Nat. Volatiles & Essent. Oils, 2021; 8(5): 2632 - 2646

NVEO
Natural Volatiles & Essential Oils

Static And Dynamic Analysis Of Functionally Graded Square And Rectangular Plates Subjected To Transverse Loading

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Abstract

The objective of this paper is to study the static and dynamic behavior of square, rectangular plates made up of Functionally Graded Material (FGM) subjected to transverse loading by Classical Plate Theory (CPT) and First Order Shear Deformation Theory (FSDT).Simply supported square and rectangular FGM plates consisting of ceramic (Alumina) at the top layer and metal (Aluminium) at the bottom layer subjected to a uniform pressure and mid-point load on the top surface of the plates were considered. Assuming the Poisson's ratios to be constant for ceramic and metal. However the Young's moduli of ceramic and metals change continuously throughout the thickness direction according to the volume fraction of the constituents materials on the basis of powerlaw. The solutions of the square and rectangular FGM plates were obtained by expanding the transverse load in terms of Fourier series expansion and then the sinusoidal terms of the Fourier series were solved by Navier's solution technique. Then, the solutions of the square and rectangular FGM plates obtained by CPT and FSDT were compared with the numerical results of finite element model which is developed using ANSYS Parametric Design Language (APDL). The parametric study includes the effect of volume fractions, aspect ratios, various thickness, support conditions, loading conditions on static and dynamic parameters. The static parameters considered were non-dimensional mid-plane displacements, non-dimensional stresses and strains where the dynamic parameter includes non-dimensional natural frequencies.

Keywords— *Functionally graded plates, CPT, FSDT, poisson's ratios , young's moduli , power law , ANSYS, static and dynamic parameters.*

I. INTRODUCTION

Functionally graded materials are new type of materials, where the volume fractions of two or more materials are varied continuously as a function of position along certain dimensions of the structure in order to achieve required functions. The composition of the

FGM plate is varied from ceramic-rich surface to metal-rich surface, with desired variation of volume fractions of the two materials in between two surfaces. FGM has the ability to control corrosion, wear and buckling. Ceramic constituent has low thermal conductivity as it provides high-temperature resistance when place at top of the material. Thus, the metal component on the other side of material prevents fracture due to thermal stresses. The effective material properties of FGM are assumed as temperature independent and gradually varying in the plate thickness direction for the plate structures.

Shyang-Ho Chi [4] used the power law, sigmoidal and exponential functionby CPT to investigate the rectangular elastic simply supported FGM plate having medium thickness and by varying the material properties in thickness direction when subjected to transverse loading.

Huu-Tai Thai and Dong-Ho Choi [2]presented the bending and free vibration analysis of FGM plates by FSDT. The equations of motion and boundary condition is derived by using Hamilton's principle.

II. CLASSICAL PLATE THEORY

A. *Introduction*

Based on the assumptions proposed by Kirchhoff, In 1888, Love developed the Classical plate theory (CPT), which is an extension of the Euler-Bernoulli beam theory. The expressions for stresses and strains of the FGM plates were derived depend on the following assumptions.

- Before and after deformations, the line elements perpendicular to middle surface of the plate remain normal and unstretched.
- The linear strain displacement relations are valid, because comparing to the thickness, the deflection of the FGM plate is very small.
- The thickness of FGM plate is assumed to be 1/20 to 1/100 of its span, which is very small. Hence the normal stress along the thickness direction has no effect on in-plane strains and it can be neglected.
- Young's modulus, Density and Poisson's ratio are functions of the spatial coordinate z.

B. *Effective Material Properties*

Consider an elastic rectangular plate as shown in Fig. 1. The plane of the plate is defined by the coordinates x and y and the z-axis originated at the middle surface of the plate is in the thickness direction. The effect of Poisson's ratio on the deformation is much less when compared to Young's modulus. Hence, in the FGM plates, with the use of power law functions, Young's moduli in the thickness direction isvaried.Based on power law, the variation through the thickness of material properties is given by,

$$g(z) = \left\{\frac{z+h/2}{h}\right\}p$$

where p defines the material variation profile and h represents the thickness of the plate.

The material property of power law based FGM plate is given by, E(z) = g(z)E1 + [1-g(z)]E2

whereE1 = Young's modulus of the bottom surface (z = - h/2)of the FGM plate,

E2 =Young's modulus of top surface (z = h/2) of the FGM plate.

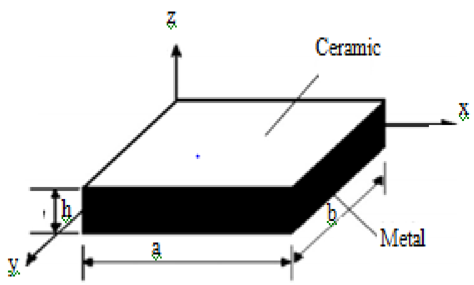

Fig.1 FGM Plate geometry

C. Static Analysis

As per the first assumption, after deformation a point A in the FGM plate will move to point $A_o$with a distance of z to the middle surfaceas shown in Fig.2. Accordingly, the transverse strain components ε_z,γ_{xz}, and γ_{yz} are trivially small [1].

