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H_∞ Analysis For Genetic Regulatory Networks

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Abstract

The H_∞ problem is explored in this study for stochastic genetic regulatory networks with Levy noise. A sufficient condition for this problem is obtained and described in terms of linear matrix inequalities (LMIs), which can be easily validated by Matlab LMI toolbox, using stability analysis and mathematical tools to examine the H_∞ performance. Finally, a numerical example is provided to demonstrate the effectiveness of the proposed methods.

Keywords: Stability analysis; Genetic regulatory networks; Levy noise; H_∞ analysis.

1 Introduction

Genetic regulatory networks (GRNs) are molecular networks that are formed by networks of regulatory interactions between DNA, RNA, and proteins that inhibit the expression of other genes. It is generally recognized that noise can disrupt GRNs at multiple stages, including transcription, translation, transport, chromatin remodelling, and pathway-specific control. When a gene is transcribed, the transcription is regulated by a collection of transcription factors in the gene promoter system. Other gene transcription mechanisms can produce transcription factors, which further form a complicated network system [1, 2]. Maintaining the network system's stability is necessary to the organism's long-term survival. As a result, one of the main challenges in biological study is the stability of GRNs.

To investigate a gene expression regulatory network, first establish a regulatory network model, and then examine the interactions between the genes in the model. The Boolean model, Petri net model and differential equation model are all well-known models of gene regulation. The differential equation model is one of the most extensively used which can effectively reflect the nonlinear dynamic behaviour of a biological system while also describing protein and mRNA concentration fluctuations.

Many researchers have been interested in studying the stability of GRNs with time delays. Chen et.al [3] established a GRNs with time delays and derived the model's necessary and sufficient conditions for stability. The uncertain GRNs for interval time delays addressed in [4]. The authors in [5, 6] discussed about the passivity performance of stochastic GRNs with time delays. Furthermore, numerous studies have contributed to the study of the Levy process see [7, 8, 9]. It is worth mentioning that there are various results on the stability analysis of stochastic differential equations with a Brownian motion in the existing literature. On the other hand, Brownian motion cannot be used to explain stochastic disturbances in many real systems.

For example, rapid environmental changes create a significant issue, and their paths may not be continuous. As stochastic systems containing Levy noise are well-suited for explaining discontinuous systems. Levy processes are stochastic processes with independent and stationary increments that describe the motion of a point whose subsequent motions are random and independent over a range of time intervals. The authors in [10] investigates asymptotic stability for stochastic differential equations with Levy noise. Zhu in [11] studied the stability of stochastic delay differential equations with Levy noise using a stability technique. Li and Xu in [12] addressed the Levy process's entire proof for exponential functions.

In light of the above discussion, the asymptotic stability of GRNs is investigated in this study. The noise term is specified as Levy type noise in the proposed model, which includes both Poisson random measurements and Brownian motion. A new set of sufficient LMI conditions is proposed to ensure the asymptotic stability of the considered system. Then, to deal with the system's disturbance, a H_∞ performance is introduced. Finally, a numerical example is provided to demonstrate the applicability of the proposed model.

Notations: $\mathbb{R}^n$ represents n dimensional Euclidean space, $\mathbb{R}^{n\times n}$ denotes set of all $n \times n$ matrix. $P > 0, (P < 0)$ means positive definite (negative definite). Sym(P) denoted as symmetry and the superscripts T denotes transpose and (-1) represents inverse of the matrix., E stands for expectation operator, $*$ shows that terms induced by symmetry.

2 Problem description

The following nonlinear genetic regulatory networks are considered in this work

$$\dot{x}(t) = -Ax(t) + Bf(y(t-\tau(t))) + \mathrm{L}, \tag{1}$$

$$\dot{y}(t) = -Cy(t) + Dx(t-\sigma(t)), \tag{2}$$

where $x(t) \in \mathbb{R}^n$ denotes the concentrations of mRNA and $y(t) \in \mathbb{R}^n$ denotes concentrations of proteins. Then $A = diag\,\{a_1, a_2, \dots, a_n\}$ and $C = diag\,\{c_1, c_2, \dots, c_n\}$ represents the dilution rates of mRNA and proteins. The coupling matrices of the considered networks are defined by $D = diag\,\{d_1, d_2, \dots, d_n\}$ and $B = (b_{ij}) \in \mathbb{R}^{n\times n}$. The nonlinear function f which is a monotonic function in Hill form, describes the protein's feedback regulation of transcription. Furthermore, L denotes the transcriptional degradation rates at their most basic level. We have dropped L by relocating the equilibrium point towards the origin of system (1).

