines, to increase productivity several times, as well as the coefficient of efficiency. These characteristics can be achieved when designing some parts according to the given parameters using closed helical surfaces instead of the existing cylindrical and conical surfaces. In engineering practice, one of the most important tasks is the task of obtaining a

mathematical model of a technical surface. This task becomes much more complicated if the technical surface has a complex spatial configuration. Mathematical models with the derivation of equations for surfaces with a developed form make it possible to obtain the necessary information for the engineering calculation of parts from the compartments of such surfaces on a computer.

Relevant for applied geometry is the applied graph problem - the analytical description of closed helical surfaces according to given conditions.

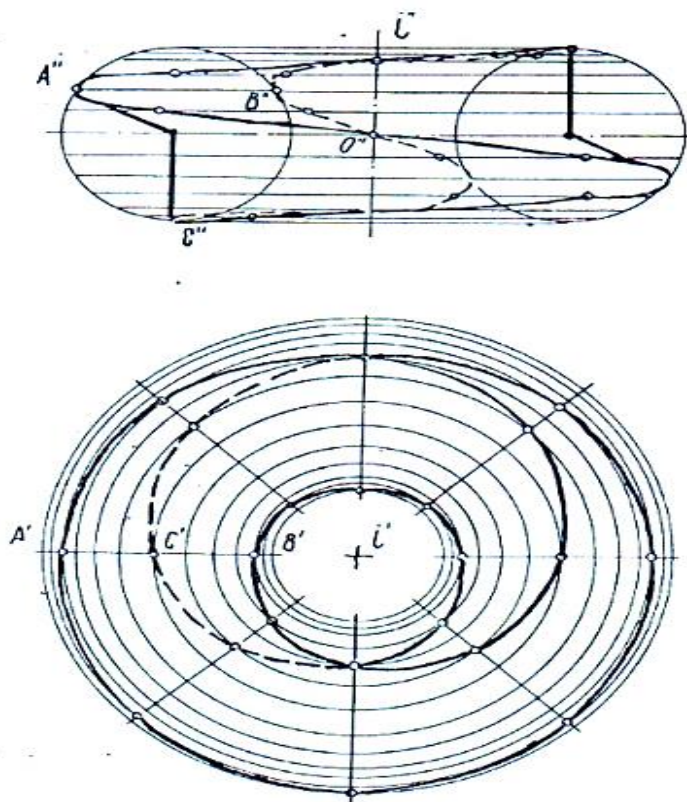
When performing design work in which petal closed helical surfaces are used, it is necessary to know the rules for graphical execution of these surfaces. On the basis of their step-by-step implementation, the geometry of the step-by-step manufacture of their volumetric models from various materials will be determined [2].

We list those characteristic determinants, in the presence of which it is possible to graphically execute the VIZ:

1. Number  $\kappa = P/m$ ,  $\kappa$  - the ratio of the number of parallels  $P$  to the number of meridians  $m$ , which make it possible to obtain an image of the edge (or edges) of the VEP on the surface of the enclosing torus;
2. VIZ type: right or left;
3. Type of VEP: petal, prismatic or other form of meridional section;
4. The type of the enclosing torus, into which the VEP is inscribed: an open torus, a torus with a pinhole or a sphere.

Let us dwell on examples in which specific arithmetic quantities appear in the generalized formulation of the condition of the problem for the graphical construction of VEP.

Figure 1. A drawing of the petal VEP of the right stroke is shown (for  $\kappa=3/1$ ), inscribed in an open torus, in which  $D=55\text{mm}$ ;  $d=30\text{mm}$ .



**Fig 1**

The algorithm for the process of image of this surface in the drawing, consisting, for example, of frontal and horizontal projections, can be written as follows:

Draw the torus according to the given dimensions. In this case, the most traditional position of the torus in relation to the main direction of orthogonal projection is assumed to be when the axis is horizontal.

We build a grid consisting of  $P$  parallels and  $m$  meridians ( $P = 3, m = 1$ ), choose one of the torus meridians, and divide it into 3 equal parts, draw parallels through each division mark. Of course, such a grid does not allow obtaining the exact edges of the VEP, so the number  $k$  must be multiplied by another number. Figure 1. accepted  $m = 8$ , as a result  $K_1 = 3 \cdot 8 / 1 \cdot 8 = 24 / 8$ . So we are dealing with a torus grid consisting of 24 parallels and 8 meridians.

In this case, we begin the construction of the meridians and parallels from a common point, which it is desirable to place on the characteristic elements of the drawing.

3. In the projection in which the equator of the torus is depicted in full size, the edge of the WEP is depicted as a successive row of diagonals of the cells of the torus grid. As you know, there are two such diagonals. Then, that the diagonal on the visible part of the torus goes from the large circle to the small ones, then this will be a link of the edge of the WEP of the right move. For our example, we will choose this option. If the movement starts from point A, and the sought VEP has only one edge, then it ends after three

revolutions at the same point. Considering this fact and having verified its correctness performed on the drawing, we obtain a horizontal projection of the SGP edge.