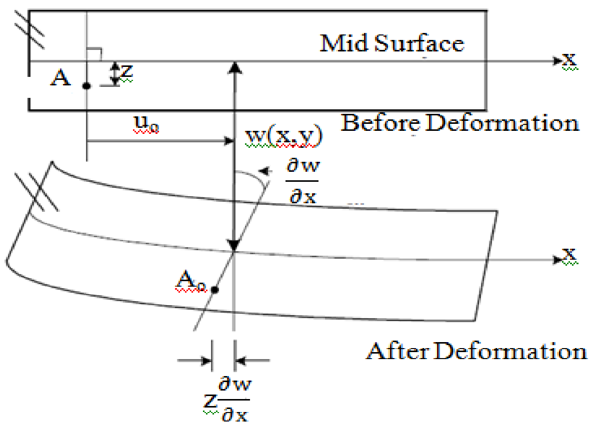

Fig.2 Deformed Configuration of FGM Plate Based on CPT

The in-plane displacements u,v and w at the generic point in the FGM plate can be expressed in the following way,

$$u(x,y,z) = u_o(x,y) - z\frac{\partial w}{\partial x}$$

$$v(x,y,z) = v_o(x,y) - z\frac{\partial w}{\partial y}$$

$$w(x,y,z) = w_o(x,y)$$

where$u_o(x,y)$, $v_o(x,y)$ and $w_o(x,y)$ are the mid-plane displacements in x, y, and z directions respectively. The strain field of the plate is defined based on the assumptions of small deformation and is given as,

$$\varepsilon_x = \frac{\partial u}{\partial x} = \varepsilon_{xo} - z\frac{\partial^2 w}{\partial x^2}$$

$\varepsilon_y = \frac{\partial v}{\partial y} = \varepsilon_{yo} - z \frac{\partial^2 w}{\partial y^2}$

$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \gamma_{xyo} - 2z \frac{\partial^2 w}{\partial x \, \partial y}$

$\varepsilon_z = \gamma_{xz} = \gamma_{yz} = 0$

By considering the third and fourth assumptions,for the plane stress problem the stress–strain relationship is given by,

$\sigma_x = \frac{E(z)}{1-\mu(z)^2}\{\varepsilon_x + \mu(z)\, \varepsilon_y\}$

$\sigma_y = \frac{E(z)}{1-\mu(z)^2}\{\varepsilon_y + \mu(z)\, \varepsilon_x\}$

$\tau_{xy} = \frac{E(z)}{2(1+\mu(z))}\{\gamma_{xyo} - 2z \frac{\partial^2 w}{\partial x \, \partial y}\}$

Assuming that q_x, $q_y$and $q_z$be the distributed load in FGM platesalong the three directions. By considering a small solid element havingthe dimensions dx, dy and dz. The equilibrium equation in terms of bending momentsfor FGM plate is given by,

$\frac{\partial^2 Mx}{\partial x^2} + 2 \frac{\partial^2 Mxy}{\partial x \, \partial y} + \frac{\partial^2 My}{\partial y^2} = - q_z\,(x,y)$

The governing equation for stress function is represented by using the compatibility equation.

$\frac{\partial^2 \varepsilon xo}{\partial y^2} + \frac{\partial^2 \varepsilon yo}{\partial x^2} = \frac{\partial^2 \gamma xyo}{\partial x \, \partial y}$

Consider an FGM plate of length=a, width=b and uniform thickness=h , which is subjected to the lateral load $q_z(x,y)$.By Fourier series, the above load is expanded as,

$q_z(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} qmn \, \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$

where,$q_{mn} = \frac{4}{ab} \int_0^a \int_0^b qz\,(x,y) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} dx\, dy$

If the uniformly distributed load (q_z (x,y)) is acting on the top surface of the FGM plate, then

$q_{mn} = \frac{16q}{mn\pi^2}$for m and n = 1,3,5,7,…..

q_{mn} =0for m and n = 2,4,6,8,…..

Suppose,the point load (P) is acting at x = u, y = v, then

$q_{mn} = \frac{4P}{ab} \sin \frac{m\pi u}{a} \sin \frac{n\pi v}{b}$ for m,n = 1,3,…

q_{mn} =0 for m,n = 2,4,...

For simply supported plate, the boundary conditions are given by,

@x =0 to a, w=0 ;$M_x = \frac{\partial^2 w}{\partial x^2} = 0$

@y =0 to b, w=0 ; $M_y = \frac{\partial^2 w}{\partial y^2} = 0$

So as to satisfy the loading conditions and boundary conditions, the displacement function ‘w’ is in the form of,

$w\,(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} qmn \, \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$

where$w_{mn} = \frac{qmn}{D\,(z)[(\frac{m\pi}{a})^2\,(\frac{n\pi}{b})^2]^2}$

$$w(x,y)=\frac{1}{D(z)}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{qmn}{[\left(\frac{m\pi}{a}\right)^2+(\frac{n\pi}{b})^2]^2}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

The above solution is the Navier's solution for square and rectangular FGM plate which is simply supported.