We discuss stochastic differential equations in the form of Levy noise in this study in the following way:

$$\begin{aligned} dx(t) &= [-Ax(t) + Bf(y(t-\sigma(t))]dt + [H(t, x(t-\tau(t), y(t)]dL(t) \\ dy(t) &= [-Cy(t) + Dx(t-\tau(t))]dt, \end{aligned} \tag{3}$$

to discuss the H_∞ performance of the proposed model, the output of the system is defined as

$$\mathcal{Z}_x(t) = Mx(t), \qquad \mathcal{Z}_y(t) = Ny(t), \tag{4}$$

where the Levy process is represented by

$$dL(t) = \mathbb{B}(t) + \int_{|y|<d} \mathbf{H}(x(u^-), y(u^-), v)\overline{\mathbb{N}}(du, dv),$$

where $\mathbb{B}$ is the Brown motion of the independent m dimension, as specified on the entire probability space (Ω, F, P). The Poisson random measure $\mathbb{N}$ is defined in $\mathbb{R}_+ \times (\mathbb{R}^d - \{0\})$ with the compensator $\overline{\mathbb{N}}$. Assume that $\overline{\mathbb{N}}(du,dv) = N(du,dv) - \eta(dv)du$ and $\int_{\mathbb{R}^n - 0} (|v|^2 \wedge 1)\eta(dv) < \infty$ and the constant $d \in (0,\infty]$, also $\mathbb{N}$ is independent of $\mathbb{B}$.

The following assumptions are essential to attain our main results.

1. The time-varying delays $\tau(t)$ and $\sigma(t)$ satisfies $0 \leq \tau(t) \leq \tau$ and $0 \leq \sigma(t) \leq \sigma$ and $\dot{\tau}(t) \leq l_1,\ \dot{\sigma}(t) \leq l_2$, where τ, σ, l_1 and l_2 are positive constants.
2. The nonlinear intensity is assumed to satisfy the following condition
   $\text{trace}(H^T(t)H(t)) \leq (x^T(t-\tau(t))\mathcal{A}_1^T\mathcal{A}x(t-\tau(t)) + y^T(t)\mathcal{B}^T\mathcal{B}y(t))$.
3. The function $f(.)$ satisfy the condition $f(x)(f(x) - Vx) \leq 0$, where
   $V = diag\,\{V_1, V_2, \dots, V_n\} > 0$.
4. The there exist constant $\mathfrak{a} > 0$, for each $x \in \mathbb{R}^n$, $p > 0$, such that
   $\int_{|y|<d} |\mathbf{H}((x,y)|^p \eta(dv) \leq \mathfrak{a}|x|^p$.

Lemma 1 *[13] For symmetric matrix $\mathcal{P} \in \mathbb{R}^{n\times n}$ and a scalar $\tau > 0$ and a vector function $\Theta(s) \in \mathbb{R}^n$ such that the integrations concerned are well defined*

$$-\int_{t-\tau}^{t} \Theta^T(s)\mathcal{P}\Theta(s)ds \leq -\frac{1}{\tau}\left(\int_{t-\tau}^{t} \Theta^T(s)ds\right)P\left(\int_{t-\tau}^{t} \Theta(s)ds\right).$$

