Nat. Volatiles & Essent. Oils, 2021; 8(5): 3377 - 3383

The Stress Intensity Factor Of An Exterior Crack In A Stress Free Strip With Forces At Crack Face

Rajendra Kumar Tripathi[1], C.K. Dixit[2]

[1]*Associate Professor, Department of Applied Sciences and Humanities (Mathematics)*
Khwaja Moinuddin Chishti Language University, Lucknow
[2]*Prof. Dr. Shakuntala Misra National Rehabilitation University Mohaan Road, Lucknow*

ABSTRACT

The determination of the stress and the displacement fields in the vicinity of a stress-free Griffith crack is reduced to the solution of Fredholm integral equation of second kind by using Fourier transform, finite and integral. The solution of Fredholm integral equation is obtained by the method of expanding the unknown function g() in decreasing powers of a-the half width of the strip. The expression of stress intensity factor, crack shape and the maximum shearing stress at (x,y) in the strip are presented analytically for two types of point forces.

Introduction

The problem of exterior crack in infinite isotropic homogeneous medium has been solved by Lowengrub [1] when crack was opened by pressures applied at crack faces only. He got the solution in closed form. However, there are only few problems of crack (s) in a finite strip. The problem of an interior crack opened by pressure at crack faces has been solved by Sneddon and Srivastava [2], who reduced the problem to dual integral equations whose solution. was given in the form of Fredholm integral equation by the method of [3].

The problem of exterior crack in a stress free strip has physical importance as one can perform the experiments to determine the stress field or crack shape. Therefore, the problem solved in the present paper is of physical importance.

of our concern here is the problem of crack occupying the space y=0, c≤AxA≤ in the xstrip [-a,a] x(-∞,∞), with edges stress free and the crack axis normal to edges. x-axis is

x axis and y-axis is normal to x-axis. The boundary condition of the problem are

$$\sigma_{xy} = \sigma_{xy}(\pm a, y) = 0, \quad 0\leq AyA<\infty, \tag{1.1}$$

$$\sigma_{xx}(\pm a, y) = 0, \quad 0\leq AyA<\infty, \tag{1.2}$$

$$\sigma_{xy}(\quad x, 0) = 0, \quad 0\leq AxA<a, \tag{1.3}$$

and the mixed boundary conditions

$$U_y(x,0) = 0, \qquad 0\leq AxA<c \tag{1.4}$$

$$\sigma_{yy}(x,0) = -Px, \qquad c\leq AxA<a \tag{1.5}$$

where (U_x, U_y) and $(\sigma_{xx}, \sigma_{xy,}\sigma_{yy})$ are the components of displacement vector and of stress tensor, respectively. Through out the analysis we checked that [4].

$$U_y\text{(x,0)>0,} \qquad \text{c<AxA} \leq a \tag{1.6}$$

Which means that the faces do not meet other than at crack tips. We followed the notation convention for transform as

$F_{cs}(a_{n,}\xi) = \int_0^a \int_0^\infty cos a_n x F(x_1 y) \sin \xi y \, dxdy,$

With a_n= nπ/a=nq. The plan of the paper is as follows : in next section we present the solution of the problem an reduce of Freehold integral equation of second kind. Section 3 Formulates the expressions for physical quantities in terms of the functions which is solution of Fredholm integral equation. Section 4 gives the solution of Fredholm integral equation. Section 5 presents some particular cases of loading. The physical quantities are calculated analytically for these different cases.

Formulation

The title problem is reduced to the displacement boundary value problem. The symmetry of geometry reduces the problem to first quadrant only. The solution of the problem follows from that of Sneddol and Srivastava [3] and written as

$$\text{Ux (x,y)} = \frac{2(1+\eta)}{aE} \sum_{n=1}^\infty \frac{\sin a_n x}{a_n}[(1-\eta)\frac{\partial^2 \emptyset_1}{\partial y^2} + \eta a n^2 \emptyset_1] + \frac{2(1+\eta)}{\pi E}\int_0^\infty \frac{cos \xi y}{\xi^2}$$

$$+[(1-\eta)\frac{\partial^2 \emptyset_2}{\partial x^3}(\eta - 2)\frac{\partial \emptyset_2}{\partial x}]\text{d}\,\xi$$