The strain fields of FGM plates are given by,

$$\varepsilon_x=\frac{z+Q11}{D(z)}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{qmn(\frac{m\pi}{a})^2}{[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2]^2}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$\varepsilon_y=\frac{z+Q11}{D(z)}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{qmn(\frac{n\pi}{b})^2}{[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2]^2}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$\gamma_{xy}=\frac{-2(z+Q11)}{D(z)}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{qmn(\frac{m\pi}{a})(\frac{n\pi}{b})}{[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2]^2}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}$$

The stress fields of FGM plates are given by,

$$\sigma_x=\frac{12(z+Q11)}{h^3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{qmn[\left(\frac{m\pi}{a}\right)^2+\mu(\frac{n\pi}{b})^2]}{[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2]^2}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$\sigma_y=\frac{12(z+Q11)}{h^3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{qmn[\left(\frac{n\pi}{b}\right)^2+\mu\left(\frac{m\pi}{a}\right)^2]}{[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2]^2}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$\tau_{xy}=\frac{-12(z+Q11)}{h^3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{(1-\mu)qmn(\frac{m\pi}{a})(\frac{n\pi}{b})}{[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2]^2}\cos\frac{m\pi x}{a}\cos\frac{\mathbf{n\pi y}}{\mathbf{b}}$$

D. *Dynamic Analysis*

The propagation of waves in the thin plate is determined by the dynamic theory. The mathematical model of continuous elastic dynamic plates can be done, either based on Newton's laws by using partial differential equations or by considering virtual work using integral equations. According to D'Alembert's principle, the forcing function of the governing differential equation for the plate becomes,

$$D(z)[\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}]+\rho(z)h\frac{\partial^2 w}{\partial t^2}(x,y,t)=0$$

For a freely vibrating plate, the natural frequencies may be determined by assuming the displacement function as,

$$w(x,y,t)=(A\cos\omega t+B\sin\omega t)W(x,y)$$

This is a separable solution of the shape function W(x,y) for modes of the vibration. ω is the natural frequency of the plate vibration. The natural frequency and vibration period T are related by $\omega = {}^{2\pi}/_{T}$.

By substituting the shape function in the governing differential equation, we have

$$\{D(z)[\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}]-\rho(z)h\omega^2\}W(x,y)=0$$

The shape function for a simply supported rectangular FGM plate may be taken as,

$$W(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}Cmn\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

where a,b are the plate dimensions and C_{mn} is the amplitude of vibration for all values of m and n. Then the governing differential equation is modified as,

$$D(z)[\frac{m^4\pi^4}{a^4}+2\frac{m^2\pi^2}{a^2}\frac{n^2\pi^2}{b^2}+\frac{n^4\pi^4}{b^4}]-\omega^2\rho(z)h=0$$

For the square and rectangular FGM plates, the natural frequency(ω) is obtained by solving the above equation,

$$\omega = \sqrt{\frac{D(z)}{\rho(z)h}}\left(\pi^2\{\frac{m^2}{a^2}+\frac{n^2}{b^2}\}\right)$$

Then the non-dimensional natural frequency($\overline{\omega}$) is given by,

$$\overline{\omega}=\omega\left(\frac{a^2}{h}\right)\sqrt{\frac{\rho 1}{E1}}$$

where$\rho_1$and $E_1$are the density and Young's modulus of the metal respectively.

III. FIRST ORDER SHEAR DEFORMATIONTHEORY

A. *Introduction*

First Order Shear Deformation Theory (FSDT) is the simplest plate theory which describes constant transverse shear strains through the plate thickness. The shear correction coefficients are required for calculating the transverse shear force. It basically depends on the following assumptions,

- The plate deflections are small.
- Plane sections which are perpendicular to middle plane of the plate remains plane but not necessarily normal to the middle plane.
- Stresses normal to the mid-surface of plates are negligible.

B. *Static Analysis*

The FGM plate displacement field is shown in Fig.3. The displacement field can be expressed by including the effect of transverse shear deformations as,

$$u(x,y,z) = u_o(x,y) - z\,\psi_x(x,y)$$
$$v(x,y,z) = v_o(x,y) - z\,\psi_y(x,y)$$
$$w(x,y,z) = w_o(x,y)$$

whereψ_x, ψ_y are the line element rotations about x axes and y axes.

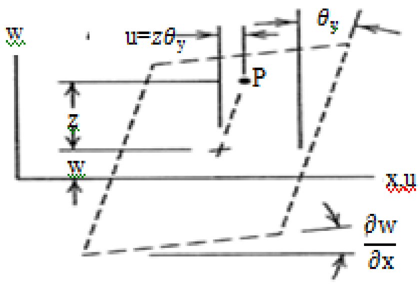

Fig.3 FGM Plate Deformed Configuration Based on FSDT

The strain–displacement equations are given by,

$$\varepsilon_x = \frac{\partial u}{\partial x} = \varepsilon_{xo} - z\,\frac{\partial^2 w}{\partial x^2}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} = \varepsilon_{yo} - z\,\frac{\partial^2 w}{\partial y^2}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \gamma_{xyo} - 2z\,\frac{\partial^2 w}{\partial x\,\partial y}$$

$$\varepsilon_z = 0$$

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \psi_x + \frac{\partial w}{\partial x}$$

$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \psi_y + \frac{\partial w}{\partial y}$