Lemma 2 *[14] For any matrices $X, Y \in \mathbb{R}^n$, matrix $Q > 0$, the following inequality is established*

$$2X^TY \leq X^TQX + Y^TQ^{-1}Y.$$

3 Main Results

Theorem 3.1 *For the given scalars τ, σ, l_1, l_2 $\mathfrak{a}$, k then system (3) is asymptotically stable if there exist a positive definite matrices $P_i > 0$, $S_i > 0$, $Q_j > 0$, $R_j > 0$, $G^T = [G_{11}^T\ G_{12}^T,\ G_{13}^T,$ $0\ 0\ 0\ 0\ 0\ 0\ 0\ ]$ and positive constants λ_1, λ_2, μ_1 , μ_2 which satisfies the following LMIs for all $i =$ 1,2, $j = 1,2,3$:*

$$P_1 \leq \lambda_1 I, \quad R_3 \leq \lambda_2 I, \tag{5}$$

$$\begin{bmatrix} [\Omega]_{10\times10} & \sqrt{\tau}\varphi^T & \sqrt{\tau}G & G \\ & -S_2 & 0 & 0 \\ & * & -S_2 & 0 \\ & * & * & -R_3 \end{bmatrix} < 0, \tag{6}$$

where

$$\Omega_{1,1} = -\,sym\,(P_1A + G_{11}) + Q_1 + Q_2 + \tau R_1,\quad \Omega_{1,2} = G_{11} - G_{12}^T,\quad \Omega_{1,3} = -G_{13}^T,\quad \Omega_{1,10} = P_1B$$
$$\Omega_{2,2} = sym\,(G_{12}) - Q_2,\quad \Omega_{2,3} = G_{13}^T,\quad \Omega_{3,3} = -Q_1(1-l_1) + (\lambda_1 + \tau\lambda_2)\mathcal{A}_1^T\mathcal{A},\quad \Omega_{3,5} = D^TP_2^T,$$
$$\Omega_{4,4} = -\frac{1}{\tau}R_1 + \mathfrak{a}kR_3,\quad \Omega_{5,5} = -\,sym\,(P_2C) + Q_3 + Q_4 + \sigma R_2 + (\lambda_1 + \tau\lambda_2)\mathcal{B}^T\mathcal{B} + \mu_1V^TV + \tau S_1,$$
$$\Omega_{6,6} = -Q_4,\quad \Omega_{7,7} = -Q_3(1-l_2) + \mu_2V^TV,\quad \Omega_{8,8} = -\frac{1}{\sigma}R_2 - \frac{1}{\tau}S_1,\quad \Omega_{9,9} = Q_5 - \mu_1,$$
$$\Omega_{10,10} = -Q_5(1-l_2) - \mu_2,\quad \varphi = [-A\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ B]$$

and the remaining elements are zero. Throughout the calculation $V(t,x(t),y(t))$ and $H(t,x(t-\tau(t),y(t))$ are abbreviated as $V(\cdot)$, $H(t)$ respectively.

Proof : *To analyze the stability conditions, the Lyapunov-Krasovskii functional is constructed* *as follows*

$$V_1(\cdot) = x^T(t)P_1x(t) + y^T(t)P_2y(t),$$
$$V_2(\cdot) = \int_{t-\tau(t)}^{t} x^T(s)Q_1x(s)ds + \int_{t-\tau}^{t} x^T(s)Q_2x(s)ds,$$
$$V_3(\cdot) = \int_{t-\sigma(t)}^{t} y^T(s)Q_3y(s)ds + \int_{t-\sigma}^{t} y^T(s)Q_4y(s)ds + \int_{t-\sigma(t)}^{t} f^Ty(s)Q_5fy(s)ds,$$
$$V_4(\cdot) = \int_{-\tau}^{0}\int_{t+\theta}^{t} x^T(s)R_1x(s)dsd\theta + \int_{-\sigma}^{0}\int_{t+\theta}^{t} y^T(s)R_2y(s)dsd\theta,$$
$$V_5(\cdot) = \int_{-\tau}^{0}\int_{t+\theta}^{t} y^T(s)S_1y(s)dsd\theta + \int_{-\tau}^{0}\int_{t+\theta}^{t} [-Ax(\theta) + Bf(y(\theta-\tau(\theta)))]^TS_2[-Ax(\theta) + Bf(y(\theta-\tau(\theta)))]dsd\theta,$$
$$V_6(\cdot) = \int_{-\tau}^{0}\int_{t+\theta}^{t} trace\ (H^T(s)R_3H(s))dsd\theta.$$