(2.1)

$$\text{Uy (x,y)} = 1/2 Uy_c(o,y) + \sum_{n=1}^\infty Uy_c\,(a_n, y) cos\;\; a_r x + \frac{2(1+\eta)}{\pi E}\int_0^\infty \frac{\sin\xi y}{\xi}$$

$$+[(1-\eta)\frac{\partial^2 \emptyset_2}{\partial x^2} + \eta\xi^2\emptyset_2]d\xi, \tag{2.2}$$

with

$$\text{U}y_c\,(a_n,\text{y}) = \frac{2(1+\eta)}{aEan^2}[(1-\eta)\frac{\partial^3\emptyset_1}{\partial y^3} + (\eta - 2)\frac{\partial \emptyset_1}{\partial y_1} an_2] \tag{2.3}$$

and

$$\emptyset_1 = An(1 + a_{ny})e^{-any}, \tag{2.4}$$

$$\emptyset_2 = \text{A}(\xi)[\cosh \xi x - \tanh \xi\, a \sinh \xi x]$$

(2.5)

Where {An} and A (ξ) are arbitrary constants to be determined through the boundary conditions, E and η are Young's modules and Poisson ratio of the medium of the strip respectively. We see from the assumptions of the solution (2.1)-(2.5) that the boundary conditions (1.1) and (1.3) are identically satisfied. The boundary (1.2) gives, after Fourier inversion as

$$\text{A}(\xi) = -\frac{4}{a}\cosh \xi a \sum_{n=1}^\infty (-1)^n \frac{{a_n}^3 An}{({a_n}^3 + \xi^2)^2},$$

(2.6)

and the mixed boundary conditions (1,4)-(1.5) give the following dual trigonometrically series relations

$$\frac{A_0}{2} + \sum_{n=1}^\infty B_n cos\, a_n x = 0, \qquad 0 \leq \text{AxA} < c \tag{2.7}$$

$$\sum_{n=1}^\infty a_n B_n cos\, a_n x = p(x), c < x \leq a, \tag{2.8}$$

With

B_n = a_n An and A_0 is a constant,

The solution of the above dual series relations is obtained through the method of Parihar [5] for triple series relations and is given by

$$g(t) + \frac{4}{\pi a^2}\int_c^a g(y)k(y,t)dy = P(t), c < t \le a, \quad (2.9)$$

$$P(t) = 2q^{-1}\frac{\cos(\frac{qt}{2})}{\sqrt{(G(c,t))}}\int_c^a \frac{\sin(qx/2)\sqrt{(G(c,x)p(x)}}{G(x,t)} \quad (2.10)$$

$$K(y,t) = \frac{\cos(\frac{qt}{2})}{\sqrt{(G(c,t))}}\int_c^a \frac{\sin(qx/2)\sqrt{(G(c,x)}}{G(x,t)} \quad \sum_{n=1}^{\infty}(-1)^n a_n sin a_n y \int_0^{\infty} \frac{cosh\, \xi(a-x)d\xi}{(a_n{}^3+\xi^2)^2 \sinh \xi a}, \quad (2.11)$$

and

$$G(x, t) = \cos(qx) - \cos(qt), \quad (2.12)$$

Where g(t) and An are related by

$$An = \frac{1}{a_n{}^3}\int_c^a g(t)\sin(a_n t)\; dt, \quad (2.13)$$

$$Ao = a_n{}^{-1}\int_0^a tg(t)\, dt, \quad (2.14)$$

and

$$\int_0^a g(t)\, dt = 0 \quad (2.15)$$

Physical Quantities

The important physical quantities are stress components σ_{xx}, σ_{xy}, σ_{yy} at general point (x,y) In the neighbourhood of crack tip, are components of strees, and of displacement written as [σ_{xx} (x,0), σ_{xy} (x,0), σ_{yy}(x,0)] and [Ux (x,o), Uy (x,0)] respectively. The component Uy (x,0) gives shapes of the crack while σ_{yy} (x,0) is developed because of opening out of this crack. Therefore of importance are the following quantities.