The governing differential equations of FSDT are got from the principle of virtual work. The expression for the virtual work done due to internal forces by considering the transverse shear stresses may be given as,

$\delta w_i = -\int_V^{\cdot}(\sigma_x . \delta\varepsilon_x + \sigma_y . \delta\varepsilon_y + \tau_{xy} . \delta\gamma_{xy} + \tau_{xy} . \delta\gamma_{xy} + \tau_{yz} . \delta\gamma_{yz} + \tau_{xz} . \delta\gamma_{xz})dV$

$\delta w_i = -\int_{-h/2}^{h/2}[\iint_R^{\cdot}((\sigma_x . \delta\varepsilon_x + \sigma_y . \delta\varepsilon_y + \tau_{xy} . \delta\gamma_{xy} + \tau_{xy}\delta\gamma_{xy} + \tau_{yz} . \delta\gamma_{yz} + \tau_{xz} . \delta\gamma_{xz})dx.dy]dz$

V=volume of the plate

R= Middle surface of the plate

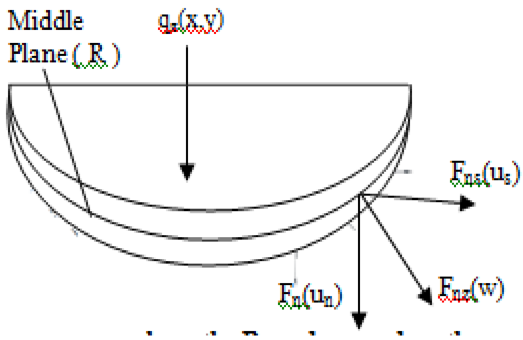

Fig.4 Forces along the Boundary and on the Surface of the Plate

To find the virtual work due to external forces, the plate with arbitrary geometry having curved boundary may be considered as shown in Fig.4.Virtual work due to external forces,

$\delta w_e = -\int_{-h/2}^{h/2}[\int_C^{\cdot}((F_n . \delta u_n + F_{ns} . \delta u_s + F_{nz} . \delta w_o)ds] dz + \iint_R^{\cdot}(q . \delta w_o)dx.dy$

$q_z(x,y)$ – lateral load per unıt area

F_n- Normal force component per unit area

F_{ns}, F_{nz} - Tangential force components along x and y directions per unit area respectively

u_n, u_s and w- Normal, tangential and vertical deflections along the respective directions.

By substituting δw_iandδw_ein the virtual work equation, $\delta w_i + \delta w_e = 0$ and equating the terms independently to zero, the governing differential equations for FSDT are obtained as,

$$\frac{\partial^2\psi x}{\partial x^2}+\left(\frac{1-\mu}{2}\right)\frac{\partial^2\psi x}{\partial y^2}+\left(\frac{1+\mu}{2}\right)\frac{\partial^2\psi y}{\partial x\partial y}-\frac{6(1-\mu)k^2}{h^2}\left(\psi_x+\frac{\partial w}{\partial x}\right)=0$$

$$\frac{\partial^2\psi y}{\partial y^2}+\left(\frac{1-\mu}{2}\right)\frac{\partial^2\psi y}{\partial y^2}+\left(\frac{1+\mu}{2}\right)\frac{\partial^2\psi x}{\partial x\partial y}-\frac{6(1-\mu)k^2}{h^2}\left(\psi_y+\frac{\partial w}{\partial y}\right)=0$$

$$G(z)hk^2\left[\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\left(\frac{\partial\psi x}{\partial x}+\frac{\partial\psi y}{\partial y}\right)\right]+q_z(x,y)=0$$

In order to satisfy loading and boundary condition, the displacement and rotation are in the form of

$$w(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} wmn \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$\psi_x=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Amn \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$\psi_y=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Bmn \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

The value of w_{mn} is obtained by solving the above equations.

$$w_{mn}=\frac{\{1+\frac{D(z)}{k^2G(z)h}\left(\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right)\}}{D\ (z)\left(\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right)^2}q_{mn}$$

Then the expression for deflection is given by,

$$w(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{qmn*K}{D(z)\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right]^2}\sin\frac{m\pi X}{a}\sin\frac{n\pi y}{b}$$

where $K=1+\frac{D(z)}{k^2G(z)h}\left(\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right)$

The strain fields of FGM plates are given by,

$$\varepsilon_x=\frac{z+Q11}{D(z)}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{qmn\left(\frac{m\pi}{a}\right)^2*K}{[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2]^2}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$\varepsilon_y=\frac{z+Q11}{D(z)}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{qmn\left(\frac{n\pi}{b}\right)^2*K}{[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2]^2}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$\gamma_{xy}=\frac{-2(z+Q11)}{D(z)}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{qmn(\frac{m\pi}{a})(\frac{n\pi}{b})*K}{[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2]^2}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}$$

The stress fields of FGM plates are given by,

$$\sigma_x=\frac{12(z+Q11)}{h^3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{qmn[\left(\frac{m\pi}{b}\right)^2+\mu\left(\frac{n\pi}{a}\right)^2]*K}{[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2]^2}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$\sigma_y=\frac{12(z+Q11)}{h^3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{qmn[\left(\frac{n\pi}{b}\right)^2+\mu\left(\frac{m\pi}{a}\right)^2]*K}{[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2]^2}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$\tau_{xy}=\frac{-12(z+Q11)}{h^3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{(1-\mu)qmn\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)*K}{[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2]^2}\cos\frac{m\pi x}{a}\cos\frac{\mathbf{n\pi y}}{\mathbf{b}}$$

where Q11 is the integration constant.