By using Ito stochastic formula, we get it as

$$\mathrm{L}V_1(\cdot) = 2x^T(t)P_1[-Ax(t) + Bf(y(t-\sigma(t))] + trace\ (H^T(t)P_1H(t)) +2y^T(t)P_2[-Cy(t) + Dx(t-\tau(t)], \quad (7)$$
$$\mathrm{L}V_2(\cdot) = x^T(t)(Q_1+Q_2)x(t) - x^T(t-\tau)Q_2x(t-\tau) - x^T(t-\tau(t)Q_1x(t-\tau(t))(1-l_1), \quad (8)$$
$$\mathrm{L}V_3(\cdot) = y^T(t)(Q_3+Q_4)y(t) - y^T(t-\sigma)Q_4y(t-\sigma) - y^T(t-\sigma(t))Q_3y(t-\sigma(t))(1-l_2) +f^Ty(t)Q_5fy(t) - f^Ty(t-\sigma(t))Q_5fy(t-\sigma(t))(1-l_2), \quad (9)$$
$$\mathrm{L}V_4(\cdot) = \tau x^T(t)R_1x(t) + \sigma y^T(t)R_2y(t) - \int_{t-\tau}^{t} x^T(s)R_1x(s)ds - \int_{t-\sigma}^{t} y^T(s)R_2y(s)ds, \quad (10)$$
$$\mathrm{L}V_5(\cdot) = \tau y^T(t)S_1y(t) - \int_{t-\tau}^{t} y^T(s)S_1y(s)ds + \tau[-Ax(t) + Bf(y(t-\tau(t))]^TS_2 \times [-Ax(t) + Bf(y(t-\tau(t))] - \int_{t-\sigma}^{t} [-Ax(s) + Bf(y(s-\tau(s))]^T \times S_2[-Ax(s) + Bf(y(s-\tau(s))]ds, \quad (11)$$
$$\mathrm{L}V_6(\cdot) = \tau\ trace\ H^T(t)R_3H(t) - \int_{t-\tau}^{t} trace\ (H^T(s)R_3H(s)ds. \quad (12)$$

By using Jensen's inequality for the integral terms appeared in above inequality

$$-\int_{t-\tau}^{t} x^T(s)R_1x(s)ds \le -\frac{1}{\tau}(\int_{t-\tau}^{t} x(s)ds)^TR_1(\int_{t-\tau}^{t} x(s)ds),$$
$$-\int_{t-\sigma}^{t} y^T(s)R_2y(s)ds \le -\frac{1}{\sigma}(\int_{t-\sigma}^{t} y(s)ds)^TR_2(\int_{t-\sigma}^{t} y(s)ds),$$
$$-\int_{t-\tau}^{t} y^T(s)S_1y(s)ds \le -\frac{1}{\tau}(\int_{t-\tau}^{t} y(s)ds)^TS_1(\int_{t-\tau}^{t} y(s)ds).$$

We assume the following conditions, that we do in many stochastic systems studies

$$trace\ (H^T(t)P_1H(t)) \le \lambda_1(x^T(t-\tau(t))\mathcal{A}_1^T\mathcal{A}_1x(t-\tau(t)) + y^T(t)\mathcal{B}_1^T\mathcal{B}_1y(t)),$$
$$trace\ (H^T(t)R_3H(t)) \le \lambda_2(x^T(t-\tau(t))\mathcal{A}_1^T\mathcal{A}_1x(t-\tau(t)) + y^T(t)\mathcal{B}_1^T\mathcal{B}_1y(t)).$$

For any constants $\mu_1 > 0, \mu_2 > 0$ the function $f(\cdot)$ satisfies the following condition

$$-\mu_1[f^Ty(t)fy(t) - y^T(t)V^TVy(t)] \ge 0, \quad (13)$$
$$-\mu_2[f^Ty(t-\sigma(t))f^Ty(t-\sigma(t)) - y^T(t-\sigma(t))V^TVy(t-\sigma(t))] \ge 0. \quad (14)$$

The following Newton-Leibnitz formula is adopted from [15]

$$-2\alpha^T(t)G[x(t) - x(t-\tau) - \int_{t-\tau}^{t} [-Ax(s) + Bf(y(s-\tau(s))]ds - \int_{t-\tau}^{t} H(s,x(s-\tau(s),y(s))dL(s)] = 0. \quad (15)$$