Crack Shape

We calculate the value Uy (x, o)from the equations (2.2), (2.6), (2.13)-(2.14) and given as

$$Uy(x, 0) = \frac{2(1-\eta^2)}{E}\int_x^a g(t)dt, c < x \le a, \quad (3.1)$$

The equation (1.6) along with (3.1) gives

$$\int_x^a g(t)dt > 0 \quad c < x \le a \quad (3.2)$$

When ensures the non overlapping of the faces. g(t) is given by the equation (2.9).

Normal Component of Stress

We calculate σ_{yy} (x, 0) through the value

$$\sigma_{yy}(x, y) = -\frac{2}{a}\int_c^a g(t)\sum_{n=1}^{n} cos a_n x \sin a_n t(1 + a_n y)^{-a} n^y \frac{4}{y}\int_0^{\infty} cos\xi y cosh\, \xi a$$

$$+ [\cosh\ \xi x\ -\ \sinh\ \xi x\ \tan\ \xi a]\ \sum_{n=1}^{\infty}(-1)^n \frac{a_n\xi^2}{(a_n{}^2+\xi^2)}\int_c^a g(y) sin a_n y\, dy\, d\xi, \quad (3.3)$$

By putting y= 0 and evaluating the value of series etc., and written as

$$\sigma_{yy}(x, 0) = -\frac{2}{a}\int_c^a \frac{g(t)\sin(qt)}{G(x,t)} + \frac{4}{\pi a^2}\int_0^{\infty}\int_c^a g(y)\{\cos\xi(a-x)\ \ cosech\, \xi a$$

$$[y \cosh\ \xi y \sinh \xi\, \pi - \pi \sinh \xi y \cosh \xi\pi]\ d\ \xi\}\ dy, \quad (3.4)$$

While the stress intensity factor is defined at

$$K_c = \lim_{x\to c}\sqrt{(c-x)\sigma_{yy}}(x, o) \quad (3.5)$$

We see form the expression in equation (3.4) that it is the only first summand which will contribute to singularity of σ_{yy}(x, 0). There fore the second is not important in the evaluation of stress intensity factor. We see from the boundary condition only that σ_{yy} (x,0) = 0 Hence the stress intensity factor corresponding to this streets component will be zero.

Crack Energy

The work done by p (x) in opening the crack Uy (x, 0) is given as

$$W = \frac{4(1-\eta^2)}{\pi a E}\int_c^a p(x)\int_x^a g(t)dt,$$

(3.6)

where g(t) is given by (2.9). The quantity which is important for yielding of the Medium is shearing stress and given by

Maximum shearing stress

where τ (z)= $\frac{1}{2}(\sigma_{yy}-\sigma_{xx})$ + i σ_{xy}

(3.7)

and z = x + iy,

(3.8)

σ_{yy} is given by (3.3) and the rest are defined as

$$\sigma_{xx} = +\frac{2}{a}\int_c^a g(t)\sum_{n=1}^{\infty}\cos a_n x\, sin a_n t\,(1+a_n y)e^{-}a_n y - \frac{4}{\pi}\int_0^{\infty} cos\xi y \cosh\xi a[\cosh\xi x$$

$$-\tan\ \xi a \sinh\xi x]\sum_{n=1}^{\infty}(-1)^n\frac{a_n\xi^2}{a_n{}^2+\xi^2}\ d\xi \int_c^a g(y) sin a_n y dy, \quad (3.9)$$

and

$$\sigma_{xy} = \frac{4}{\pi}\int_c^a g(y)dy\int_0^{\infty} sin\ \xi y \cosh\xi a\,[\sinh\xi x - \tanh a\ cos\ \xi x]\, d\xi \sum_{n=1}^{\infty}(-1)^n$$

$$\frac{a_n \sin a_n t\xi^2}{(an^2+\xi^2)^2}$$

(3.10)