C. Dynamic Analysis

For thin plates vibrating at higher modes shear deformation effects are.The expression for kinetic energy can be written as,

$$KE=\int\int_R\int_{z=-h/2}^{h/2}\frac{\rho(z)}{2}[(\frac{\partial u}{\partial t})^2+(\frac{\partial v}{\partial t})^2+(\frac{\partial w}{\partial t})^2]dz.dx.dy$$

Total potential energy can be written as,

$$TPE=\int\int_R\int_{z=-h/2}^{h/2}\frac{1}{2}[\ \sigma_x.\varepsilon_x+\sigma_y.\varepsilon_y+\tau_{xy}.\gamma_{xy}+\tau_{xy}.\gamma_{xy}+\tau_{yz}.\gamma_{yz}+\tau_{xz}.\gamma_{xz}]dz.dx.dy-\int_0^a\int_0^b q\ .w\ dx.dy$$

By using Hamilton's principle [7],

$$\int_{t1}^{t2}\delta(KE-TPE)\ dt=0$$

$\delta =$ Variation of energy with respect to x and y only

$t_1$and t_2 =values of time variable at the start and end of the time interval.

The governing differential equations are given as,

$$-D(z)\ [\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}]\ W(x,y)\ +\frac{\rho h^3}{12}\nabla^2\omega^2\ W(x,y)+\rho(z)h\omega^2\ W\ (x,y)=0$$

$$\text{Natural frequency,}(\ \omega)\ =\ \frac{\sqrt{\frac{D(z)}{\rho(z)h}}\ (\ \pi^2\{\frac{m^2}{a^2}+\frac{n^2}{b^2}\})}{(\frac{h^2}{12}\left\{\frac{m^2}{a^2}+\frac{n^2}{b^2}\right\}+1)}$$

IV. FINITE ELEMENT METHOD

The numerical technique which is used to find out the approximate solutions in terms of partial differential equations for the boundary value problem is the Finite element analysis (FEA). FEA is applied in engineering as a computational tool in which a complex problem is subdivided into smaller and simpler parts called finite elements by the mesh generation technique.

A. *Finite element modeling of FGM plate using ANSYS 15.0*

The simply supported rectangular FGM plates subjected to lateral load was analyzed using the commercially available software ANSYS APDL [5].The element chosen for this analysis is shell 63, which is a four-nodded linear elastic structural shell having both bending and membrane capabilities.

B. *ANSYS Model*

The FGM square and rectangular plates along with its deformation profile are shown in Figures 5 and 6 respectively.

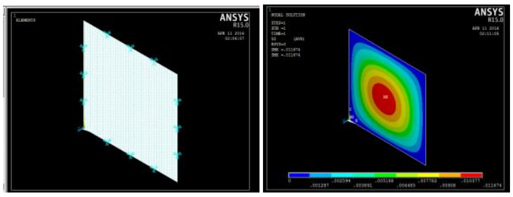

Fig.5FGM Square Plate and its Deformation Profile

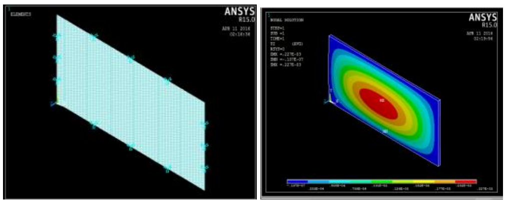

Fig. 6FGM Rectangular Plate and its Deformation Profile

V. RESULTS AND DISCUSSIONS

For the numerical calculations, FGM plate consisting of ceramic (Alumina) at the top layer and metal (Aluminium) at the bottom layer subjected to a uniform pressure of 100kN/m^2 and mid-point load of 100kN were considered. The constant Poisson's ratio of 0.3 was assumed for both ceramic and metal and the Young's modulus of the FGM plate bottom surface which consists of metal is 70GPa, whereas the top surface which consists of ceramic is 380GPa. The constant length of 1m was considered and the aspect ratio (i.e. length to width ratio) was varied as 1, 2 and 4 respectively. For all the above combinations, the thickness of the plate was also changed as 0.01m, 0.02m and 0.05m respectively. Throughout the analysis, the volume fraction (ratio of volume of metal to volume of ceramic) was varied from zero to two with an increment of 0.2. Then, the analytical solutions of the square and rectangular FGM plates obtained by CPT and FSDT were compared with the numerical results of APDL finite element model.

A. Simply Supported Square and Rectangular FGM Plate Subjected to UDL using CPT

The non-dimensional mid-plane displacement values for the FGM plates having various power law indices (p), aspect ratios (a/b) and thickness (h) were tabulated in Table 1.