By using Lemma 2, we obtained as

$$2\alpha^T(t)G\int_{t-\tau}^{t} [-Ax(s) + Bf(y(s-\tau(s))]ds \le \tau\alpha^T(t)GS_2^{-1}G^T\alpha(t) + \int_{t-\tau}^{t} [-Ax(s) + Bf(y(s-\tau(s)))]^TS_2[-Ax(s) + Bf(y(s-\tau(s)))]ds, \quad (16)$$
$$2\alpha^T(t)G\int_{t-\tau}^{t} H^T(s)dL(s) \le \alpha^T(t)GR_3^{-1}G^T\alpha(t) + (\int_{t-\tau}^{t} H(s)dL(s))^TR_3(\int_{t-\tau}^{t} H(s)dL(s)). \quad (17)$$

where $\alpha(t) = [x^T(t)\, x^T(t-\tau)\, x^T(t-\tau(t)) \int_{t-\tau}^{t} x^T(s)ds\, y^T(t) y^T(t-\sigma) y^T(t-\sigma(t)) \int_{t-\sigma}^{t} y(s)ds f^T y(t) f^T y(t-\sigma(t))]$.

Taking mathematical expectations for the stochastic terms appeared in (17)

$$E\left[\int_{t-\tau}^{t} H^T(s)d\mathrm{L}(s)\, R_3 \int_{t-\tau}^{t} H(s)d\mathrm{L}(s)\right] =$$
$$E[\int_{t-\tau}^{t} H^T(s)dw(s)R_3 \int_{t-\tau}^{t} H(s)dw(s)$$
$$+2\int_{t-\tau}^{t} H(s)dw(s)R_3 \int_{t-\tau}^{t} \int_{|y|<d} \mathbf{H}(x(u^-), y(u^-), v)\bar{N}(du,\ dv)$$
$$+\int_{t-\tau}^{t} \int_{|y|<d} \mathbf{H}(x(u^-), y(u^-), v)\bar{N}(du,\ dv)$$
$$\times R_3 \int_{t-\tau}^{t} \int_{|y|<d} \mathbf{H}(x(u^-), y(u^-), v)\bar{N}(du,\ dv)]. \quad (18)$$

From the assumption (A4), we can obtained as

$$E[\int_{t-\tau}^{t} \int_{|y|<d} \mathbf{H}(x(u^-), y(u^-), v)\bar{N}(du,\ dv)]^2 R_3 \le$$
$$\mathfrak{a}E[(\int_{t-\tau}^{t} \int_{|y|<d} (\mathbf{H}(x(u^-), y(u^-), v))^2 \eta(dv)\ du)R_3]$$
$$\le \mathfrak{a}\mathbb{E}[(\int_{t-\tau}^{t} (\int_{|y|<d} (\mathbf{H}(x(u^-), y(u^-), v))^2 \eta(dv))\ ds)R_3]$$
$$\le \mathfrak{a}kE[(\int_{t-\tau}^{t} x^2(s)ds)R_3].$$

By using the Lemma 1

$$E[\int_{t-\tau}^{t} H^T(s)dw(s)\ R_3 \int_{t-\tau}^{t} H(s)dw(s)] \le E[\int_{t-\tau}^{t}\ trace\ (H^T(s)R_3H(s))ds].$$

Finally, obtained as

$$E[\int_{t-\tau}^{t} H^T(s)dL(s)\ R_3 \int_{t-\tau}^{t} H(s)dL(s)] \le E[\int_{t-\tau}^{t}\ trace\ (H^T(s)R_3H(s))ds] +$$
$$\mathfrak{a}kE[(\int_{t-\tau}^{t} x(s)ds)^T R_3 (\int_{t-\tau}^{t} x(s)ds)]. \quad (19)$$

Now, combining the equation from (7) to (19), we have obtained as

$$E\mathrm{L}V(\cdot) \le E\alpha^T(t)\Psi\alpha(t)$$

where $\Psi = \Omega + \tau\phi^T S_2 \phi + \tau G S_2^{-1} G^T + G R_3^{-1} G^T$ and using Schur complement it is easy to attain LMIs (5) and (6). Then $\Psi < 0$ holds, it follows that $EV(\cdot) \le 0$. Therefore, the considered system (3) is asymptotically stable.