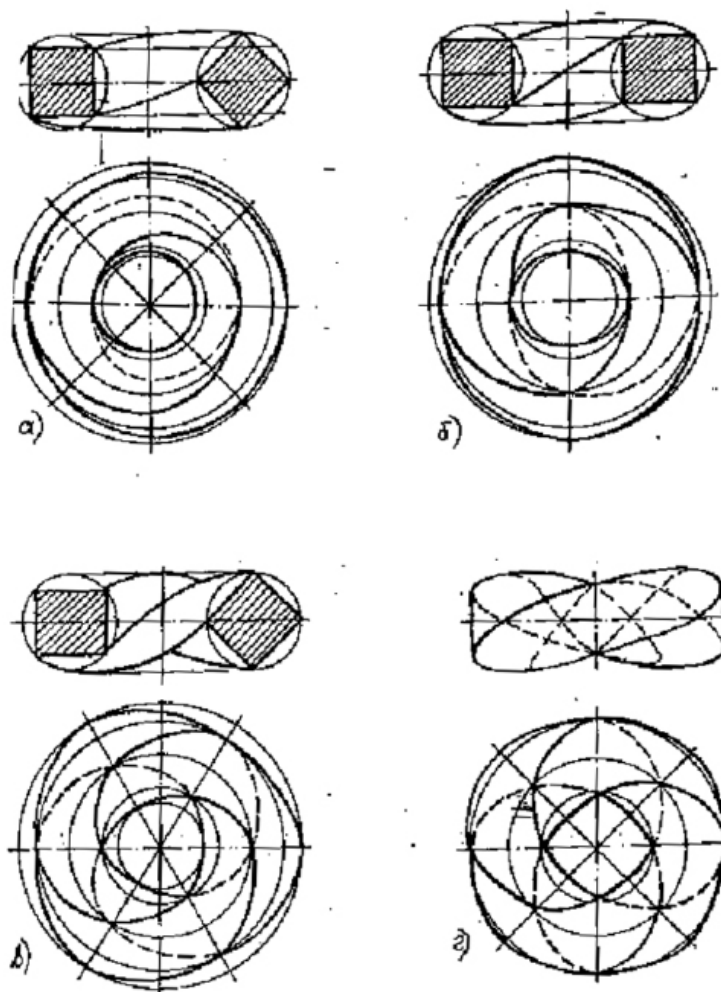


Fig. 2

4. We find the frontal projection of each working vertex of the torus mesh, and then, sequentially connecting them, we obtain the frontal projection of the edge of the SGP.

5. We form a surface by drawing a common edge, which the petal VEP is a circle that coincides with the circle of the torus axis. Distinguish between visible and invisible areas of the rib projection. If necessary, not petal, but prismatic VEP, all these rules remain in force, with the exception of paragraph 5.

Figure 2. the drawings of the prismatic VEP of the left move are shown, in which the enclosing torus is characterized as  $D=40$  мм  $d=20$  мм. The following options are given: а)  $\kappa = P/m=4/1$ ; б)  $\kappa = P/m = 4/2$ ; в)  $\kappa = P/m = 4/3$ ;

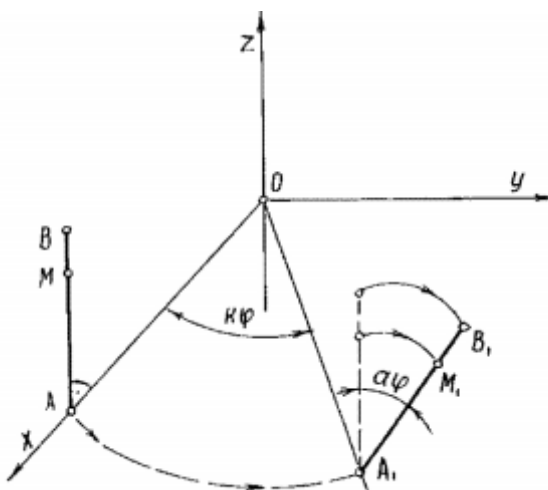
г)  $\kappa = P/m = 4/4$ . This means for the prismatic SGP shown in Fig. 2. а, б, the number of ribs is one; in Fig. 2. в, the number of ribs is two in Figure 2.5. д, the number of ribs 4.

Consider mathematical models of closed lobe helical surfaces. The generatrices of the petal closed helical surfaces are the segments of the bundle of straight lines. In some cases, from one generatrix it is