TABLE. 1 NON-DIMENSIONAL MID-PLANE DISPLACEMENT(w/h)

	h=0.01m			**h=0.02m**			**h=0.05m**		
P	**a/b =1**	**a/b =2**	**a/b =4**	**a/b =1**	**a/b =2**	**a/b =4**	**a/b =1**	**a/b =2**	**a/b =4**
0	1.19	0.19	0.01	0.07	0.01	0.001	0.001	0.0003	0.00
0.2	1.33	0.21	0.01	0.08	0.01	0.001	0.002	0.0003	0.00
0.4	1.48	0.23	0.02	0.09	0.01	0.001	0.002	0.0003	0.00
0.6	1.65	0.26	0.02	0.10	0.01	0.001	0.002	0.0004	0.00
0.8	1.83	0.29	0.02	0.11	0.01	0.001	0.002	0.0004	0.00
1.0	2.01	0.32	0.02	0.12	0.02	0.001	0.003	0.0005	0.00
1.2	2.21	0.35	0.03	0.13	0.02	0.001	0.003	0.0005	0.00
1.4	2.42	0.38	0.03	0.15	0.02	0.002	0.003	0.0006	0.00
1.6	2.63	0.42	0.03	0.16	0.02	0.002	0.004	0.0006	0.00
1.8	2.85	0.45	0.03	0.17	0.02	0.002	0.004	0.0007	0.00
2.0	3.08	0.49	0.04	0.19	0.03	0.002	0.004	0.0007	0.00

It was noticed that, the mid-plane displacement increases with increase in volume fraction of the plate, because of increase in the metal content. Since stiffness of the metal is less compared to that of ceramic, increase in metal content leads to increase in deflection value. But the deflection decreases with increase in thickness of the plate. Irrespective of thickness of the plate the deflection decreases, when the aspect ratio increases. The results obtained from CPT, FSDT and ANSYS excellently agree with the results obtained from Navier-type three dimensional solution [6] for the FGM square plate with volume fraction (p=0) and thickness 0.01m. Hence, it was inferred that ceramic plates show lesser displacement values compared to that of metal plate. The comparison of CPT, FSDT and ANSYS results showed that the deviation of results were within 3%.

The non-dimensional normal stresses in x-direction and y-direction were tabulated in Tables 2 and 3 respectively.It was observed that fully ceramic plates give the smallest normal stress values. The normal stresses$\sigma_x$andσ_y increased with increase in the volume fraction but decreased with increase in aspect ratio and thickness of the plate. Since bending stiffness of the ceramic is more compared to metal, fully metallic plates give largest value of normal stress [5].As the plate becomes more and more metallic, the normal stress (σ_y) showed greater value than the corresponding value of normal stress (σ_x).

TABLE. 2NON-DIMENSIONAL NORMAL STRESS($\frac{\sigma x*h}{a*q}$) IN X-DIRECTION

	h=0.01m			h=0.02m			h=0.05m		
P	**a/b =1**	**a/b =2**	**a/b =4**	**a/b =1**	**a/b =2**	**a/b =4**	**a/b =1**	**a/b =2**	**a/b =4**
0	29.5	8.7	2.0	16.0	4.3	1.0	6.4	1.7	0.4
0.2	31.3	8.7	2.0	16.2	4.4	1.0	6.5	1.7	0.4
0.4	32.8	8.9	2.0	16.4	4.4	1.0	6.6	1.8	0.4
0.6	33.4	9.0	2.1	16.7	4.5	1.0	6.7	1.8	0.4
0.8	34.1	9.2	2.1	17.0	4.6	1.1	6.8	1.8	0.4
1.0	34.8	9.4	2.1	17.4	4.7	1.1	7.0	1.9	0.4
1.2	35.5	9.6	2.2	17.7	4.8	1.1	7.1	1.9	0.4
1.4	36.2	9.8	2.2	18.1	4.9	1.1	7.2	2.0	0.4
1.6	36.9	10.0	2.3	18.5	5.0	1.1	7.4	2.0	0.5
1.8	37.6	10.2	2.3	18.8	5.1	1.2	7.5	2.0	0.5
2.0	38.3	10.4	2.4	19.2	5.2	1.2	7.7	2.1	0.5

TABLE. 3NON-DIMENSIONAL NORMAL STRESS($\frac{\sigma y*h}{a*q}$) IN Y-DIRECTION

	h=0.01m			h=0.02m			h=0.05m		
P	**a/b =1**	**a/b =2**	**a/b =4**	**a/b =1**	**a/b =2**	**a/b =4**	**a/b =1**	**a/b =2**	**a/b =4**
0	29.5	17.0	5.6	16.0	8.5	2.8	6.4	3.4	1.1
0.2	31.3	17.1	5.6	16.2	8.5	2.8	6.5	3.4	1.1
0.4	32.8	17.4	5.7	16.4	8.7	+2.8	6.6	3.5	1.1
0.6	33.4	17.7	5.8	16.7	8.8	2.9	6.7	3.5	1.2
0.8	34.1	18.0	5.9	17.0	9.0	3.0	6.8	3.6	1.2
1.0	34.8	18.4	6.0	17.4	9.2	3.0	7.0	3.7	1.2
1.2	35.5	18.8	6.2	17.7	9.4	3.1	7.1	3.8	1.2
1.4	36.2	19.2	6.3	18.1	9.6	3.1	7.2	3.8	1.3
1.6	36.9	19.5	6.4	18.5	9.8	3.2	7.4	3.9	1.3
1.8	37.6	19.9	6.5	18.8	10.0	3.3	7.5	4.0	1.3
2.0	38.3	20.3	6.6	19.2	10.1	3.3	7.7	4.1	1.3

The non-dimensional shear stress variation for different values of volume fractions, aspect ratios and thickness was tabulated in Table4.