4 H_∞ Analysis

This section, we have focused on H_∞ performance for perturbed version of the system (3).

$$\begin{aligned} dx(t) &= [-Ax(t) + Bf(y(t-\sigma(t)) + M\mathcal{W}_1(t)]dt + [H(t, x(t-\tau(t), y(t)]dL(t) \\ dy(t) &= [-Cy(t) + Dx(t-\tau(t)) + N\mathcal{W}_2(t)]dt, \end{aligned} \quad (20)$$

where $\mathcal{W}_1(t)$, $\mathcal{W}_2(t)$ are the disturbance inputs in $L_2([0,\infty), \mathbb{R})$ and M, N are known constant matrices. Further, the H_∞ performance is introduced to analyze the disturbance attention level $\gamma > 0$

$$J = [\int_0^\infty \mathfrak{z}^T(t)\mathfrak{z}(t) - \gamma^2 \mathcal{W}^T(t)\mathcal{W}(t)]dt], \quad (21)$$

where $\mathfrak{z}(t) = [x^T(t) \quad y^T(t)]^T, \mathcal{W}(t) = [\mathcal{W}_1^T(t) \quad \mathcal{W}_2^T(t)]^T$.

Definition 1 *The system (3) is said to be asymptotically stable with given disturbance attenuation level $\gamma > 0$, if it is stochastically asymptotically stable under zero initial condition and satisfies* $\| \mathfrak{z}(t) \|_2 \le \| \mathcal{W}(t) \|_2$ for every non-zero $\mathfrak{z}(t) \in L_2[0,\infty)$.

Theorem 3.2 *For the given scalars τ, σ, l_1, l_2 $\mathfrak{a}$, k then system (3) is asymptotically stable* *also satisfies the performance index (21) if there exist a matrices $P_i > 0$, $S_i > 0$, $Q_j > 0$, $R_j > 0$, $G^T = [G_{11}^T\ G_{12}^T,\ G_{13}^T,\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ ]$ and positive constants λ_1, λ_2, μ_1 , μ_2 which satisfies the following LMIs for all $i = 1,2$, $j = 1,2,3$:*

$$P_1 \leq \lambda_1 I, \quad R_3 \leq \lambda_2 I, \tag{22}$$

$$\begin{bmatrix} [\widetilde{\Omega}]_{10\times10} & \mathcal{R}_1 & \mathcal{R}_2 & \sqrt{\tau}\varphi^T & \sqrt{\tau}G & G \\ & -\gamma^2 I & 0 & 0 & 0 & 0 \\ & * & -\gamma^2 I & 0 & 0 & 0 \\ & * & * & -S_2 & 0 & 0 \\ & * & * & * & -S_2 & 0 \\ & * & * & * & * & -R_3 \end{bmatrix} < 0, \tag{23}$$

where $\widetilde{\Omega}_{1,1} = \Omega_{11} + M^T M$, $\widehat{\Omega}_{5,5} = \Omega_{5,5} + N^T N$, $\mathcal{R}_1 = [P_1 \quad 0_{n,9n}]^T$, $\mathcal{R}_2 = [0_{n,4n},\ P_2,\ 0_{n,5n}]^T$ and the remaining elements are same as in $[\Omega]_{10\times10}$ of Theorem 3.1.

Proof : *The proof is followed as in Theorem (3.1), from zero initial conditions, $V(0) = 0$ and $V(\infty) \geq 0$, so*

$$J = E[\int_0^\infty \mathfrak{z}^T(t)\mathfrak{z}(t) - \gamma^2 \mathcal{W}^T(t)\mathcal{W}(t) + \mathrm{L}V(t, x(t), y(t))]dt - E[V(t, x(t), y(t))],$$
$$= E[\int_0^\infty [\mathfrak{z}^T(t)\mathfrak{z}(t) - \gamma^2 \mathcal{W}^T(t)\mathcal{W}(t) + \mathrm{L}V(t, x(t), y(t))]dt],$$
$$\leq E[\int_0^\infty [\alpha_1^T(t)\Theta\alpha_1(t)]dt],$$

where Θ is defined in (23) and $\alpha_1(t) = [\alpha^T(t) \quad \mathcal{W}^T(t)]^T$. Therefore, we can conclude that the performance index $J \leq 0$ whenever $\Theta < 0$, it can be verified with $E[\| \mathfrak{z} \|_2] \leq \| \mathcal{W} \|_2$. Hence the system (20) is stochastically stable with a disturbance attenuation level $\gamma > 0$.