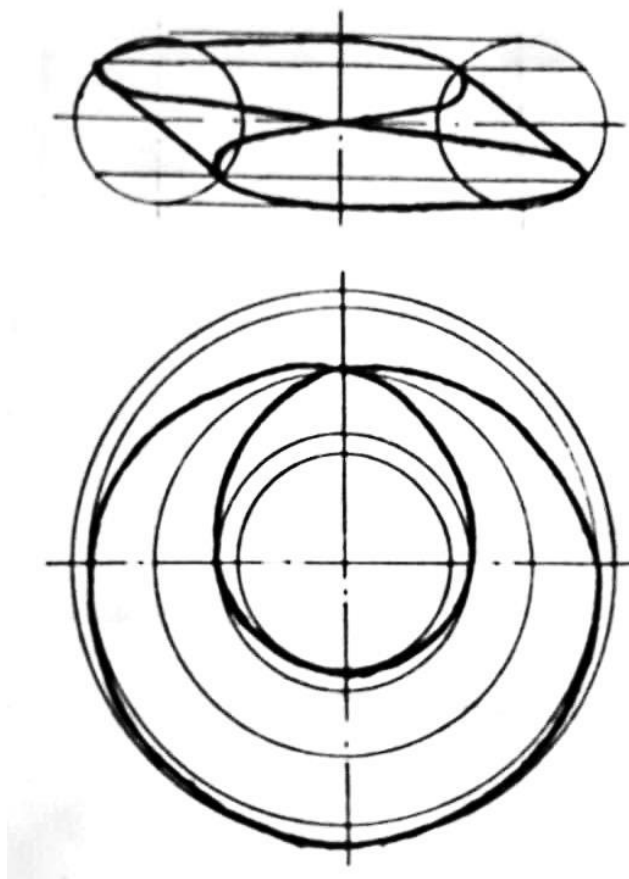
possible to construct a self-intersecting petal closed helical surfaces, in which the meridional section also has several bundles of straight lines. They are generators of this surface [3].

In space, the straight line segment AB (Figure 2.1) moves so that point A describes a circle, point B moves along a closed curve - the geodesic line of an open torus. Point A makes two complete revolutions around the OZ axis, point B passes along a closed curve and takes its original position ... Rotating, point B makes one complete revolution around point A. As a result, a two-lobed closed surface is obtained, having only one side and one edge. The kinematic formation of a multi-lobe VEP includes

**Fig 3.**



i have a number of options. One of the options - the two-lobed VEP actually represents the well-known model of the Möbius strip in Fig. 4, the Möbius strip is given in a torus mesh.



**Fig 4.**

In the second variant, we have the case with self-intersection of the Möbius strip. Which is an example of obtaining a four-lobed SGP.

Using the kinematic method of surface formation, we will try to compose its parametric equations in the XYZ Cartesian coordinate system for the case of a two-lobe model, i.e. Mobius sheet [4].

The generatrix AB with point A is located on the OX axis, parallel to the OZ axis and at a distance R from the origin. The position of point A is determined by turning it through an angle  $2\phi$  relative to the OZ axis, and the position of the segments  $A_1B_1$  - the same turning angle  $2\phi$  relative to the XOY plane. Then for any point  $M(x,y,z)$  located on the segment AB at a distance t from point A (which can be changed), the equation will have the form:

$$\left. \begin{aligned} x &= R \cos 2\phi + t \cos \phi \cdot \cos 2\phi \\ y &= R \sin 2\phi + t \cos \phi \cdot \sin 2\phi \\ z &= t \sin \phi \end{aligned} \right\} \quad (2.1)$$

Hence:

$$\left. \begin{aligned} x &= (R + t \cos \phi) \cos 2\phi \\ y &= (R + t \cos \phi) \sin 2\phi \\ z &= t \sin \phi \end{aligned} \right\} \quad (2.2)$$

Here  $R \geq 0$ ;  $0 \leq t \leq d$ ;  $0 \leq \phi \leq 2\pi$ ;  $AB=d$ .

Let us find the equation of the trace of the surface of the petal closed helical surfaces on the surface  $z=h$ .

petal closed helical surfaces

From the third equation of system (2.2) we have:

$$t = \frac{h}{\sin\phi}$$

Substituting the value of  $t$  in the first two equations, we get

$$\left. \begin{aligned} x &= \left(R + \frac{h}{\sin\phi} \cos\phi\right) \cos 2\phi \\ y &= \left(R + \frac{h}{\sin\phi} \cos\phi\right) \sin 2\phi \end{aligned} \right\} \quad (2.3)$$

Further

$$\left. \begin{aligned} x &= (R + h \operatorname{ctg}\phi) \cos 2\phi \\ y &= (R + h \operatorname{ctg}\phi) \sin 2\phi \end{aligned} \right\} \quad (2.4)$$

After simple mathematical transformations, we obtain the equation of the trace of the surface of the petal closed helical surfaces;

$$x^2 + y^2 = (R + h \operatorname{ctg}\phi)^2. \quad (2.5)$$

or polar coordinates ;

$$\rho = R + h \operatorname{ctg}\phi. \quad (2.6)$$

## CONCLUSION

The paper deals with the issues of geometric and mathematical modeling of closed lobe helical surfaces. In connection with the tasks set, the following results were obtained.

1. Based on the analysis of methods for constructing lobe closed helical surfaces, it is proposed to minimize the volume of geometric constructions, which makes it possible to simplify mathematical modeling.
2. A universal toric mesh is proposed for obtaining petal closed helical surfaces, which makes it possible to accurately construct drawings and models of petal closed helical surfaces

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