TABLE.4NON-DIMENSIONAL SHEAR STRESS($\frac{\tau xy*h}{a*q}$)

	h=0.01m			h=0.02m			h=0.05m		
P	**a/b =1**	**a/b =2**	**a/b**	**a/b =1**	**a/b =2**	**a/b =4**	**a/b =1**	**a/b =2**	**a/b**

			=4						=4
0	17.3	5.5	1.0	8.6	2.8	0.5	3.5	1.2	0.2
0.2	17.4	5.6	1.0	8.7	2.8	0.5	3.5	1.1	0.2
0.4	17.7	5.7	1.0	8.8	2.8	0.5	3.6	1.2	0.2
0.6	18.0	5.8	1.0	9.0	2.9	0.5	3.6	1.2	0.2
0.8	18.3	5.9	1.0	9.2	2.9	0.5	3.7	1.2	0.2
1.0	18.7	6.0	1.0	9.4	3.0	0.5	3.8	1.2	0.2
1.2	19.1	6.1	1.1	9.6	3.1	0.5	3.8	1.2	0.2
1.4	19.5	6.2	1.1	9.8	3.1	0.5	3.9	1.3	0.2
1.6	19.9	6.4	1.1	9.9	3.2	0.6	4.0	1.3	0.2
1.8	20.3	6.5	1.1	10.1	3.2	0.6	4.1	1.3	0.2
2.0	20.6	6.6	1.1	10.3	3.3	0.6	4.1	1.3	0.2

It was observed that smallest shear stress values occurs when the plate is of fully ceramic. The increase inshear stress increases with volume fraction and decreases with increase in aspect ratio and thickness because the FGM platestiffnessdecreases. Mostly ductile materials like aluminium fail in shear. Ductility increases by increasing the volume fraction, also increases the shear stress value.

The normal strain (ε_x and ε_y) variations in x and y direction and shear strain variation (γ_{xy}) were tabulated in Tables 5, 6 and 7 respectively.

TABLE. 5 NORMAL STRAINS(ε_x) IN X-DIRECTION

	h=0.01m			**h=0.02m**			**h=0.05m**		
P	**a/b =1**	**a/b =2**	**a/b =4**	**a/b =1**	**a/b =2**	**a/b =4**	**a/b =1**	**a/b =2**	**a/b =4**
	$*10^{-4}$	**$*10^{-5}$**	**$*10^{-6}$**	**$*10^{-4}$**	**$*10^{-5}$**	**$*10^{-6}$**	**$*10^{-5}$**	**$*10^{-6}$**	**$*10^{-7}$**
0	5.90	9.44	8.17	1.48	2.36	2.04	2.36	3.78	3.27
0.2	5.95	9.52	8.24	1.49	2.38	2.06	2.38	3.81	3.29
0.4	6.04	9.66	8.36	1.51	2.42	2.09	2.42	3.87	3.34
0.6	6.15	9.84	8.51	1.54	2.46	2.13	2.46	3.94	3.41
0.8	6.27	1.00	8.68	1.57	2.51	2.17	2.51	4.02	3.47
1.0	6.40	1.02	8.86	1.60	2.56	2.22	2.56	4.10	3.55
1.2	6.54	1.05	9.05	1.63	2.61	2.26	2.61	4.18	3.62
1.4	6.67	1.07	9.23	1.67	2.67	2.31	2.67	4.27	3.69
1.6	6.80	1.09	9.41	1.70	2.72	2.35	2.72	4.35	3.76
1.8	6.93	1.11	9.59	1.73	2.77	2.40	2.77	4.43	3.84
2.0	7.06	1.13	9.77	1.76	2.82	2.44	2.82	4.52	3.91

TABLE. 6NORMAL STRAIN (ε_y) IN Y-DIRECTION

	h=0.01m			**h=0.02m**			**h=0.05m**		
P	**a/b =1**	**a/b =2**	**a/b =4**	**a/b =1**	**a/b =2**	**a/b =4**	**a/b =1**	**a/b =2**	**a/b =4**

	$*10^{-4}$	$*10^{-4}$	$*10^{-4}$	$*10^{-4}$	$*10^{-5}$	$*10^{-5}$	$*10^{-5}$	$*10^{-5}$	$*10^{-6}$
0	5.90	3.78	1.31	1.48	9.44	3.27	2.36	1.51	5.23
0.2	5.95	3.81	1.32	1.49	9.52	3.29	2.38	1.52	5.27
0.4	6.04	3.87	1.34	1.51	9.66	3.34	2.42	1.55	5.35
0.6	6.15	3.94	1.36	1.54	9.84	3.41	2.46	1.57	5.45
0.8	6.27	4.02	1.39	1.57	1.00	3.47	2.51	1.61	5.56
1.0	6.40	4.10	1.42	1.60	1.02	3.55	2.56	1.64	5.67
1.2	6.54	4.18	1.45	1.63	1.05	3.62	2.61	1.67	5.79
1.4	6.67	4.27	1.48	1.67	1.07	3.69	2.67	1.71	5.91
1.6	6.80	4.35	1.51	1.70	1.09	3.76	2.72	1.74	6.02
1.8	6.93	4.44	1.53	1.73	1.11	3.84	2.77	1.77	6.14
2.0	7.06	4.52	1.56	1.76	1.13	3.91	2.82	1.81	6.25