Solution of Freehold Integral Equation

First we interchange the order of integration and of summation and use

$$\sum_{k=1}^{\infty}(-1)^k = \frac{\cos kx}{k^2+a^2} = \frac{\pi}{2a}.\frac{\cosh ax}{\sinh a\pi} - \frac{1}{2a^2} \quad (4.1)$$

and then exponentials incoming functions into series of descending orders exponentials and evaluating the integrals we get

$$x_{me}(x,y) = \sum_{m=0}^{\infty}\sum_{l=0}^{\infty}[(y-a)\sum_{n=1}^{6} d_r + (y+a)\sum_{r=7}^{12} d_r] \quad (4.2)$$

with

$d_1 = d_1{}^{-2}$ (a, 2m, 21, -y), $d_2 = d_2{}^{-2}$ (2a, 2m, 21, -y),

$d_3 = d_3{}^{-2}$ (2a, 2m, 21, -y, -x), $d_4 = d_4{}^{-2}$ (3a, 2m, 21, y),

$d_5 = d_5{}^{-2}$ (4a, 2m, 21, y), $d_6 = d_6{}^{-2}$ (4a, 2m, 21, y-x),

$d_7 = d_7{}^{-2}$ (a, 2m, 21, y), $d_8 = d_8{}^{-2}$ (2a, 2m, 21, y),

$d_9 = d_9{}^{-2}$ (2a, 2m, 21, y, x), $d_{10} = d_4$, $d_{11} = d_5$,

$d_{12} = d_{12}{}^2$(4a, 2m, 2L,yx)

$d_j{}^2 = (a_1, a_{2,} a_3, a_4, a_5)$, $(\sum_{r=1}^{5} a_r)^2$, j= 1, 2, -----12. (4.3)

Thus the evolution for kernel K (y, t) are given by

$$K(y,t) = \frac{4}{\pi a^2}\frac{\cos(\frac{qt}{2})}{\sqrt{G(c,t)}}[\sum_{m=0}^{\infty}\sum_{l=o}^{\infty} < \{(y-a)(d_1+d_2+d_4+d_5) + (y+a)(d_7+d_8-d_{10}-d_{11})\}$$

$$+ (y-a)\{K^1{}_{ml}(y,t) + K^2{}_{ml}(y,t) + K^3{}_{ml}(y,t) - K^4{}_{ml}(y,t)\}>], \quad (4.3)$$

Wher

$$K^i{}_{ml} = \int_c^a \frac{\sin(qx/2)\sqrt{(G(c,x)d_{3i}(x,y)dx}}{G(x,t)} \quad (4.4)$$

i = 1, 2, 3, 4

Now we assume the solution g(t) at

$$g(t) = \sum_{m=0}^{\infty} g_m(t) a^{-m-1} \quad (4.5)$$

We Substitute the value of g(t) from (4.5) and K(y,t) from (4.3) into (2.9) and comparing the coefficients of equal powers of $\{a^{-m-1}\}$ we get

$$g_0(t) = 2\frac{\cos\left(\frac{qt}{2}\right)}{\sqrt{(G(c,t))}}p_0(t), g_{-1}(t) = 0 \quad (4.6)$$

and the recurrence relation for m

$$g_m(t)=\frac{4}{\pi^2}\frac{\cos\left(\frac{qt}{2}\right)}{\sqrt{G(c,t)}}[\int_c^a y g_{m-2}(y)\{M_1(y) + M_1(y,t)dy + \int_c^a g_{m-2}(y)\{M_{11}(y)$$
$$+M_{22}(y,t)\}dy], \quad (4.7)$$

with

$$Po(t)= \int_c^a \frac{\sin\left(\frac{qx}{2}\right)\sqrt{(G(c,x)p(x)dx}}{G(x,t)}, \quad (4.8)$$

$$M_1(y) = \sum_{m=0}^{\infty}\sum_{l=0}^{\infty}[\frac{a}{\sqrt{2}}(d_1 + d_2 + d_7 + d_8) + \sum_{r=1}^{4}\int_c^a \frac{\sin\left(\frac{qx}{2}\right)}{\sqrt{G(c,x)}} d_{3r}(x,y)dx]$$

(4. 9)

$$M_2(y,t) = \sum_{i=1}^{4}\frac{\sin\left(\frac{qx}{2}\right)d_{3i}(x,y)dx}{\sqrt{(c,x)G(x,t)}}]G(t, c) \quad (4.10)$$

$$M_{11}(y) = \sum_{m=0}^{\infty}\sum_{l=0}^{\infty}[-\frac{a}{\sqrt{2}}(d_7 + d_8 - 2d_4 - 2d_5 - d_1 - d_2)$$
$$+ \int_c^a \frac{\sin\left(\frac{qx}{2}\right)}{\sqrt{G(c,x)}}(d_9 - d_3 - d_6 - d_{12})dx]$$