5 Numerical Example

A numerical example is provided in this section to demonstrate the efficiency of the derived results. The parameters of the considered GRNs and their outputs are as follows:

$$B = \begin{bmatrix} 0 & 0 & 0.5 \\ 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \end{bmatrix}, \ M = \begin{bmatrix} 0.16 & 0.8 & 0 \\ 0.2 & 0 & 0.45 \\ 0 & 0.05 & 0.9 \end{bmatrix}, \ N = \begin{bmatrix} 0.25 & 0.1 & -0.28 \\ 0.2 & 0.1 & -0.19 \\ 0.15 & 0 & 0.23 \end{bmatrix},$$

$A = diag\,\{1,1,1\}$, $C = diag\,\{3,3,3\}$, $D = diag\,\{0.1,0.1,0.1\}$, $\mathcal{A}_1 = diag\,\{0.4,0.4,0.4\}$, and $\mathcal{B}_1 = diag\,\{0.2,0.2,0.2\}$, $V = diag\,\{0.65,0.65,0.65\}$ and $f(x) = x^2/(1 + x^2)$. The constant values are taken as $\mathfrak{a} = 0.15$, $k = 0.5$, $\gamma = 0.1$, $\tau = 1.2$, $\sigma = 1.1$, $l_1 = 0.2$, $l_2 = 0.1$.

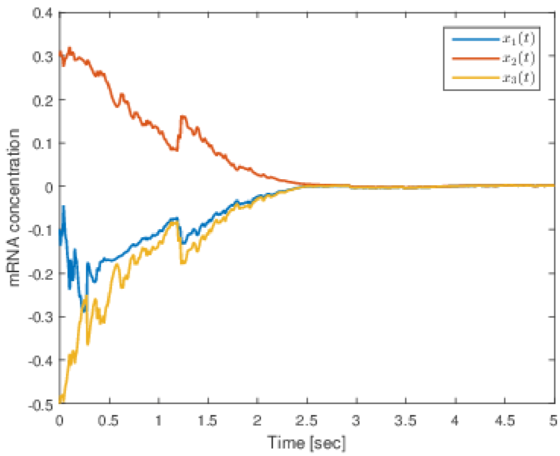

Figure 1: State trajectories of mRNA

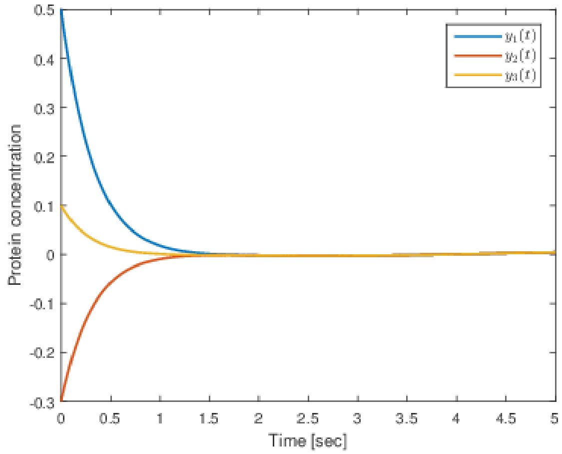

Figure 2: State trajectories of protein

Simulations are plotted in the following way based on the above parameters. Fig.1 shows the concentrations of mRNAs and protein concentrations are presented in Fig.2. We can deduce from these figures that the state mRNA contains Levy noise, implying that there will be some fluctuation before it converges to an equilibrium point. Since the protein state is free of noise, it achieves fast convergence.

6 Conclusion

The stability of GRNs with time delays and Levy noise is examined in this paper. Then the results are extended to analyze the H_∞ performance. Furthermore, the necessary conditions for obtaining stochastic stabilization are discussed, as well as a simulation example demonstrating that system (3) and (4) is globally asymptotically stable. In future investigations, the stability of GRNs with Levy noise needs to be studied further.

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