TABLE. 7 SHEARSTRAIN (γ_{xy})

	h=0.01m			**h=0.02m**			**h=0.05m**		
P	**a/b =1**	**a/b =2**	**a/b =4**	**a/b =1**	**a/b =2**	**a/b =4**	**a/b =1**	**a/b =2**	**a/b =4**
	$*10^{-4}$	$*10^{-4}$	$*10^{-4}$	$*10^{-4}$	$*10^{-5}$	$*10^{-5}$	$*10^{-5}$	$*10^{-5}$	$*10^{-6}$
0	7.87	2.52	4.38	1.97	6.34	1.11	3.20	1.05	1.99
0.2	1.03	3.30	5.72	2.58	8.28	1.45	4.17	1.36	2.53
0.4	1.30	4.36	7.23	3.26	1.05	1.83	5.26	1.71	3.13
0.6	1.59	5.10	8.84	3.98	1.28	2.23	6.42	2.08	3.76
0.8	1.88	6.02	1.04	4.70	1.51	2.62	7.56	2.44	4.39
1.0	2.14	6.86	1.19	5.36	1.72	2.99	8.62	2.78	4.96
1.2	2.36	7.56	1.31	5.91	1.89	3.29	9.49	3.06	5.44
1.4	2.53	8.09	1.40	6.32	2.02	3.52	1.01	3.27	5.80
1.6	2.63	8.42	1.46	6.58	2.11	3.66	1.06	3.40	6.02
1.8	2.68	8.58	1.49	6.71	2.15	3.73	1.08	3.46	6.12
2.0	2.69	8.60	1.50	6.72	2.15	3.73	1.08	3.47	6.12

Increase in volume fraction leads to increase in normal strain, but it decreases when the aspect ratio and thickness of the plate increases. Since the stiffness of ceramic is high compared to that of metal, metal plate gives larger strain value. As the volume fraction increases, the difference in value for normal strain(ε_x) increases, but the normal strain(ε_y) decreases. The comparison of CPT, FSDT and ANSYS results showed that the deviation was within 2%.

The non-dimensional natural frequency values were tabulated in Table 8.

TABLE. 8 NON-DIMENSIONAL NATURAL FREQUENCY($\bar{\omega}$)

P	**a/b=1**	**a/b=2**	**a/b=4**
0	11.77	29.43	100.08

0.2	11.34	28.36	96.44
0.4	10.93	27.32	92.91
0.6	10.53	26.32	89.50
0.8	10.14	25.36	86.24
1	9.78	24.45	83.14
1.2	9.43	23.59	80.56
1.4	9.11	22.78	77.46
1.6	8.81	22.02	74.89
1.8	8.53	21.32	72.50
2	8.27	20.67	70.29

Increase in volume fraction leads to decrease in natural frequency, but it increases with increase in aspect ratio and thickness of the plates. Since the natural frequency is directly proportional to stiffness, increase in volume fraction decreases the stiffness value which also decreases the natural frequencies. The natural frequency increases with increase in the aspect ratio, because of the decrease in the width of the FGM plate.

In the same manner, the various parameters like non-dimensional mid-plane displacements, non-dimensional stresses, strains and non-dimensional natural frequencies were found for square and rectangular simply supported FGM plates subjected to mid-point load by CPT, FSDT as well as ANSYS. It was observed that the results of square and rectangular plates subjected to mid-point load was two times that of the same plate subjected to uniform pressure.

VI. CONCLUSIONS

In this paper, the investigations were made on the square and rectangular simply supported FGM plates by using CPT, FSDT and ANSYS. The effect of volume fractions, aspect ratios, various thickness, support conditions, loading conditions on static and dynamic parameters were analysed. The static parameters considered were non-dimensional mid-plane displacements, non-dimensional stresses and strains where the dynamic parameter considered was non-dimensional natural frequencies. It was found that the response of the simply supported square and rectangular FGM plates obtained from CPT, FSDT and ANSYS was intermediate to that of fully ceramic plate and fully metallic plate. It was also observed that as the volume fraction increases, the FGM plate stiffness decreases. Hence increase in the volume fraction, increases the static parameter values, but decreases the dynamic parameter values. From the investigations, it was observed that the deflections, stresses, strains and natural frequencies were highly dependent on the volume fraction. The numerical results obtained from the various plate theories such as CPT, FSDT and ANSYS simulation were compared with the results published in the literatures.

ACKNOWLEDGEMENT

The authors wish to thank the Principal and the Head of the Department of Civil Engineering, PSG College of Technology for providing the necessary facilities and guidance to complete the work successfully.

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