(4.11)

$$M_{22}(y,t) = G(t,c)\int_c^a \frac{\sin\left(\frac{qx}{2}\right)}{\sqrt{G(c,x)G(x,t)}}(d_9 - d_3 - d_6 - d_{12}\}dx]$$

(4.12)

Thus the solution of Fredholm integral equation (2.9) has got solution given through the equation (4.6)-(4.12). The only variable in this solution is $p_0(t)$ which changes according to loading. Therefore, in the next section we consider the different types of loading.

Special Cases of Loading

In the present section we are going to deal with there types of loading, namely

Case I- Polynomial (in cosine function)

Case II- Constant through at the crack faces

Case III- Point loads at cracks faces

we shall discuss one by one.

Case I- we take the loading p(x) as follows

$$p(x) = p_0 \sum_{r=0}^{\infty}\beta_r cos^r x, \quad (5.1)$$

M 10

Where P_0 is Polynomial of stress, β_r are arbitrary but known constants. Now we substitute this value of p(x) from equation (5.1) into equation (4.8) and evaluate the integrals we get

$$P_0(t) = p_0 \sum_{r=0}^{\infty} a_r I_r(t), I_r(t) = D_{r-1} + \cos(qt)\, I_{r-1}(t), (t) \quad (5.2)$$

with D_{r-1} as constant and defined as

$$D_{r-1} = \int_c^a \sin(qx/2)\sqrt{G(c,x)cos^{r-1}(qx)}\, dx \quad (5.3)$$

$D_0= \sqrt{(2a)}$

$$I_0 = -\frac{a}{\sqrt{2}} + \frac{a}{2}G(c,t)\{\frac{1}{G(t,c)}, 0\le t\le c$$

(5.4)

0, a≥t≥c.

Substituting this know value P_0(t) into equation (4.6) and get $g_0(t)$

$$g_1(t)=\frac{8\,p_0\cos(qt/2)}{\pi^2\sqrt{(G(c,t))}}\sum_{r=0}^{\infty}[\alpha_r T_1 + L_r{}^{(1)}(t)], \tag{5.5}$$

where T_1 is and constant and defined

$$T_1=\int_c^a \frac{\cos\left(\frac{qy}{2}\right)M_{11}(y)}{\sqrt{(G(c,t))}}I_r(y)dy, \tag{5.6}$$

$$L_r{}^1\,(\text{t})=\int_c^a \frac{\cos\left(\frac{qy}{2}\right)M_{11}(y)}{\sqrt{(G(c,t))}}I_r(y) \text{ x } M_{22}(y,t)dy \tag{5.7}$$

With I_r defined by second of equation (5.2)

Similarly

$$g_2(t)=\frac{64p_0}{\pi^4}\frac{\cos(qt/2)}{\sqrt{G(c,y))}}\sum_{r=0}^{\infty}[\,(a_r T_2 + T_3) + L_r{}^{(2)}(t)], \tag{5.8}$$

where T_2 and T_3 are constants and defined as

$$T_2=\int_c^a y\,\frac{\cos(qy/2)}{\sqrt{G(c,y))}}I_\alpha(y)M_1(y)dy,$$

(5.9)

$$T_3=\int_c^a \frac{\cos(qy/2)}{\sqrt{G(c,y))}}[(a_r T_1 + L_r{}^1\,(\text{y})]\text{ dy}$$

(5.10)

Hence we can easily calculate other value of {g_m(t)} and get total value substtituting these in the expressions of physical quantities we get the physical quantities to the required

Case II - we Consider the case

p(x) = P_0 (Constant)

Thus we get from equations (4.8) and (5.11)

$$P_0(t)=\frac{aP_0}{\sqrt{2}};\, g_0(t)=\frac{2aP_0}{\sqrt{2}}\frac{\cos(qt/2)}{\sqrt{G(c,y))}}$$

(5.12)

$$g_1(t)=\frac{8a}{\pi^2\sqrt{2}}P_0[T_{11}+T_{12}(t)]\,\frac{\cos(qt/2)}{\sqrt{G(c,y))}},$$

(5.13)

where

$$T_{11}=\int_c^a \frac{\cos\left(\frac{qy}{2}\right)}{\sqrt{G(c,y))}}M_{11}(t)dy,$$

(5.14)

$$T_{12}=\int_c^a \frac{\cos\left(\frac{qy}{2}\right)}{\sqrt{G(c,y))}}M_{22}(y,t)dy,$$

(5.15)

Similarly

$$g_2(t)=\frac{8a}{\pi^2\sqrt{2}}\frac{\cos\left(\frac{qy}{2}\right)}{\sqrt{G(c,y))}}[T_{21}+T_{22}(t)+\frac{4}{\pi^2}\{T_{23}+T_{24}(t)\}], \tag{5.16}$$

with

$$T_{21}=\int_c^a y\frac{\cos\left(\frac{qy}{2}\right)}{(G(c,y))}M_1(y)dy, \qquad T_{22}(t)=\int_c^a y\frac{\cos\left(\frac{qy}{2}\right)}{\sqrt{(G(c,y))}}M_2(y,t)dy,$$

(5.17)

$$T_{23}= \int_c^a \{T_{11} + T_{12}(y)\} \frac{\cos\left(\frac{qy}{2}\right)}{(G(c,y))} M_{11}(y)\, dy,$$

(5.18)

$$T_{24}(t)= \int_c^a \{T_{11} + T_{12}(y)\} \frac{\cos\left(\frac{qy}{2}\right)}{\sqrt{(G(c,y))}} M_{22}(y,t)\, dy,$$

(5.19)

Thus we can calculate other value of $\{g_m\}$'s. Hence substituting these value in expression of physical quantities we get the expression for physical quantities for this case of loading.

Case III- We Consider the loading defend as

$$\text{p(x)}\, \frac{1}{2} p_0 \delta(y)\{\delta(x-d) + \delta(x+d)\} \tag{5.20}$$

Substituting this value of p(x) in (4.8) we get

$$g_0(t)= \frac{2P_0 \cos\left(\frac{qt}{2}\right)}{\sqrt{(G(c,t))}} \frac{\sin\,(qd/2)\sqrt{(G(c,d))}}{G(d,t)},\ g_{-1}(t) = 0 \tag{5.21}$$

Calculating from equation (4.7)

$$g_1(t) = \frac{8}{\pi^2} \frac{p_0 \cos\left(\frac{qy}{2}\right)}{\sqrt{(G(c,y))}} [\sin \text{qd}/2 \sqrt{(G(c,d))}\ \{T_{31} + T_{32}(t)\}], \tag{5.22}$$

where

$$T_{31} = \int_c^a \frac{\cos\left(\frac{qy}{2}\right)}{\sqrt{(G(c,y))}} \frac{M_{11}(y)dy}{G(d,y)}, \tag{5.23}$$

$$T_{32} = \int_c^a \frac{\cos\left(\frac{qy}{2}\right)}{\sqrt{(G(c,y))}} \frac{M_{22}(y,t)dy}{G(d,y)}, \tag{5.25}$$

Thus we can calculate {gm},m≥2 from equation (4.7) and (5.21)- (5.24). Therefore, it is easy to evaluate physical quantities.

REFERENCES

1. M. Lowengrub, Some dual trigonometrically equations with an application to elasticity, Int. J. Engng Sci., 4 (1969)
2. I. N. Snedlon and R. P. Srivastava, The stress in the vicinity of an infinite row of Collinear cracks in an elastic body. *Proc. Roy. Soc.* Edin. A 67 (1965).
3. I. N. Sneddon, Dual integral equation with trigonometric kernels. *Proc. Glasgow Math. Assoc.,* 5 (1962).
4. E. E Burnistrn, An example of a partially closed Griffth crack, Int. *J. Fractive Mech.,* (1969).
5. K. S Parihar, Some triple trigonometrical series equation and their application, *Proc. Roy. Soc. Edinb.,* A, 69 (1971).